The Effect of the Interbank Network Structure on Contagion and Common Shocks

Abstract

This paper proposes a dynamic multi-agent model of a banking system with central bank. Banks optimize a portfolio of risky investments and riskless excess reserves according to their risk, return, and liquidity preferences. They are linked via interbank loans and face stochastic deposit supply. Evidence is provided that the central bank stabilizes interbank markets in the short-run only. Comparing different interbank network structures, it is shown that money-center networks are more stable than random networks. Systemic risk via contagion is compared to common shocks and it is shown that both forms of systemic risk require different optimal policy responses.

Keywords: systemic risk, contagion, common shocks, multi-agent simulation

JEL classification: C63, E52, E58, G01, G21

1. Introduction

The recent financial crisis has highlighted the necessity to understand systemic risk both qualitatively and quantitatively in order to safeguard financial stability. Bandt et al. (2009) provide a categorization of systemic risks, distinguishing between a broad and a narrow sense. In their nomenclature, contagion effects on interbank markets pose a systemic risk in the narrow

sense, whereas the broad sense of systemic risk is characterized as a common shock that affects many institutions at once. The crisis has shown that systemic risk not only can take many forms, but is also highly dynamic: slowly building up in normal times, but rapidly emerging during times of distress. The insolvency of the US investment bank Lehman Brothers in September 2008 marked the tipping point between the build up and rapid manifestation of systemic risks and lead to a freeze in interbank markets. As a consequence, the risk premia for unsecured interbank loans increased drastically, which resulted in a massive impairment of banks' liquidity provision. Governments and central banks were forced to undertake unprecedented non-standard measures to reduce money market spreads and ensure liquidity provision to the banking system. This shows that central banks are key actors for the functioning of interbank markets, even though they do not directly participate in them. To motivate central bank interventions, Allen et al. (2009) and Freixas et al. (2010) show that central bank intervention can increase the efficiency of interbank markets. It is thus clear, that every realistic model of interbank markets has to feature the central bank as one key actor.

Interbank markets exhibit what Haldane (2009) denotes as a knife-edge, or robust-yet-fragile property. In normal times, the connections between banks lead to an enhanced liquidity allocation and increased risk sharing amongst financial institutions. This was shown by Allen and Gale (2000) who extend the classical bank-run model by Diamond and Dybvig (1983) and show that highly interconnected banking systems are less prone to bank-runs. In times of crisis, however, the same interconnections can amplify shocks that

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spread through the system. This was shown i.e. by Gai and Kapadia (2008), who investigate systemic crises with a network model and show that on the one hand, the risk of systemic crises is reduced with increasing connectivity on the interbank market. On the other hand, however, the magnitude of systemic crises increases at the same time. This knife-edge property of interbank markets can be attributed to a counterparty risk externality. Acharya and Bisin (2010) compare over-the-counter (OTC) and centralized clearing markets in a general equilibrium model. They show that the intransparency of OTC markets is ex-ante inefficient and attribute this to a counterparty risk externality. This externality can best be illustrated in a small example. Assume a simple banking network that consists of three banks (A,B, and C) where bank A has issued uncollateralized interbank loans to banks B and C. The interest rate on the interbank loans will include a risk premium to capture counterparty risk. Now assume that B has issued another interbank loan to C. This will increase the counterparty risk of bank B, as B is now vulnerable to a default of bank C. However, bank A is not aware of this increase and will thus underprice the counterparty risk. Thus, the structure of financial networks and especially interbank networks is relevant for the analysis of systemic risk. Taking this into account, the question arises, if there exist network structures that are less prone to the counterparty externality and hence more resilient to financial distress.

The counterparty risk externality makes it clear that the network structure of financial system plays an important role when assessing systemic risk. An overview of the existing literature on financial networks can be found i.e. in

Allen et al. (2010). The network structure of interbank markets can be best captured in an exposure matrix where the issuance of a loan from bank i to bank j is denoted as the loan size in row i and column j. Using such a matrix, Eisenberg and Noe (2001) show that a unique clearing payment vector exists and analyze the spreading of contagious defaults in general network topologies. The difference to this paper is that we develop a dynamic model of cascading bank defaults, while Eisenberg and Noe (2001) calculate the impact of a default in a static network structure. Empirical analyses of the interbank network structure exist for for a number of countries (see i.e. European Central Bank (2010) for a recent overview). It is shown that interbank networks often exhibit a scale-free topology, i.e. they are characterized by few money center banks with many interconnections and many small banks with few connections. Sachs (2010) follows the static approach of Eisenberg and Noe, but also compares contagion effects in scale-free networks and random networks and finds that contagion is more pressing in scale-free networks. What is missing in the literature, however, is a dynamic analysis of the financial stability properties of different network topologies.

The crisis revealed that there also exist other externalities besides the counterparty risk externality. One of them being a correlation externality between banks' portfolios. Securitization was designed to distribute risks from within the banking system to investors outside the banking system. A thorough analysis, however, shows that a significant part of the securitized risk was still residing within the banking system at the peak of the crisis (see i.e. Krishnamurthy (2008)). As a consequence, a strong correlation between banks'

assets arised. As banks are unaware of the portfolio of competing banks, they cannot assess this correlation and thus choose non-optimal levels of correlation for their portfolios. This externality could thus be best described as a correlation externality. A large extend of the literature on systemic risk in interbank markets has focused on the analysis of contagion effects (i.e. studying the counterparty risk externality). Recently, more attention has been given to the correlation externality and the analysis of common shocks as sources of systemic risk. Acharya and Yorulmazer (2008) point out how banks are incentivized to increase the correlation between their investments and thus the risk of an endogenous common shock in order to prevent costs arising from potential information spillovers. The increasing correlation in the financial sector is also verified empirically. De Nicolo and Kwast (2002) analyze the increase in the correlation between large and complex financial organizations during the 1990s, a development that was further fueled by securitization. The new insights on common shocks give rise to the question which form of systemic risk poses the greater threat to financial stability: interbank contagion caused by the counterparty externality, or common shocks caused by the correlation externality. Thus far, no comparison of the different systemic risk manifestations in a single model has been conducted in the literature. This paper aims to close this gap by explicitly comparing the impact of different shocks resulting from the two externalities.

