A Quantitative Model of Corporate Reputation Building in Debt Markets and Firm Dynamics*

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Abstract

I estimate a heterogeneous firm model on dynamic adverse selection with screening and signaling in public debt markets for privately informed persistent productivity, and quantify the effect of asymmetric information on misallocation of capital and financial liabilities. The firm's manager chooses between public debt (e.g., corporate bonds), priced on the accumulation of borrower's history, and monitored private debt (e.g., bank loans), allowing costly monitoring to be an imperfect substitute for reputation building. Cross-subsidization caused by asymmetric information in public debt markets leads to overinvestment of low productivity firms and capital misallocation. A good reputation lowers borrowing costs from public lenders. The firm's manager uses leverage and equity to send a good signal. However, neither reputation building nor monitored private debt substantially remove capital misallocation. I find precise information associated with productivity increases total factor productivity, decreases the variance of the marginal product of capital, and increases consumption by 1.4%. Less demand for monitoring lowers aggregate bank debt ratio by 6 percentage points. In the counterfactual policy analysis, the taxation of debt forgiveness under Chapter 11 reorganization generates a rise in consumer welfare by reducing low productivity firms' incentive to overborrow.

Keywords: Adverse Selection; Debt Structure; Firm Heterogeneity

JEL Classification: E22, E23, G21, G23, G33

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1 Introduction

Since the seminal works of Jaffee and Russell (1976) and Stiglitz and Weiss (1981), a large and growing body of theoretical literature emphasizes the importance of asymmetric information in debt markets. Generally, the presence of private information at the firm leads to adverse selection problems and a misallocation of debt financing. Understanding the size of misallocation created by asymmetric information in debt markets can guide policymakers to regulate industries (e.g., the credit rating industry) and to design policies (e.g., U.S. corporate tax). While there is a considerable difference in the abilities of creditors to acquire information associated with debtor activities, scholars have spent decades theoretically studying the dynamic roles of informed and uninformed lenders. However, quantitative models have mostly focused on a single type of debt: fully informed or uninformed lenders examining static costs of short-lived hidden information. In this paper, I jointly explore the role of two types of debt: fully informed lenders and uninformed lenders subject to dynamic costs of long-lived hidden information.

I address three questions. First, what is the effect of asymmetric information in corporate bond markets on misallocation of capital and financial liabilities? Next, how does reputation building —the assessment of unobserved firm characteristics —affect financial liabilities, capital structure, and investment? Finally, can tax policy improve capital allocation in the presence of asymmetric information in corporate bond markets? These questions are critical as reputation building could restore efficiency in credit markets and fix misallocation of real resources to borrowers. However, there is little, if any, direct evidence that using reputation building to reduce asymmetric information will impact the optimal firm choice of financial liabilities, capital structures, and investment.^{2,3,4} The evidence is scarce, not only because reputation building and firm choices are endogenous, but also because the severity of information asymmetry is unobservable to bondholders, shareholders, and the econometrician. Under these circumstances, the magnitude of the effect of information asymmetry

¹Diamond (1989, 1991)

²There is a growing literature in industrial organization and macroeconomics that studies the effect of adverse selection on welfare. Corporate bonds are one of the largest markets studied in the literature since unmonitored debt is the primary source of debt financing for the U.S. non-financial corporate sector (\$5.8 trillion in 2019). Insurance markets (survey by Einav and Finkelstein (2011)) accounted for \$1.3 trillion in 2019 (hereafter market size in the U.S.); online credit markets (Kawai et al. (2014); Xin (2020)) \$9 billion in 2016; and unsecured consumer credit markets (Chatterjee et al. (2020)) \$0.9 trillion in 2019 (credit card markets).

³Since the 2000's in the U.S., over 70% of non-financial corporate sector's debt financing in aggregate is publicly traded debt, predominantly corporate bonds (Appendix Figure A1).

⁴While most papers on the reputational concerns study credit rating agencies (Mathis et al., 2009; Bolton et al., 2012; Baghai and Becker, 2019), private lenders (Boot and Thakor, 1993; Chemmanur and Fulghieri, 1994a,b), and activists (Johnson and Swem, 2020), I study the reputational concerns of corporate borrowers.

in corporate bond markets is still an open question.⁵

An important innovation of the model is the microfoundation of key driving forces of debt substitution. Without bank debt in the economy, the effect of reputation building would be overstated. In the model, market debt and bank debt have three technological characteristics: (i) information availability from bank monitoring; (ii) costly bank financial intermediation; and (iii) recovery of defaulted debt associated with limited debt enforcement. In addition, I introduce Chapter 11 (Ch. 11) and Chapter 7 (Ch. 7) bankruptcies to the model for characterizing the bankruptcy decision and debt recovery in the equilibrium (Antill and Grenadier, 2019; Corbae and D'Erasmo, 2020). Even though Ch. 7 bankruptcy is rarely observed in both real-world and simulated data (14bps per annum), Ch. 7 bankruptcy plays a central role in determining bank debt recovery under Ch. 11 bankruptcy.