One particularly useful class of models to analyze the above mentioned questions are multi-agent simulations. Iori et al. (2006) develop a network model of a banking system, where agents (banks) can interact with each other via

interbank loans. The balance sheet of banks consists of risk-free investments and interbank loans as assets, and deposits, equity and interbank borrowings as liabilities. Banks channel funds from depositors towards productive investment. They receive liquidity shocks via deposit fluctuations and pay dividends if possible. Nier et al. (2008) describe the banking system as a random graph where the network structure is determined by the number of nodes (banks) and the probability that two nodes are connected. The banks' balance sheet consists of external assets (investments) and interbank assets on the asset side and net worth, deposits, and interbank loans as liabilities. Net worth is assumed to be a fixed fraction of a bank's total assets and deposits are a residual, designed to complete the bank's liabilities side. Shocks that hit a bank and lead to its default are distributed equally amongst the interbank market. The authors find, that (i) the banking system is more resilient to contagious defaults if its banks are better capitalized and this effect is non-linear; (ii) the effect of the degree of connectivity is non-monotonic; (iii) the size of interbank liabilities tend to increase the risk of a knock-on default; and (iv) more concentrated banking systems are shown to be prone to larger systemic risk. More recently, Ladley (2011) analyzes the impact of the interbank network heterogeneity on systemic risk in a multi-agent setting. The balance sheet of banks consists of equity, deposits, cash reserves, loans to the non-bank sector and interbank loans. Ladley considers risky investment opportunities and explicitly models how banks attract deposits by choosing their offered deposit interest rates. Banks determine the optimal structure of their portfolio via a genetic algorithm. He finds that that for small shocks, high interconnectivity helps stabilizing the system, while for

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large shocks high interconnectivity amplifies the initial impact.

This paper wants to answer the aforementioned questions about the impact of the network structure on financial stability by developing a dynamic model of a banking system. Banks optimize a portfolio of risky investments and riskless excess reserves. Risky investments are long-term investment projects that fund an unmodelled firm sector while riskless excess reserves are short-term and held at the deposit facility of the central bank. Banks face a stochastic supply of household deposits and stochastic returns from risky investments. This gives rise to liquidity fluctuations and initiates the dynamic formation of an interbank loan network. Banks have furthermore access to central bank liquidity if they can provide sufficient collateral. This model is used to first analyze the impact that the provision of central bank liquidity has on financial stability. It is shown that the central bank can stabilize the financial system in the short-run. In the long-run, however, the system always converges to the equilibrium state. Possible network structures will be given at the beginning of each simulation. They reflect contractual agreements amongst banks and determine the set of possible interbank loans. The realized network structure at each point in time is a subset of the possible network structure (i.e. the set of existing edges at any point in time is a subset of the set of possible edges). This closely resembles the situation in reality, where the day-to-day topology of interbank networks also varies from the monthly or quaterly aggregated network structures that are analyzed in the literature. Different possible network structures are compared, and it is shown that in random graphs, the relationship between the degree of inter-

connectivity and financial instability is non-monotonic. Scale-free networks are seen to be more stable than small-world networks, which in turn tend to be more stable than random networks. Thus, the effect of contagion is exagerrated in the literature, as most papers assume random networks and most real-world interbank networks are scale-free. The model captures key effects of the dynamics of interbank networks and can thus be used to analyze the impact of different externalities on financial stability. The counterparty risk externality is compared to the correlation externality and it is shown that, contrary to their importance in the literature, common shocks are not subordinate to interbank contagion. Finally, a number of policy conclusions for the optimal reaction to financial crises, as well as for the monitoring and regulation of systemic risk are drawn from the model.

The remainder of this paper is organized as follows. After this introduction, section two describes the dynamic model that has been used to analyze the aforementioned questions. Section three will present the main results, while section four derives some policy implications and concludes.

2. The Model

This section wants to outline some key features that all models of systemic risk should incorporate and develop a dynamic model of a banking system that can be used to analyze the impact of the interbank network structure on financial stability. Firstly, deposit fluctuations have to be included for two reasons: (i) Because of the maturity transformation that banks perform and since deposits usually have a short maturity, deposit fluctuations can lead to

illiquidity. Banks that become illiquid will have to liquidate their long-term investments at steep discounts (for a model that describes this mechanism, see i.e. Uhlig (2010)). Due to marked-to-market accounting, these steep discounts will lead to losses in banks' trading books and have to be compensated by banking capital. Thus, illiquidity can lead to insolvency. (ii) As deposit fluctuations are generally considered to be one of the reasons why banks engage in interbank lending (see i.e. Allen and Gale (2000)), they have to be included into all models of systemic risk. Without deposit flucutations as a driving force for the formation of interbank networks, it is impossible to describe the counterparty risk externality in a dynamic setting. Secondly, as fluctuations in investment returns have to be compensated by banking capital, risky investments are a major cause of bank insolvencies. Without risky investments, it is impossible to model the correlation externality as it arises precisely in a situation when the returns of risky assets of a number of banks have negative realizations at the same time. In order to model common shocks, risky investments have thus to be taken into account.

Iori et al. (2006) and Nier et al. (2008) develop multi-agent models of a banking system, but assume a risk-free investment opportunity. Nier et al. (2008) further assume deposits to be residual. I follow both papers in some aspects and develop a network model of interbank markets. However, I explicitly allow the possibility of risky investments and deposit fluctuations. I furthermore include a central bank in the model, since it is evident from the literature that monetary policy has a large influence on the stability of interbank markets. This model allows the investigation of direct contagion

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effects as well as common shocks. This is another difference to the existing literature, which exclusively focuses on individual forms of systemic risk.

2.1. Balance Sheets

The balance sheet of a bank k holds risky investments I^k and riskless excess reserves E^k as assets at every point in (simulation-) time $t = 1 \ldots \tau$. The investments of bank k have a random maturity[1] $\tau_I^k > 0$ and I assume that each bank finds enough investment opportunities according to its preferences. The bank refinances this portfolio by deposits D^k (which are stochastic and have a maturity of zero), from which it has to hold a certain fraction rD^k of required reserves at the central bank, fixed banking capital BC^k (which is assumed to be held in a highly liquid form), interbank loans L^k and central bank loans LC^k. Interbank loans and central bank loans are assumed to have a maturity of $\tau_L^k = \tau_{LC}^k = 0$. The maturity mismatch between investments and deposits is the standard maturity transformation of commercial banks. Interbank loans can be positive (bank has excess liquidity) or negative (bank has demand for liquidity), depending on the liquidity situation of the bank at time t. The same holds for central bank loans, where the bank can use either the main refinancing operations to obtain loans, or the deposit facility to loan liquidity to the central bank. The balance sheet of the commercial bank therefore reads as:

$$I_t^k + E_t^k = (1 - r)D_t^k + BC_t^k + L_t^k + LC_t^k \tag{1}$$

The interest rate for deposits at a bank is r^d and the interest rate for central bank loans is r^b. Note that there is no distinction between an interest rate

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[1] Maturity τ implies that the asset matures in $\tau + 1$ update steps.

for the lending and deposit facility and therefore the interest rate on the interbank market will be equal to the interest rate for central bank loans.

The banks decide about their portfolio structure and portfolio volume. A constant relative risk aversion (CRRA) utility function is assumed to model the bank's preferences:

$$u^k = \frac{1}{1-\theta^k} \left(V^k (1 + \lambda^k \mu^k - \frac{1}{2}\theta^k(\lambda^k)^2(\sigma^k)^2) \right)^{(1-\theta^k)} \tag{2}$$

where λ^k is the fraction of the risky part of the portfolio, μ^k is the expected return of the portfolio and θ^k is the banks risk aversion parameter. $V_t^k = I_t^k + E_t^k$ denotes the bank's portfolio volume. The risky part of the portfolio follows from utility maximisation and reads as:

$$(\lambda^k)^* = \min\left\{ \frac{\mu^k}{\theta^k(\sigma^2)^k}, 1 \right\} \quad \in [0,1] \tag{3}$$

The portfolio volume can be obtained by similar measures as:

$$(V^k)^* = \left[\frac{1}{r^b} \left(\left(1 + \lambda^k\mu^k - \frac{1}{2}\theta^k(\lambda^k)^2(\sigma^2)^k\right)^{(1-\theta^k)} \right) \right]^{1/\theta^k} \tag{4}$$

where r^b denotes the refinancing cost of the portfolio. Since banks obtain financing on the interbank market and at the central bank at the same interest rate, this refinancing cost is equal to the main refinancing rate. It is possible to introduce a spread between the lending and deposit facility and therefore allowing the interest rate on the interbank market to stochastically vary around the main refinancing rate. If a bank now plans its optimal portfolio volume, it calculates with a planned refinancing rate. This refinancing rate follows from the banks plan about how much interbank loans it wants
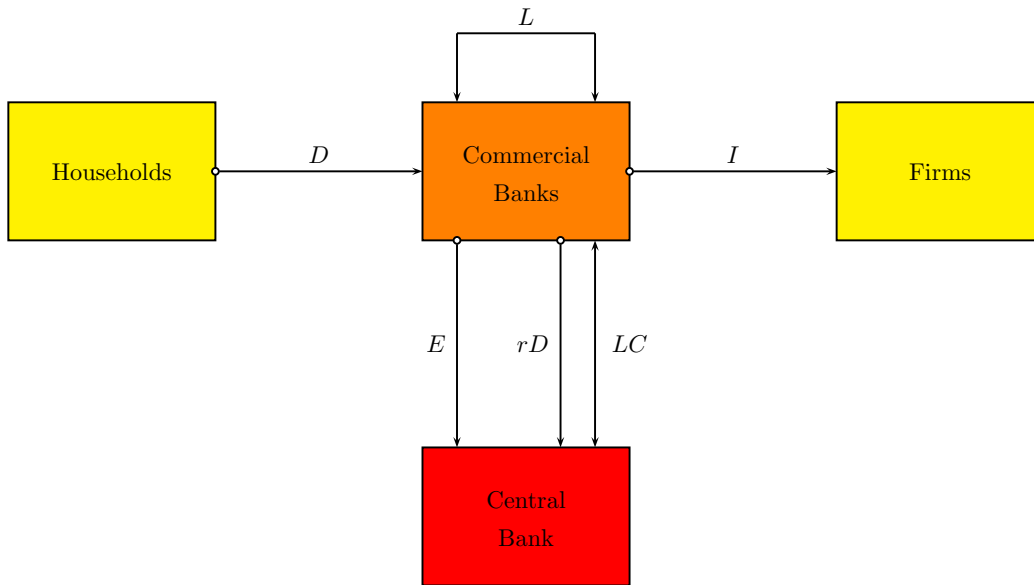
Figure 1: Interaction dynamics of the model. The private sector (household/firms), the banking sector (commercial banks) and the central bank interact via the exchange of deposits, investments, loans, excess- and required reserves and central bank loans. Arrows indicate the direction of fund flows.

to obtain on the interbank market at a planned refinancing rate and how much central bank loans it plans to obtain at the main refinancing rate. If this plan cannot be realized (e.g. if a bank's liquidity demand is unsatisfied on the interbank market), banks make a non-optimal portfolio choice. This possibility is excluded for the sake of simplicity. Note, that a market for central bank money is not explicitly modelled. The central bank rather accomodates all liquidity demands of commercial banks, as long as they can provide the neccessary securities. This assumption is not unrealistic in times of crises, as for example the full allotment policy of the ECB at the peak of the crisis shows.

2.2. Update Algorithm

In the simulation I have implemented an update algorithm that determines how the system evolves from one state to another. The algorithm is divided up into three phases that are briefly described here. Every update step is done for all banks for a given number of sweeps. At the beginning of phase 1 the bank holds assets and has liabilities from the end of the previous period:

$$\underline{I}^k_{t-1} + \underline{E}^k_{t-1} + r\underline{D}^k_{t-1} = \underline{D}^k_{t-1} + \underline{BC}^k_{t-1} + \underline{L}^k_{t-1} + \underline{LC}^k_{t-1} \tag{5}$$

where an underline denotes realized quantities. In period 0 all banks are endowed with initial values. The update step starts with banks getting the required reserves $r\underline{D}^k_{t-1}$ and excess reserves $\underline{E}^k_{t-1}$ plus interest payment from the central bank (it is assumed that for both required and excess reserves an interest of r^b is paid). The banks obtain a stochastic return for all investments $\underline{I}^k_{t-1}$ which might be either positive or negative. The firms furthermore pay back all investments $\underline{I}^k_f$ that were made in a previous period and have a maturity of $\tau^k_I = 0$. The banks then pay interest for all deposits that were deposited in the previous period. After that the banks can either receive further deposits from the households, or suffer deposit withdrawings ΔD^k_t. At the end of the first period, all interbank and central bank loans plus interests are paid either to, or by bank k.

At the beginning of phase 2, the bank's liquidity $\widehat{Q}^k$ is therefore given as:

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\begin{aligned}
\widehat{Q}^k_t \;=\; & (1+r^b)\left[r\underline{D}^k_{t-1} + \underline{E}^k_{t-1}\right] + \mu^k \underline{I}^k_{t-1} + \underline{I}^k_f - r^d\underline{D}^k_{t-1} \pm \Delta D^k_t \\
& -(1+r^b)\left[\underline{L}^k_{t-1} + \underline{LC}^k_{t-1}\right] + \underline{BC}^k_{t-1}
\end{aligned}
\tag{6}
$$

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where the banking capital has to be taken into account as it was assumed to be highly liquid. All banks with $\widehat{Q}_t^k < 0$ are marked as illiquid and removed from the system. Banks that pass the liquidity check now have to pay required reserves $r\underline{D}_t^k$ to the central bank.