Monitoring technology, the ability to observe the firm's marginal product of capital (MPK) directly, provides private lenders with the benefits and costs of borrowing, which are dependent on the types of debtors. The model's salient feature is that past credit history, type scores in the terminology of the literature, enters as an endogenous state variable into the firm manager's optimization problem. Since private lenders (e.g., bank lenders) can gain access to monitoring technology, I assume that the persistent (but not fully permanent) firm's productivity is privately observed with monitoring costs. On the other hand, public lenders (e.g., bondholders) have no access to private information on the firm's productivity in the benchmark model. Therefore, the firm's productivity is interpreted as the agent's type in the principal-agent problem. Adverse selection is addressed through the screening of borrowers —uninformed lenders offer a set of contracts in order to price discriminate. In turn, the signaling takes place in public debt markets in which subjective beliefs are revised endogenously as information about the firm's investment, financing, and bankruptcy are made public. Then, the firm's financing policy takes into account the trade-offs between the benefits and costs of financing options and building a good reputation through signaling. Since dynamic learning is not a perfect substitute for monitoring, enhanced monitoring skills of bondholders reduce capital misallocation.

⁵Ivashina (2009); Darmouni (2020) estimate the size of information asymmetry in bank loan markets. Both papers use shocks to banks as an exogenous variation. However, finding a plausible exogenous variation in corporate bond markets is harder due to data limitations on bondholders.

⁶See Diamond (1984); Holmstrom and Tirole (1997) for the role of bank monitoring.

⁷See Lummer and McConnell (1989); Gilson et al. (1990); Asquith et al. (1994); Bolton and Scharfstein (1996); Bolton and Freixas (2000) for limited debt enforcement.

⁸This paper's definition of monitoring is more closely related to that of a screening device in adverse selection than a preventative device in moral hazard or punishing device in costly state verification.

⁹Chatterjee et al. (2020) introduce type scores as a counterpart to credit scores in the consumer credit market.

Another significant difference between arm's-length corporate bonds and informed bank loans is the strength of debt enforcement technology. The model characterizes recovery at default, which depends on the value of cash flow for corporate bonds and the value of asset for bank loans. This is in line with notions of "cash flow-based debt" which includes the majority of corporate bonds (Lian and Ma, 2020). Recovery at default of bank loans depends on the liquidation value of assets. This is implemented by modeling the renegotiation process, in which the firm's manager uses the threat of liquidation via Ch. 7 bankruptcy to extract full surplus from bank lenders. On the other hand, the renegotiation process between the debtor and dispersed bondholders is different from bank lenders since bondholders suffer from a free-rider problem (Rajan, 1992). Taking the model to real-world data, debt enforcement technology is essential to rationalize distributional characteristics of realized market debt recovery rates at default. Unlike the collateral constraints in the theoretical model (Kiyotaki and Moore, 2012) and quantitative models (e.g., Khan and Thomas (2013)), bankruptcy happens on-the-equilibrium path in my model. This feature allows me to compare real-world and simulated data in the event of default and to perform deep "tests" of theory.

By establishing a theory that jointly explains the likelihood of default and recovery of defaulted bonds under asymmetric information, the model provides a theoretical framework for credit ratings and recovery ratings for corporate bonds issued by Credit Rating Agency (CRA).^{11,12} While the model interprets CRA as a function of information processing, I find the model with adverse selection closely replicates predictive powers in forecasting actual recovery rates at default in real-world data. Moreover, the model is rich enough to capture untargeted dimensions of the micro and macro data: (i) cross-sectional patterns of bankruptcy, realized recovery rates at default, and recovery ratings; (ii) aggregate bank debt ratio and average debt-to-EBITDA; and (iii) dynamics of leverage and credit ratings.

I solve for a stationary recursive competitive equilibrium. The key point is that one does not need to account for off-the-equilibrium beliefs as the manager's utility is subject to extreme value shocks. These shocks result in strategies on every path having strictly positive probability (i.e., no unreached information sets). This makes the equilibrium notion simpler than standard signaling games.

 $^{^{10}}$ Drechsel (2019); Greenwald (2019) propose a model with both flow-based and stock-based constraints on borrowings.

¹¹Survey evidence shows CFOs consider credit ratings to be the most important factor of debt financing (Appendix A.2.2).

¹²The recovery rating, which has been introduced in the mid-2000s since practitioners recognized the importance of recoveries to pricing, is a complementary credit risk indicator. Moody's provides information associated with recovery rates at default in the form of "Loss Given Default Assessments" similar to S&P "Recovery Ratings".

Then, I structurally estimate the model via Simulated Method of Moments (SMM) by fitting the model to a sample of Compustat firms. Identification of the degree of information asymmetry is essential to derive quantitative results. I estimate five parameters to target the same number of the data moments. Out of these parameters, two parameters characterize the size of the variance of extreme value shocks to the manager's utility. The dual role of utility shocks is (i) to control the size of unobservable noise to signals sent by informed borrowers to uninformed lenders and (ii) to generate observable heterogeneity across firms including bankruptcy decisions. Namely, I use the likelihood of bankruptcy and the variance of debt to assets in the data to identify these parameters. The remaining three parameters are the costs of production, bankruptcy, and external equity financing.