In phase 3 the bank k determines its planned level of investment $I_t^k = (\lambda^k)^*(V^k)^*$ and excess reserves $E_t^k = (1-(\lambda^k)^*)(V^k)^*$ according to equations (3) and (4). From this planned level and the current level of investments (all investments that were done in earlier periods and have a maturity $\tau_I^k > 0$), as well as the current liquidity (6) the bank determines its liquidity demand (or supply). If a bank has a liquidity demand, it will go first to the interbank market, where it asks all banks i that are connected to k (denoted as $i : k$), if they have a liquidity surplus. In this case the two banks will interchange liquidity via an interbank loan. The convention is adpotet that a negative value of L denotes a demand for liquidity and therefore the interbank loan demand of bank k is given by:

$$L_t^k = \widehat{Q}_t^k - I_t^k \tag{7}$$

From this, one can obtain the realized interbank loan level, via the simple rationing mechanism:

$$\underline{L}_t^k = \min \begin{cases} L_t^k & , & -\sum_{i:k} L_t^i & | & L_t^i \cdot L_t^k < 0 & ; & \text{if } L_t^k > 0 \\ -L_t^k & , & \sum_{i:k} L_t^i & | & L_t^i \cdot L_t^k < 0 & ; & \text{if } L_t^k < 0 \end{cases} \tag{8}$$

Now there are three cases, depending on the bank's liquidity situation. If a bank has neither a liquidity demand nor excess liquidity, it will not interact

with the central bank and this step is skipped. However, if the bank still has a liquidity demand, it will ask for a central bank loan:

$$LC_t^k = \underline{L}_t^k - L_t^k \tag{9}$$

The central bank then checks if the bank has the neccessary securities and if so, it will provide the loan:

$$\underline{LC}_t^k = \max\left(LC_t^k, -\alpha^k \underline{I}_{t-1}^k\right) \tag{10}$$

where $\alpha^k \in [0,1]$ denotes the fraction of investments of bank k that are accepted as securities by the central bank. If a bank has insufficient securities, the central bank will not provide the full liquidity demand and the bank has to reduce the planned investment and excess reserve level. If the bank has no securities (no investments $\underline{I}_{t-1}^k$), it cannot borrow from the central bank. This rationing mechanism maps planned investment levels to realized ones.

The second case is that a bank has a large liquidity surplus even if all planned investments can be realized. In this case, the bank is able to pay dividends A_t^k and the dividend payment is determined by:

$$A_t^k = \min\left\{LC_t^k, \beta^k \underline{I}_t^k\right\} \tag{11}$$

where $\beta^k \in [0,1]$ is the dividend level of bank k. The dividend level will typically be very close to 1 as shareholders will push the bank to rather pay dividends than use the money to deposit it at the central bank at low interest rates. The remaining:

$$\underline{LC}_t^k = LC_t^k - A_t^k \tag{12}$$

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is transferred to the central bank's deposit facility. Finally the realized investments are transferred to the firm sector and the realized excess reserves are transferred to the central bank.

These steps are done for all $k = 1 \ldots N$ banks in the system for $t = 1 \ldots \tau$ time steps. As there are two stochastic elements in the simulation (the return of investments and the deposit level) two channels for a banks insolvency are modelled. The first channel is via large deposit withdrawals. As deposits are very liquid and investments are illiquid for a fixed, but random investment time, this maturity transformation might lead to illiquidity and therefore to insolvency. The second channel for insolvency is via losses on investments. If the banks banking capital is insufficient to cover losses from a failing investment, this bank will be insolvent. If a bank fails, all the banks that have loaned to this bank will suffer losses, which they have to compensate by their own banking capital. This is a possible contagion mechanism, where the insolvency of one bank leads to the insolvency of other banks, that would have survived if it was not for the first bank's insolvency. The impact of the contagion effect will depend on the precise network structure of the interbank market at the time of the insolvency.

2.3. Network theory

A financial network consists of a set of banks (nodes) and a set of relationships (edges) between the banks. Even though many relationships exist between banks, this paper focuses on relationships that stem from interbank lending. For the originating (lending) bank the loan will be on the asset side of its balance sheet, while the receiving (borrowing) bank will hold the loan as

a liability. To describe the toplogy of a network, some notions from graph theory are helpful. The starting point is the definition of a graph.

Definition 1. *A (un)directed graph $G(V, E)$ consists of a nonempty set V of vertices and a set of (un)ordered pairs of vertices E called edges. If i and j are vertices of G, then the pair ij is said to join i and j.*

One sometimes speaks of graphs as networks and the two terms are often used interchangably. Since the focus of this paper is on interbank markets, the nodes of a network are (commercial) banks and the edges are interbank loans between two banks. For every graph a matrix of bilateral exposures which describes the exposure of bank i to bank j can be constructed.

Definition 2. *The matrix of bilateral exposures $W(G) = [w_{ij}]$ of an interbank market G with n banks is the $n \times n$ matrix whose entries w_{ij} denote bank i's exposure to bank j. The assets a_i and liabilities l_i of bank i are given by $a_i = \sum_{j=1}^{n} w_{ij}$ and $l_j = \sum_{j=1}^{n} w_{ji}$.*

Closely related to the matrix of bilateral exposures is the adjacency matrix that describes the structure of the network without referring to the details of the exposures.

Definition 3. *The entries a_{ij} of the adjacency matrix $A(G)$ are one if there is an exposure between i and j and zero otherwise.*

One can define the interconnectedness of a node as the in- and out-degree of the node.

Definition 4. *The in-degree $d_{in}(i)$ and out-degree $d_{out}(i)$ of a node i are defined as:*

$$d_{in}(i) = \sum_{j=1}^{n} a_{ji} \quad , \quad d_{out}(i) = \sum_{j=1}^{n} a_{ij} \tag{13}$$

and give a measure for the interconnectedness of the node i in a directed graph $G(V, E)$. The two degrees are equal for directed graphs.

One can define the size of a node i analogously to its interconnectedness in terms of the value in- and out-degree.

Definition 5. *The value in- and out-degree of a node are defined as:*

$$vdc_{in}(i) \quad = \quad \frac{\sum_{j=1}^{n} w_{ji}}{\sum_{k=1}^{n} \sum_{j=1}^{n} w_{kj}} \in [0, 1] \tag{14}$$

$$vdc_{out}(i) \quad = \quad \frac{\sum_{j=1}^{n} w_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} w_{jk}} \in [0, 1] \tag{15}$$

and give a measure for the size of the node. The value in-degree is a measure for the liabilities of a node while the value out-degree is a measure for its assets.

A quantity that can be used to characterise a network is its average path length. The average path length of a network is defined as the average length of shortest paths for all pairs of nodes $i, j \in V$. Another commonly used quantity to describe the topology of a network is the clustering coefficient, introduced by Watts and Strogatz (1998) in their seminal work on small-world networks. Given three nodes i, j and k, with i lending to j and j lending to k, then the clustering coefficient can be interpreted as the probability that i lends to k as well. For $i \in V$, one define the number of opposite edges of i as:

$$m(i) := |\{j, k\} \in E : \{i, j\} \in E \text{ and } \{i, k\} \in E| \tag{16}$$

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and the number of potential opposite edges of i as:

$$t(i) := d(i)(d(i) - 1) \tag{17}$$

where $d(i) = d_{in}(i) + d_{out}(i)$ is the degree of the vertex i. The clustering coefficient of a node i is then defined as:

$$c(i) := \frac{m(i)}{t(i)} \tag{18}$$

and the clustering coefficient of the whole network $G = (V, E)$ is defined as:

$$C(G) := \frac{1}{|V'|} \sum_{i \in V'} c(i) \tag{19}$$

where V' is the set of nodes i with $d(i) \geq 2$. The average path length of the whole network can be defined for individual nodes. The single source shortest path length of a given node i is defined as the average distance of this node to every other node in the network.

It is possible to distinguish between a number of networks by looking at their average path length and clustering coefficient. One extreme type of networks are regular networks which exhibit a large clustering coefficient and a large average path length. The other extreme are random networks which exhibit a small clustering coefficient and a small average path length. Watts and Strogatz (1998) define an algorithm that generates a network which is between these two extremes. They could show that the so-called *"small-world networks"* exhibit both, a large clustering coefficient and small average path length. A large number of real networks like the neural network of the worm Caenorhabditis elegans, the power grid of the western United States, and

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the collaboration graph of film actors are small-world networks. From a systemic risk perspective, small-world networks are interesting, as it is reasonable to assume that the short average path length and high clustering of small-world networks make them more vulnerable to contagion effects than random or regular networks. Small-world networks can be created by using the algorithm defined in Watts and Strogatz (1998). Starting point is a regular networks of N nodes where each node is connected to its m neighbours. The algorithm now loops over all links in the network and rewires each link with a probability β. For small values of β (about 0.01 to 0.2) the average path length drops much faster than the clustering coefficient so one can have a situation of short average path length and high clustering. A small-world network is shown on the left side of Figure (2) with $N = 50$, $k = 4$, $\beta = 0.05$.

Another interesting class of networks are scale-free networks. They are characterized by a logarithmically growing average path length and approximately algebraically decaying distribution of node-degree (in the case of an undirected network). They were originally introduced by Barabási and Albert (1999) to describe a large number of real-life networks as e.g. social networks, computer networks and the world wide web. To generate a scale-free network one starts with an initial node and continues to add further nodes to the network until the total number of nodes is reached. Each new node is connected to k other nodes in the network with a probability that is proportional to the degree of the existing node. When thinking about financial networks, this preferential attachment resembles the fact that larger and more interconnected banks are generally more trusted by other market

participants and therefore form central hubs in the network. On the right side of Figure (2) a scale-free network with $N = 50$ and $k = 2$ is shown.

A typical feature of scale-free networks is their degree-distribution, as it typically follows a power-law. The exponent of the power-law can be measured and characterises the network topology for different networks. Boss et al. (2004) show that the degree distribution of the Austrian interbank market follows a power law with an exponent of $\gamma = -1.87$. Cajueiro and Tabak (2007) analyze the topology of the Brazilian interbank market. They show that the Brazilian interbank market employs a scale-free toplogy and is characterized by money-center banks. Iori et al. (2008) and Manna and Iazzetta (2009) report that the Italian interbank market shows a similiar scale-free behaviour. Cont and Moussa (2009) show that a scale-free interbank network will behave like a small-world network when Credit Default Swaps (CDS) are introduced. In this sense a CDS acts as a "short-cut" from one part of the network to another. This paper therefore focuses on these three classes of networks (random, scale-free and small-world) to analyze their effect on systemic risk through contagion effects.

2.4. Model Parameters

There are eighteen model parameters that control the numerical simulation. If not stated otherwise, numerical simulations were performed with the parameters given in this section. The simulations were perfomred with $N = 100$ banks and $\tau = 1000$ update steps each. Note that the simulation results do not change if the number of banks is increased. It has to be ensured, however, that the number is large enough so that differences in the network topologies
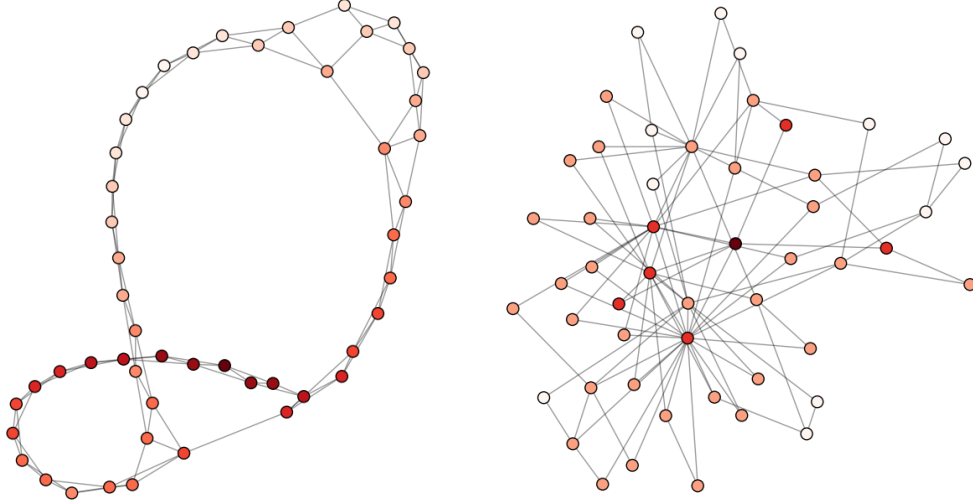
Figure 2: On the left: a small-world network that was created using the algorithm of Watts and Strogatz (1998) with $N = 50$, $k = 4$ and $\beta = 0.05$. On the right: a scale-free network that was created using the methodology introduced in Barabási and Albert (1999) with $N = 50$ and $m = 2$. The colour is an indication for the single source shortest path length of the node and ranges from white (large) to red (short).

become significant enough to be visible in the simulation results. The number of update steps has to be large enough for the system to reach a steady state from where on the results only change little. Every simulation was repeated `numSimulations=100` times to average out stochastic effects. The interest rate deposits was chosen to be $r^d = 0.02$ and the main refinancing rate as $r^b = 0.04$, which resembles the situation in the Eurozone prior to the crisis. The required reserve rate is $r = 0.02$ which is in line with legal

requirements. The interbank connection level for random graphs is denoted as `connLevel`$\in [0, 1]$. At a `connLevel=0` there is no interbank market and at `connLevel=1` every bank is connected to every other bank. For scale-free networks the parameters $m = 1, 2, 4, 10$ and for small-world networks the parameters $\beta \in [0.001, 0.1]$ were used.