I explore quantitative implications of monitoring technology adopted by bondholders from a normative perspective. My model works as a laboratory for analyzing counterfactual exercises where bondholders are informed about unobserved productivity.¹³ The rich model with endogenous decisions of bankruptcy and financing is insusceptible to the Lucas critique. In terms of economic magnitude, precise information associated with productivity increases measured Total Factor Productivity (TFP), decreases the variance of the MPK, and increases consumption by 1.4%. Less demand for information lowers aggregate bank debt ratio by 6 percentage points. Cross-sectional variation in recovery rates at default among unobserved types has a direct consequence on the severity of asymmetric information. My model rationalizes large dispersion of market debt recovery rates at default in the data, and I show weak debt enforcement technology in corporate bond markets is essential to match this feature. Lastly, I measure 7bps of monitoring costs (12% of intermediation costs) for corporate bonds as the break-even point for welfare improvement.

Finally, I conduct a variety of policy experiments. I find that the taxation of Cancellation Of Debt (COD) income, the amount of debt forgiveness, improves resource allocation and welfare. Asymmetric information impedes the efficient allocation of capital which leads to a decrease in TFP and consumption. This happens because ex-ante costs of borrowing become more expensive when the low productivity firm largely reduces debt repayment under Ch. 11 bankruptcy. In the benchmark model, the low productivity firm receives information rents from the high productivity firm due to information asymmetry through public debt markets. This leads to inefficient use of resources. The taxation of COD fixes this misallocation by penalizing the low productivity firm that issues corporate bonds to overinvest in capital and encouraging the high productivity firm to borrow more by reducing the screening costs.

¹³The monitoring ability of bondholders could be improved by undertaking policy actions. Amihud et al. (1999) suggest introducing a representative agent of dispersed bondholders to mimic the ability of the bank lender by providing incentives for delegated monitoring.

Reallocation of capital improves TFP and increases the representative household's welfare as measured by consumption.

Contribution. The first contribution of this paper is the development of a dynamic adverse selection model in corporate finance that jointly accounts for patterns of debt structure, leverage, and credit qualities. ¹⁴ The model is connected to four strands of literature. First, the model applies discrete choice frameworks (McFadden et al., 1973; Rust, 1987) in which the firm's manager has substantial real controls over financing decisions but is subject to unobserved transitory preference shocks. This second layer of information asymmetry between the inside manager and outsiders (creditors and equity holders) adds noise to signals sent by the inside manager to outside debt holders. Without this noise, the Bayesian inference problem about the firm's type becomes trivial, and therefore these preference shocks are important to measure the magnitude of information asymmetry. Second, my model resembles that used by Hennessy and Whited (2007) in which firms choose debt, internal finance, bankruptcy, and distributions in the presence of costly equity financing. The model is augmented to incorporate both public and private debt financing (Crouzet, 2017; Xiao, 2019). 15 Third, the model embeds signaling and screening on dynamic adverse selection where firms are allowed to substitute financing options. In this context, my model is closely related to Chatterjee et al. (2020), which applies dynamic adverse selection to the unsecured consumer credit market. Fourth, this paper also contributes to the literature on estimating dynamic models in corporate finance. A quantitative dynamic contracting model is studied by Li et al. (2016) and is extended into a moral hazard problem by Nikolov et al. (2020).

Second, my paper contributes to the literature studying rational learning in corporate finance. I contribute to the literature by solving a model in which private information has a long-run effect under recursive updating of beliefs with endogenous signaling. Jovanovic (1982) presents a model of the life cycle of firm growth and learning. His view is that firms are unaware of their real efficiency. Thus, firms learn over time about their true abilities from a noisy signal. In my paper, firms are informed, but bondholders must learn about the firm's type. Diamond (1991) introduces a theoretical model showing that, by repeatedly borrowing from banks, firms can build their reputation and use it to access public debt markets under favorable terms. In contrast, my model can be solved numerically and provides quantitative predictions using real-world data. Hennessy et al. (2010) solve a model of a single debt

¹⁴Houston and James (1996); Krishnaswami and Subramaniam (1999); Cantillo and Wright (2000); Denis and Mihov (2003); Faulkender and Petersen (2006)

¹⁵These models are used to explain empirical facts including imperfect debt substitution during Great Recession (De Fiore and Uhlig, 2011; Becker and Ivashina, 2014; De Fiore and Uhlig, 2015).

¹⁶Another version of incomplete information is that the firm's type is known, but lenders gather information with noise and a delay (Benzoni et al., 2019).

instrument and equity substitution with heterogeneous firms in a separating equilibrium, essentially assuming hidden information is short-lived. My model extends to long-lived hidden information.

Structure of the Paper. The remainder of the paper is organized as follows. Section 2 explains the dynamic programming of the model. Section 3 describes the estimation of the model while Section 4 elaborates on equilibrium results. Section 5 performs counterfactual experiments. Finally, Section 6 concludes.

2 Model

Time t is a discrete infinite sequence. Let current variable x_t be denoted x and next periods' variable x_{t+1} be denoted x'.

2.1 Technologies, Preferences, and Firm Entry and Exit

Technology. Firms produce homogeneous goods with decreasing returns to scale production technology $F: \mathcal{R}^+ \times \mathcal{Z} \to \mathcal{R}^+ \mid F(k,z) = \exp(z)k^{\alpha_k}, \alpha_k \in (0,1)$ where z is the firm's productivity and k is physical capital. The price of capital is set to one for simplicity. Earnings Before Interest, Taxes, Depreciation, and Amortization (EBITDA) $\Pi_k: \mathcal{R}^+ \times \mathcal{Z} \to \mathcal{R}$ is $\Pi_k(k,z) = F(k,z) - f$, where $f \in \mathcal{R}^+$ is fixed costs for production.¹⁷ The rate of depreciation in the capital stock is $\delta \in [0,1]$.

Preference. Firm managers and financial institutions are risk-neutral with a discount factor β . The manager makes a financing decision that maximizes his/her lifetime utility but not necessarily maximizes shareholder value.

The manager effectively receives utility from equity share (cash flow per shareholding from fixed equity compensation) and action-specific transitory utility shocks. I remain silent about the optimality of compensation structures of the firm's manager as in Nikolov and Whited (2014), and defer to a separate body of work for that. Instead, my paper's focus is to quantify the trade-off between informed and uninformed debtors. Rational inattention provides a microfoundation of additively separable utility shocks (Matějka and McKay (2015)). In the equivalent model, the manager pays costs of information-processing to investigate payoffs

 $^{^{17}}$ The interpretation of the firm specific productivity z is not restricted to the efficiency of physical capital, but also could be understood as a matching quality between the firm's manager and employee in the firm. A good matching quality might mean as a good fit of the firm's manager talent and experience to the firm's culture, the industrial structure, the nature of business, and the business strategy.

(e.g., communication costs in the board meeting, purchasing database, running surveys, paying people to monitor). Then, the manager's choice follows the Logit model. Other interpretations of these utility shocks are (i) the manager's unobserved effort, (ii) private benefits and costs of control (Grossman and Hart, 1980; Barkalay and Holderness, 1989), (iii) extraction of corporate resources (Hall and Murphy, 2003), and (iv) costs of control including covenant violations which trigger a technical default. Direct evidence of private benefits is difficult to obtain. However, these utility shocks help to explain cross-sectional variations in the data.

Firm Entry and Exit. Firm entering and exiting happen each period. Continuing firm, which consists of both firms that do not file bankruptcy and firms that file Ch. 11 bankruptcy, exit exogenously with a probability $\eta \in [0, 1]$. The exiting firm liquidates a fraction $\chi \in [0, 1]$ of physical capital. Examples of an exiting event are a mass of claims for defective products or an expropriation by a government. Section 2.9 describes endogenous exiting via Ch. 7 bankruptcy and Section 2.13 explains entrants.

2.2 Financial Markets

Firms finance investment by debt and equity. Firms choose to issue corporate bonds or bank loans in the form of debt finance.

Debt Markets. Competitive risk-neutral bondholders and bank lenders offer non-contingent one-period debt contracts. The equilibrium price menus are q_M for bonds and q_B for loans. These price menus are contingent on observable firm characteristics. Accordingly, bonds and loans are not contingent on the manager's transitory preference shocks in an incomplete market setup. The representative household supplies one-period risk-free asset at price q to bondholders and bank lenders. Creditors pay intermediation costs μ_M for bonds and μ_B for loans. A zero profit condition under perfect competition holds for each individual price menu.

Equity Market. External finance costs for equity infusion d < 0 are $\lambda(d) = \lambda_1 |d| \mathbb{1}_{d < 0}$ where $\mathbb{1}$ is the indicator function. The firm pays linear equity flotation costs $\lambda_1 \in \mathcal{R}^+$. Altinking and Hansen (2000) provide detailed evidence regarding linear equity flotation costs. This reduced form fashion captures the effect of adverse selection costs between managers and shareholders on underwriting fees in the literature (Gomes (2001); Hennessy and Whited (2007)). I do not include fixed costs of equity issuance in order to ensure the model has a well-defined unique equilibrium.

2.3 Household

The representative household maximizes lifetime utility by choosing optimal allocation of consumption, a safe asset, and stock holding. The risk-free debt price is q. Therefore, the representative household supplies funds to debt and equity markets.

2.4 Legal Environment: Bankruptcy Laws

Bankruptcy is an endogenous decision of the firm's manager. On top of that the firm's manager can choose between Ch. 11 reorganization versus Ch. 7 liquidation following the Federal Bankruptcy Reform Act of 1978:

Ch. 11 Reorganization. The firm must pay fixed costs $f_{c11} \in \mathcal{R}^+$ and variable costs $(1 - s_{c11})(1 - \delta)k$ where $s_{c11} \in [0, 1]$ is reorganization efficiency. A fraction of the debt is repaid to creditors. The firm continues to operate. The manager's continuation value decreases at the rate of $\pi \in [0, 1]$. The parameter π captures the loss of the continuation value under Ch. 11 bankruptcy. The data show a large drop in shareholder's value during Ch. 11 bankruptcy.

Ch. 7 Liquidation. The firm's assets are liquidated. After repayment to creditors, the salvage value, $s_{c7}k$, is distributed to creditors and shareholders. $s_{c7} \in [0,1]$ is liquidation efficiency.

2.5 Information Structure

The manager's transitory preference shocks and the firm's persistent productivity are private information.

Assumption 1 (Information Asymmetry) Model parameters, technologies, and information sets are public knowledge. The firm's manager (insider) knows all information associated with oneself. In contrast, bondholders, bank lenders, and a representative household (outsiders) cannot observe action-specific transitory preference shocks. Persistent productivity is private information in the absence of monitoring.

2.6 Monitoring Technology, Restructuring Technology, and Intermediation Costs

Two financial institutions exist in the economy: bondholders and bank lenders. Three assumptions explain why firms choose debts between these financiers.