Two sets of parameters are used to describe the influence of the real economy on the model. The first set is the probability that a credit is returned successful, $p_f = 0.97$. The return for a successful returned credit is taken to be $\rho_f^+ = 0.09$ and in case a credit defaults, the negative return on the investment is $\rho_f^- = -0.05$. The choice of parameters again resembles the situation in the Eurozone and will sometimes be referred to as "normal" parameters. As "crisis" parameters $\rho_f^+ = 0.97$ and $\rho_f^- = -0.08$ were used. This implies that banks have larger losses on their risky assets in times of crises. To plan their optimal portfolio, the banks have an expected credit success probability p_b and expected credit return ρ_b^+. It is assumed that these expected values correspond to the true values from the real economy. The optimal portfolio structure and volume of a bank depends also on its risk aversion parameter θ. For each bank, $\theta \in [1.67, 2.0]$ was chosen randomly to account for heterogeneity in the banking sector. The range of possible θ values is restricted by the portfolio structure of banks. For $\theta > 2$ and the selected parameters, banks would hold more then 15% of their assets as risk-free assets, which is unreasonable. For $\theta < 1.67$, portfolio theory would imply that banks hold no risk-free assets. The value of the factor of constant relative risk aversion is subject to an ongoing debate, even though a value greater than one is well

established (see i.e. the discussion in Ait-Sahalia and Lo (2000)).

Deposit fluctuations ΔD_t^k were modelled as:

$$\Delta D_t^k = (1 - \gamma^k + 2\gamma^k x)\underline{D}_{t-1}^k \tag{20}$$

with $\gamma^k = 0.02$ (in "normal" times) and $\gamma^k = 0.1$ (during a "crisis" period) can be interpreted as a scaling parameter for the level of deposit fluctuations and x being a random variable with $x \in [0, 1]$. The fraction of a banks investments that the central bank accepts as securities is set to $\alpha^k = 0.8$, assuming that banks invest only in assets which have a good rating. The level of dividends β^k determines the fraction of a banks excess liquidity (that is free funds that are available if a bank has reached its optimal investment volume) that the bank will pay out as dividends to shareholders. It is assumed that shareholders can find more profitable investment opportunities than the deposit facility of the central bank and will thus push for banks to pay out as much of the excess liquidity as possible. In order to accomodate the fact that banks in reality nonetheless make use of the deposit facility, a dividend level of $\beta^k = 0.99$ was chosen for the simulations. Note that a change in the dividend level does not qualitatively change the results.

3. Results

To answer the question which impact central bank activity has on financial stability, I first varied the level of collateral α^k that is accepted by the central bank in order to provide liquidity to banks. For $\alpha^k = 1$ the central bank will accept all assets of commercial banks as collateral, while for $\alpha^k = 0$, no
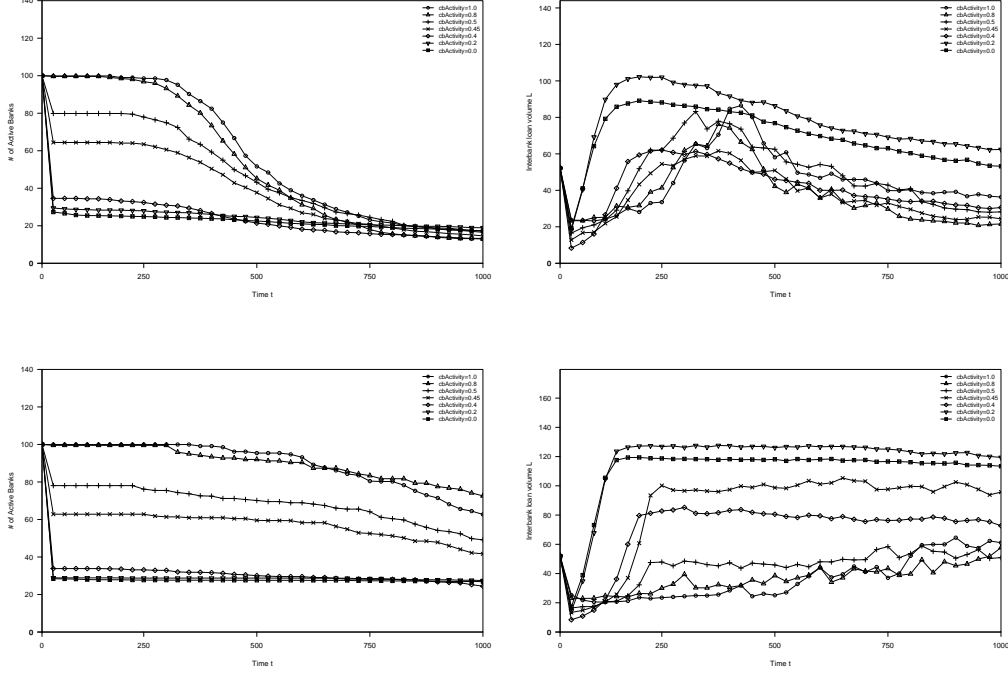
Figure 3: The effect of central bank activity for different scenarios. Top: crisis scenario. Bottom: normal scenario. Left: number of active banks over simulation time. Right: interbank loan volume over simulation time. The central bank activity α^k varied between $\alpha^k \in [0.0, 1.0]$.

assets will be accepted. Thus, α^k is used as a parameter to determine the fraction of assets that are of high enough quality to be accepted as collateral. Banks will obtain liquidity for the amount of collateral that they can deposit at the central bank. In Figure (3) it can be seen, that a significant stabilizing effect from the liquidity provision by the central bank is obtained from $\alpha^k \sim 0.45$. However, this effect is non-linear in α^k which implies that, on the one hand, even slight changes in the collateral requirements can have significant stabilizing effects if performed around the critical value. On the

other hand, even large changes can have very little effect, if performed away from the critical value. The effect on the number of active banks is similar for both, the normal and the crisis scenario. On the right hand side of Figure (3) the impact of the collateral requirements on the volume of interbank loans is displayed. It can be seen, that in both scenarios an abundant provision of central bank liquidity will lead to a crowding-out effect on interbank liquidity. It can further be seen, that a high amount of interbank liquidity is correlated with high financial instability. This is precisely the knife-edge property of interbank markets: if the exposures amongst banks are too large, an initial knock-on effect will be amplified in the system.

In Figure (4) the impact of different network topologies on financial stability in times of crisis and normal times is shown. When comparing the results for random networks, it can be seen that the difference in network topology is not significant during normal times.[2] In times of crisis, however, the different levels of interconnectedness come into play. Figure (4) also confirms the result of Nier et al. (2008), who show that the relationship between the level of interconnectedness on interbank markets and financial contagion is non-monotonic. It can furthermore be seen, that contagion effects tend to be larger in in random networks than in small-world networks, where in turn contagion effects tend to be larger than in scale-free networks. This implies that analyses that are conducted with static random networks can overestimate contagion effects when a dynamic model of systemic risk is used.