First, bank lenders are specialized in monitoring in the spirit of Diamond (1984); Holmstrom and Tirole (1997) (Assumption 2). The theoretical idea behind these papers is that a single large bank is more efficient in financing projects than each investor paying a cost to monitor. Gustafson et al. (2020) find that the empirical likelihood of monitoring is positively related to the lead bank's share of syndicated loans.

Assumption 2 (Monitoring Technology) Bank lender is a monitoring lender. On the other hand, bondholders supply debt to borrowers and a representative household owns shares of stock, both without monitoring.

The information problem arises because bondholders cannot observe action-specific transitory utility shocks nor the firm's persistent productivity z. Transitory preference shocks hinder learning about the true type. Bondholders Bayesian infer the firm's type from publicly observed choices.¹⁸

Second, bank debt has higher intermediation costs than market debt (Assumption 3). This assumption helps to explain why the safest firms issue corporate bonds. If firms are far away from bankruptcy, market debt is less expensive than bank debt to finance their projects.

Assumption 3 (Intermediation Costs) The intermediation costs of bank lenders μ_B are greater than the intermediation costs of bondholders μ_M .

This is a reduced form fashion to capture all other costs which are not explicitly modeled in this paper. Philippon (2015) provides comprehensive evidence of intermediation costs, and evaluates the difference in intermediation costs as an annual spread of 2%. At least three factors to explain the positive spread ($\mu_B - \mu_M > 0$). First factor is liquidity. Bank loans are known to be more illiquid than corporate bonds. Second factor is the regulatory

¹⁸I do not allow uninformed outsiders (bondholders and a representative household) to use information in the cash flow statement (e.g., operating income and equity payouts) for direct inference of the firm's productivity. For example, observable equity payouts have no one-to-one relationship to production function since total equity payout is the sum of dividends, repurchases, retained earnings, and seasoned equity issuance. One issue arises from the measurement problem and others arise from data limitation.

treatment (e.g., capital requirement, liquidity coverage ratio) of bank lenders.¹⁹ Last factor is the banks do not pass-through their funding costs to firms.

Third, the degree of debt enforcement under Ch. 11 bankruptcy is stronger for bank loans than bonds.

Assumption 4 (Threat of Liquidation to Restructure Bank Debt) Bargaining takes place to restructure bank debt under Ch. 11 bankruptcy with the threat of liquidation.

Assumption 4 follows the empirical characterization of bank debt offering greater flexibility in times of financial distress (Lummer and McConnell, 1989; Gilson et al., 1990; Asquith et al., 1994). On the contrary, bondholders receive residual cash flow under Ch. 11 bankruptcy. There are two reasons for limited participation in the bargaining process for market debt restructuring. First, dispersed bondholders have little incentive to participate in the bargaining game due to a free-rider problem, as in Bolton and Scharfstein (1996).²⁰ Second, the Trust Indenture Act (TIA) of 1939 limits the ability of bondholders and issuers to bargain over debt structure using the threat of liquidation.²¹ These problems stem from the lack of an agent to coordinate and enforce collective decisions. This assumption of limited participation in the bargaining process is in line with notions of "asset-based debt" and "cash flow-based debt". Since the majority of corporate bonds are cash flow-based lending (Lian and Ma, 2020), the value of debt is driven by the cash flow value. This is also similar to the methodology used in practice to evaluate market debt recovery rates at default. EBITDA multiple valuation approach, the assumption that enterprise value is equal to the proxy for company's continuing EBITDA times EBITDA multiple, is the common approach for recovery analysis. Conversely, the primary focus of bank debt is the liquidation value of physical assets.

2.7 Discrete State Space, Choice Set, and Extreme Value Shocks

The observable state and action spaces for bank lenders are $\omega_B = (e, z, s, b, \phi, e') \in \Omega_B \equiv \mathcal{E} \times \mathcal{Z} \times \mathcal{S} \times \mathcal{B} \times \Phi \times \mathcal{E}$ where Ω_B is an information set of bank lenders. Each variable is an

 $^{^{19}}$ Moreover, Graham (1999) points out that the tax disadvantage of the lender's organizational form is creating higher cost of private debt.

²⁰A microfounded model of the limited ability of participation is in Bolton and Freixas (2000).

²¹TIA of 1939 is a law to protect bondholders. In practice, the bond's principal and interest payments should not be impaired without unanimous bondholder consent. TIA prohibits collective action clauses (CACs) in bond contracts which allow a majority of bondholders to agree to cancel a portion or all of the outstanding debt. CACs are frequently included in bonds issued in Luxembourg and United Kingdom, and the amendments of Chilean and German laws in 2000s.

element of a set of discrete grid points $(e \in \mathcal{E}, z \in \mathcal{Z}, s \in \mathcal{S}, b \in \mathcal{B}, \text{ and } \phi \in \Phi)$. The sets of discrete grid points are $\mathcal{E} \equiv \{e_1, e_2, \dots, e_{N_e}\}$ for equity which consist of share offerings and retained earnings, $\mathcal{Z} \equiv \{z_1, z_2, \dots, z_{N_z}\}$ for productivity, and $\mathcal{B} \equiv \{b_1, b_2, \dots, b_{N_b}\}$ for debt outstanding, where N_e , N_z , and N_b are numbers of grid points. The set of debt types consists of corporate bonds $(\phi = M)$ and bank loans $(\phi = B)$.

In each period, the firm's manager chooses debt outstanding $b \geq 0$, composition of debt ϕ , next period equity e', and then bankruptcy decisions $\Delta \in \{0,1\}$ where $\Delta = 0$ ($\Delta = 1$) represents non-bankruptcy (bankruptcy). When the firm files bankruptcy, the firm's manager chooses filing for Ch. 11 or Ch. 7 bankruptcy.

I assume that productivity follows a symmetric two-state Markov chain ($N_z = 2$, see Appendix D.1). I denote the low (high) productivity by z_L (z_H). The model generates approximately equal numbers of low and high productivity types. The stationary distribution has fewer low productivity firms. Because endogenous exiting via Ch. 7 bankruptcy is more likely to happen in the state of low productivity.

Type score $s \equiv \Pr(z_H)$ is the subjective probability of the high productivity z_H firm (details are in Appendix B.1). The space of type score is defined on the real line $s \in \mathcal{S} = [0, 1]$ when $N_z = 2$. Appendix C.4 contains details about the settings in numerical exercises.

Price menus q_M and q_B depend on observable firm's characteristics and action spaces. I define the complementary notation of observable variables for bondholders as $\omega_M = (e, s, b, \phi, e') \in \Omega_M \equiv \mathcal{E} \times \mathcal{S} \times \mathcal{B} \times \Phi \times \mathcal{E}$. This immediately leads to $\Omega_M \subset \Omega_B$, i.e. the bank has more information than bondholders (Assumption 2). Debt price is contingent on $\overline{\omega}_{\phi} \equiv \Omega_{\phi} \setminus \Phi$ for type $\phi \in \{M, B\}$. Finally, the current state variables for the firm's manager are $\omega = (e, z, s) \in \Omega \equiv \mathcal{E} \times \mathcal{Z} \times \mathcal{S}$ (see the timing in Section 2.8).

Finally, extreme value shocks $\varepsilon_{b,\phi,e'}$ (ε_{Δ}) are conditional on action of debt b, the type of debt instrument ϕ , and equity e' (bankruptcy Δ). Transitory utility shocks $\varepsilon_{b,\phi,e'}$ and ε_{Δ} are drawn from i.i.d. Type-1 Extreme Value distribution with mean zero and scale parameters $1/\alpha$ and $1/\alpha_{\Delta}$.²² Therefore, $\varepsilon_{b,\phi,e'}$ and ε_{Δ} are all correlated with actions but independent to each other. Scale parameters are transformed into unit costs of information in the framework of rational inattention by Matějka and McKay (2015). For more details about Generalized Type-I Extreme Value distribution (Gumbel distribution), see Appendix B.2. Without these action specific disturbances, there is no inference problem to solve (reputation building becomes a perfect substitute for monitoring).

²²The value of the firm's manager depends on the mean of type 1 extreme value distribution since the outside option is firm exiting. The constant level shift of the firm's manager utility is absorbed into fixed costs for production f and fixed costs for Ch. 11 bankruptcy f_{c11} .

2.8 Timing

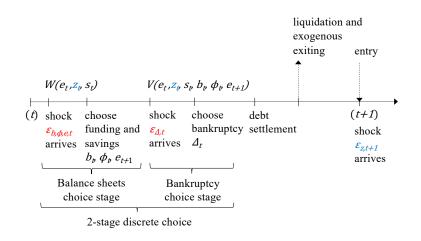
The model period is subdivided into two stages: balance sheets choice stage (List 3) and bankruptcy choice stage (List 4). Figure 1 shows the timing assumption graphically in the model.

- 1. Firm begins the period with state vector (e, z, s).
- 2. Financial Institutions (FIs) payback $q^{-1}b$ and collect b' from households.
- 3. Balance sheets choice stage
 - (a) The manager learns action specific transitory utility shocks $\varepsilon_{b,\phi,e'}$. The manager chooses how much to borrow b, the type of debt instrument ϕ , and equity e' for the end of the period (retained earning or issuance/repurchase equity).
 - (b) Firm borrows b from market or bank lenders and purchases capital k.
- 4. Bankruptcy choice stage²³
 - (a) The manager learns action specific transitory utility shocks ε_{Δ} . The manager declares bankruptcy or not.
 - (b) Continuing firms produce. Then, firm sells capital k and repays debt $q_M^{-1}b$ or $q_B^{-1}b$ to bondholders and bank lenders in debt settlement.²⁴ Firm who files Ch. 11 bankruptcy pays fixed and variable costs.
 - (c) Pay dividends or issue equity.
- 5. Exogenous exiting happens at rate χ and endogenous exiting via Ch. 7 bankruptcy happens at the end of the period.
- 6. Based on each individual firm's actions in the current period, bondholders and share-holders Bayesian update next period type scores s'.
- 7. The idiosyncratic productivity shock arrives ε'_z , and it follows to the next period's productivity level z'.

 $^{^{23}}$ The model takes into account strategic default which includes continuation value under Ch. 11 bankruptcy. On the contrary, the models of Crouzet (2017) and Xiao (2019) assume that the firm defaults when cash flow falls to negative.

²⁴Since liquidation and reorganization occur interperiod, FIs keep repaid cash until the next period.