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[2]And similarly for small-world and scale-free networks.
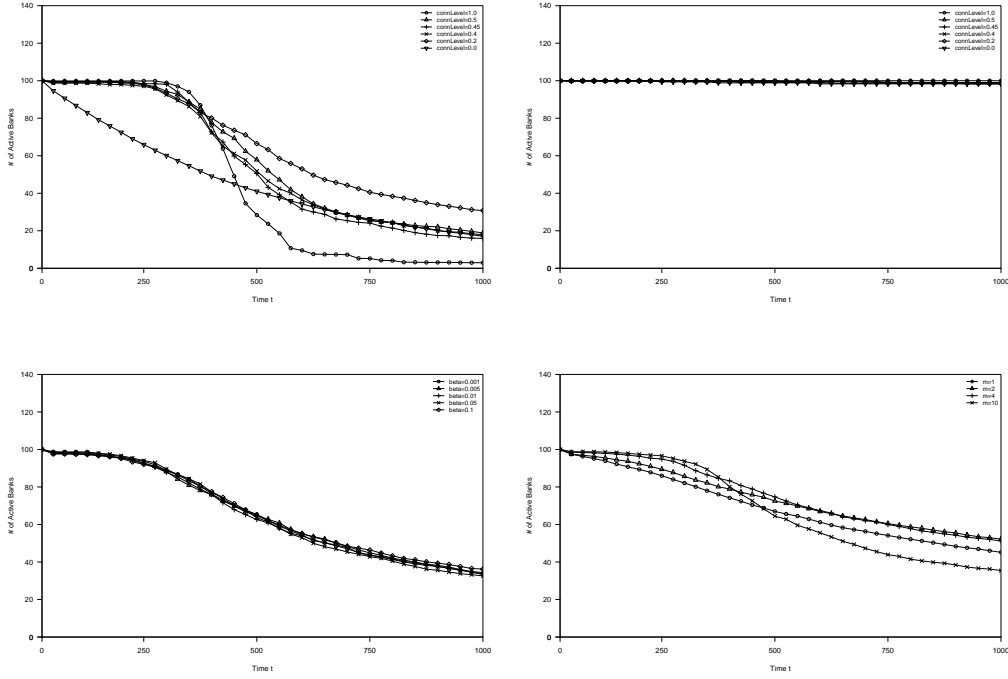
Figure 4: The effect of different network topologies on financial stability. Left top: crisis scenario and random topology. Right top: normal scenario and random topology. Connection levels of `connLevel`= $0.0, 0.2, 0.4, 0.45, 0.5, 1.0$ were used. Bottom left: crisis scenario and small-world network with $\beta = 0.001, 0.005, 0.01, 0.05, 0.1$. Bottom right: crisis scenario and scale-free network with $m = 1, 2, 4, 10$.

For increasing levels of interconnectedness in random networks, it can be seen from Figure (4) that there exists a "tipping" point, where the networks become endogenously instable. To better understand this, the interbank loan volume is depicted in Figure (5). As Ladley (2011) argues, the knife-edge property of interbank markets requires shocks to be small, in order to exihibt a stabilizing effect. Figure (5) shows an increase in interbank market volume
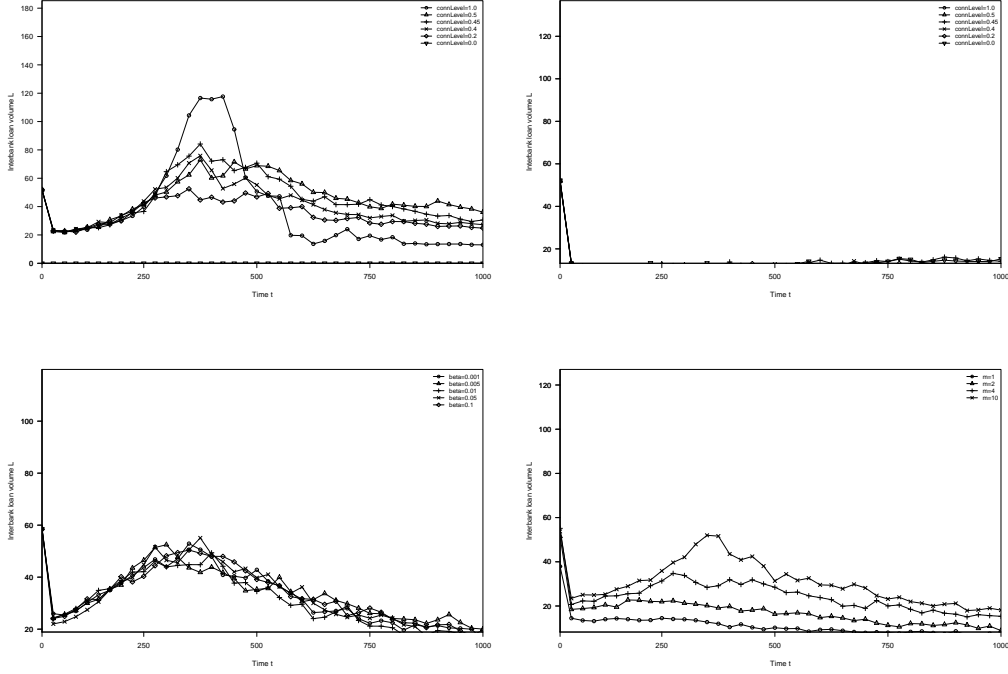
Figure 5: The effect of different network topologies on interbank loan volume. Left top: crisis scenario and random topology. Right top: normal scenario and random topology. Connection levels of `connLevel`$= 0.0, 0.2, 0.4, 0.45, 0.5, 1.0$ were used. Bottom left: crisis scenario and small-world network with $\beta = 0.001, 0.005, 0.01, 0.05, 0.1$. Bottom right: crisis scenario and scale-free network with $m = 1, 2, 4, 10$.

until a tipping point, where the amount of interbank loans becomes large and contagion effects dominate. This in turn leads to an increasing number of insolvencies that spread easier in the system if the level of interconnectedness increases. It can also be seen from Figure (5) that the volume of interbank markets in normal times is significantly smaller than the volume in times of distress. This is easily understood in the model setup, as times of distress imply larger liquidity fluctuations and therefore larger amounts of interbank