Figure 1: Timing of Individual Firm's Manager Problem



2.9 Firm Manager's Discrete Choice Problem

My model belongs to a class of dynamic discrete choice models. The manager's preference is specified using a random utility model. The conditional value functions of balance sheet choice stage $(W(\omega))$ and bankruptcy choice stage $(V(\omega_B))$ are

$$W(\omega) = \mathbb{E}_{\varepsilon_{b,\phi,e'}|\omega} \left[\max_{(b,\phi,e')\in\{\mathcal{B}\times\Phi\times\mathcal{E}\}} \{V(\omega_B) + \varepsilon_{b,\phi,e'}\} \right]$$
(1)

$$V(\omega_B) = \mathbb{E}_{\varepsilon_{\Delta}|\omega_B, \varepsilon_{b,\phi,e'}} \left[\max_{\hat{\Delta} \in \{0,1\}} \{ v_{\hat{\Delta}}(\omega_B) + \varepsilon_{\hat{\Delta}} \} \right]$$
 (2)

with an interim stage for debt restructuring:

$$v_{\Delta=1}(w_B) = \max\{v_{c11}(\omega_B), v_{c7}(\omega_B)\}$$
(3)

where v_{c11} (v_{c7}) captures the conditional value of Ch. 11 (Ch. 7) bankruptcy. These preference shocks give closed-forms of equations (1-2) introduced by McFadden et al. (1973) and Rust (1987).

$$W(\omega) = \frac{1}{\alpha} \ln \left(\sum_{(b,\phi,e')\in\{\mathcal{B}\times\Phi\times\mathcal{E}\}} \exp(\alpha V(\omega_B)) \right)$$
 (4)

$$V(\omega_B) = \frac{1}{\alpha_\Delta} \ln \left(\sum_{\hat{\Delta} \in \{0,1\}} \exp(\alpha_\Delta v_{\hat{\Delta}}(\omega_B)) \right)$$
 (5)

I separate the systematic state variables ω and ω_B from the idiosyncratic variables $\varepsilon_{b,\phi,e'}$ and ε_{Δ} following Rust (1987).

Define balance sheets decision rules $\sigma_{\omega_B}(\omega_B): \Omega_B \to [0,1]$:

$$\sigma_{\omega_B}(\omega_B) = \frac{\exp(\alpha V(\omega_B))}{\sum_{(b,\phi,e')\in\{\mathcal{B}\times\Phi\times\mathcal{E}\}} \exp(\alpha V(\omega_B))}$$
 (6)

and bankruptcy decision rules conditional on balance sheets choice $\sigma_{\Delta|\omega_B}(\omega_B): \Omega_B \times \{0,1\} \to 0$ [0, 1]:

$$\sigma_{\Delta|\omega_B}(\omega_B) \equiv \frac{\exp(\alpha_\Delta v_\Delta(\omega_B))}{\sum_{\hat{\Delta}\in\{0,1\}} \exp(\alpha_\Delta v_{\hat{\Delta}}(\omega_B))}$$
(7)

Unconditional bankruptcy decision rules can be derived from iterated law of expectation: $\sigma_{\Delta=1}(\omega_B) \equiv \sigma_{\Delta=1|\omega_B}\sigma_{\omega_B}$ where $\sigma_{\Delta=1|\omega_B} = \sum_{y\in\{c11,c7\}}\sigma_{y|\omega_B}$. $\sigma_{y|\omega_B}$ are decision rules for Ch. 11 and Ch. 7 bankruptcy in the sub-game stage such that $\sigma_{y|\omega_B}(\omega_B):\Omega_B\to[0,1]$. $y \in \{c11, c7\}$ is an index of Ch. 11 and Ch. 7 bankruptcies. $\sigma_{y|\omega_B}(\omega_B)$ is defined as $\sigma_{c11|\omega_B}(\omega_B) \equiv \sigma_{\Delta=1|\omega_B} \mathbb{1}_{v_{c11}(\omega_B) \ge v_{c7}(\omega_B)} \text{ and } \sigma_{c7|\omega_B}(\omega_B) \equiv \sigma_{\Delta=1|\omega_B} \mathbb{1}_{v_{c11}(\omega_B) < v_{c7}(\omega_B)}.$

Non-bankruptcy Firm. Corporate investment can be financed with either internal funds generated by operating income, debt issuance, or equity issuance. The continuing firm's manager receives utility:

$$v_{\Delta=0}(\omega_B) = d_{\Delta=0}(\omega_B) - \lambda(d_{\Delta=0}(\omega_B)) + \underbrace{\eta \chi e'}_{\text{exogeneous liquidation}} + (1-\eta)q \sum_{(z',s')\in\{\mathcal{Z}\times\mathcal{S}\}} g_z(z'|z)g_s(s'|\omega_M, \Delta=0)W(e',z',s')$$
(8)

subject to

$$k = b + e \tag{9}$$

$$d_{\Delta=0}(\omega_B) = \Pi_{\Delta=0}(k,z) - \underbrace{\mathcal{R}_{\phi}^{(\Delta=0)}(\overline{\omega}_{\phi})}_{\text{repayment to lenders}} -e'$$
(10)

$$\Pi_{\Delta=0}(k,z) = \Pi_k(k,z) + (1-\delta)k$$
 (11)

$$\mathcal{R}_{\phi}^{(\Delta=0)}(\overline{\omega}_{\phi}) = q_{\phi}(\overline{\omega}_{\phi})^{-1}b \tag{12}$$