loans issued between agents. However, this implies that interbank markets will be more prone to contagion effects in times of high deposit and asset return volatility. It also implies that interbank markets are more susceptible to systemic risk when the volume of the interbank market is larger.
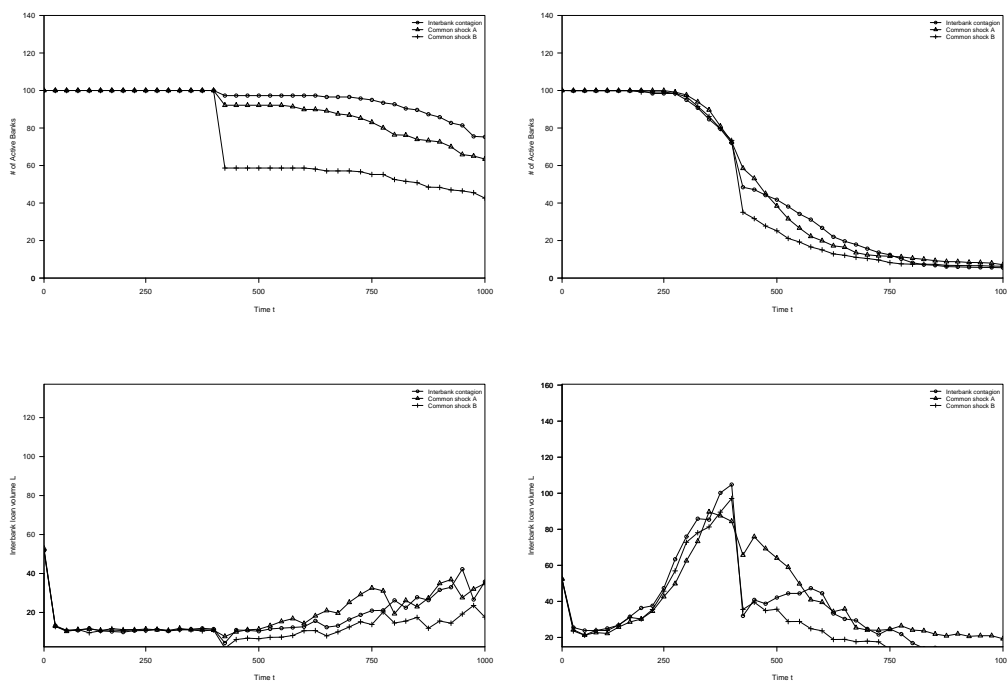

Figure 6: The impact of different forms of systemic risk on financial stability and interbank loan volume. Left: normal scenario. Right: crisis scenario. Top: number of active banks over time. Bottom: interbank loan volume voer time. Interbank contagion: the largest bank in the system at time $t = 400$ was sent into insolvency. Common shock A: all banks suffer a common shock of 10% on all their assets. Common shock B: all banks suffer a common shock of 20% on all their assets.

To understand the impact of different forms of systemic risk on financial stability, Figure (6) compares two different types of shocks. In the case of pure

interbank contagion, the largest bank in the system is selected and exoge-
nously sent into default. The impact of this default on the remaining number
of active banks in the system is depicted in Figure (6) at the top. Again, it
can be seen that the impact is larger in times of distress than in normal times.
To analyze the impact such a default has on the liquidity provision in inter-
bank markets, Figure (6) shows the interbank market volume at the bottom.
When a common shock hits the system, banks with insufficient equity will go
into insolvency. While this might only be a small number of banks, a larger
number of banks become more vulnerable to deposit and asset return fluc-
tuations. As was seen in Figure (5), shocks that exceed a certain threshold
will lead to an increased number of insolvencies in the system. When banks
become more vulnerable, this threshold is reached easier and the whole sys-
tem remains unstable as long as the volume on the interbank market (and
hence the magnitude of possible shocks) will lead to increased insolvencies.
When the crisis hits, the volume of interbank transactions drops until it has
reached a level where the endogenous deposit and asset return fluctuations
will not lead to an increased number of insolvencies. Comparing the case
of common shocks to the case of interbank contagion, it can be seen that,
while the impact of a common shock on the number of active banks is more
severe than in the contagion case, the opposite holds true for interbank mar-
ket liquidity. The pure contagion case has a substantial impact on interbank
market liquidity, which on the other hand implies a smaller size of shocks
due to endogenous fluctuations.

4. Conclusion and Policy Implications

This paper provides further evidence that central bank intervention can indeed alleviate financial distress and liquidity shortages on interbank markets, at least in the short run. Even small changes in the collateral requirements of central banks can lead to a significant enhancement of liquidity provision on interbank markets. There is, however, a large range of required collateral quality, where even a significant change in the collateral requirements will not lead to a significant enhancement of liquidity provision. The simulation results also show that an abundant provision of central bank liquidity can lead to a crowding-out of interbank liquidity. The desired impact of central bank activity on liquidity provision will thus be smaller in the long run. This is confirmed by the fact that, while the central bank has a stabilizing effect on the financial system in the short-run, the long run equilibrium will always be the equilibrium that would have been reached without central bank activity.

The model developed in this paper allows for a deeper understanding of the knife-edge property of interbank markets. The results indicate that there is an upper limit of interbank loan volume for different network topologies, where endogenous deposit and asset return fluctuations will lead to an increased number of bank insolvencies. The limit itself depends on the topology of the interbank markets and will be larger for higher interconnected banking systems. This implies that the knife-edge property of interbank markets depends on the precise market structure and level of interconnectedness. For higher connectivity on the interbank market, larger amounts of interbank liquidity can be tolerated by the system without a substantial increase in

financial fragility. However, even for complete networks, where every bank is connected to every other bank, such an upper limit exists. In fact, for higher interconnected networks, shocks will spread more rapid, which implies a higher fragility of the system once the tipping point is reached.

Already the correlation of higher interconnectedness and increasing system fragility makes it clear, that the topology of the interbank network is relevant for the assessment of financial stability. This paper also shows that the topology of the interbank network impacts the assessment of the long-run stability of the banking system. This "topology effect" is more accentuated in times of crisis, while in normal times, the topology has little impact. This result is of particular relevance for the question which interbank network structure is most resilient to financial distress. It turns out that networks with large average path length are more resilient to financial distress and that it is precisely during a crisis where the network topology matters.

Even though contagion effects are far better studied in the literature, it turns out that common shocks pose a greater threat to financial stability. This is also due to the knife-edge property of interbank markets. When a common shock strikes the entire banking system, banks become more vulnerable to endogenous fluctuations and occasional idiosyncractic insolvencies. This leads to a drastic vulnerability of the entire system and a large number of bank insolvencies. However, contagion affects interbank market liquidity more severely than common shocks. Again, the impact of the shocks is larger during times of distress, which holds especially true for the impact of contagious

defaults on interbank liquidity provision.

From the perspective of monitoring systemic risk, this paper provides evidence that the topology of the interbank network has to be taken into account. The interbank network topology, however, is highly dynamic and varies from day to day. This implies that further analyses of this dynamic behaviour are necessary in order to understand the full impact of the network topology on the propagation of shocks.

The results in this paper also have implications for the optimal reaction of central banks to financial crises, as different forms of systemic risk have a different impact on the financial system. In the case where systemic risk is mainly mainfesting in the form of contagion, central banks should resort to providing short-term liquidity to the financial system. Because of to the crowding-out of interbank liquidity by abundant central bank liquidity, however, this liquidity provision should be short- or medium-term only. In the case where systemic risk is mainly manifesting in the form of a common shock, the optimal policy reaction is to re-capitalize the financial system. Only a strengthening of the banks' equity will make them more resilient to endogenous fluctuations. This is especially relevant, as the reduction in interbank lending is smaller in the case of a common shock and the simulation results indicate a direct relation between high interbank lending (with respect to the resilience of each individual bank, i.e. the banks' capital buffer) and financial fragility. Thus, a better understanding of all forms of systemic risk is required in order for policy makers to find appropriate crisis reactions.

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