The balance sheet equates assets k to debt b and equity e (equation (9)). The equity payout in equation (10) is the firm's operating profit minus repayment to lenders and next period savings. Dividends correspond to the positive equity payout $d_{\Delta=0} > 0$. As a result, I do not allow firms to pay dividends and issue equity simultaneously. Equation (11) is the firm's operating profit. Repayment to the creditor is described in equation (12), which is equal to the book value of debt $q_{\phi}(\overline{\omega}_{\phi})^{-1}b$ where $q_{\phi}(\overline{\omega}_{\phi})^{-1}$ is the gross returns of debt of type ϕ . On the RHS of the interperiod Bellman equation (8), I take into account all of the possible combinations of (i) the next period productivity level z' which exogenously follows a 1st-order Markov process with transition $g_z(z'|z)$ and (ii) the next period type scores s' which endogenously follows the Bayesian updating process $g_s(s'|\omega_M, \Delta=0)$. I explain the Bayesian updating process in Section 2.11.

Ch. 11 Bankruptcy Firm. If the firm declares Ch. 11 bankruptcy, the manager receives utility:

$$v_{c11}(\omega_B; \varphi_B) = d_{c11}(\omega_B; \varphi_B) - \lambda(d_{c11}(\omega_B; \varphi_B)) + \underbrace{\eta \chi e'}_{\text{exogeneous liquidation}}$$

$$+ (1 - \eta)(1 - \pi)q \sum_{(z', s') \in \{\mathcal{Z} \times \mathcal{S}\}} g_z(z'|z)g_s(s'|\omega_M, \Delta = 1) \underbrace{W(e', z', s')}_{\text{going-concern value}}$$
(13)

subject to k = b + e and

$$d_{c11}(\omega_{B}; \varphi_{B}) = \Pi_{c11}(z, k, b) - \mathcal{R}_{\phi}^{(c11)}(\overline{\omega}_{B}) - e'$$

$$\Pi_{c11}(k, z) = \Pi_{k}(k, z) + s_{c11}(1 - \delta)k - f_{c11}$$

$$\mathcal{R}_{M}^{(c11)}(\overline{\omega}_{B}) = \min\{q_{M}(\overline{\omega}_{M})^{-1}b, \max\{\Pi_{c11}(k, z), 0\}\}$$

$$\mathcal{R}_{B}^{(c11)}(\overline{\omega}_{B}) = \varphi_{B}^{(c11)}(\overline{\omega}_{B})q_{B}(\overline{\omega}_{B})^{-1}b$$
(15)

Equation (14) is the repayment to bondholders. The debtor paybacks debt unless the firm's operating profit is less than the debt claim. The repayment to the bank lender in equation (15) depends on recovery rates at default $\varphi_B^{(c11)} \in [0,1]$. Recovery rates at default are determined in the sub-game stage of one-shot bargaining as described in Section 2.10. The threat points of this offer are: if there is no agreement, the borrower files Ch. 7 bankruptcy. In the U.S. law, the court may convert Ch. 11 to Ch. 7 bankruptcy if the reorganization plan is not confirmed by creditors. In this instance, Absolute Priority Rule (APR) might be violated since shareholders are paid while debtors are not paid in full. APR violation is very

common in Ch. 11 bankruptcy (e.g., Betker (1995)).

The firm's manager and shareholders incur costs associated with Ch. 11 bankruptcy from three different sources. First, the manager's continuation value of the next period declines exogenously by the factor of π and changes endogenously through the belief updating function g_s . A strictly positive parameter $\pi > 0$ discourages the firm's manager from choosing Ch. 11 bankruptcy. Second, the firm loses $1 - s_{c11}$ of the value of the asset from reorganization. Third, Ch. 11 bankruptcy creates fixed financial distress costs f_{c11} . These static costs capture legal costs and other costs regarding Ch. 11 bankruptcy. On the other hand, the benefit of Ch. 11 bankruptcy arises from Assumption 4.

Ch. 7 Bankruptcy Firm. When filing for Ch.7 bankruptcy, the firm's manager receives utility:

$$v_{c7}(\omega_B) = d_{c7}(\omega_B) - \lambda(d_{c7}(\omega_B)) \tag{16}$$

subject to k = b + e and

$$d_{c7}(\omega_B) = \Pi_{c7}(k) - \mathcal{R}_{\phi}^{(c7)}(\overline{\omega}_{\phi})$$

$$\Pi_{c7}(k) = s_{c7}k$$
(17)

$$\mathcal{R}_{\phi}^{(c7)}(\overline{\omega}_{\phi}) = \min\{q_{\phi}(\overline{\omega}_{\phi})^{-1}b, \max\{\Pi_{c7}(k), 0\}\}$$

$$\tag{18}$$

where s_{c7} is a parameter for liquidation efficiency. The available funds to repay debt are $\Pi_{c7}(k)$ (equation 18), and funds are linearly increasing in total assets k. There is neither production nor a continuation value since Ch. 7 bankruptcy ceases business operations.

2.10 Bank Debt Recovery Rates at Default $\varphi_B^{(c11)}$ under Ch. 11 Bankruptcy

When the firm decides to renegotiate bank debt under Ch. 11 bankruptcy, the manager enters into a bargaining stage with bank lenders (Danis and Gamba, 2018; Corbae and D'Erasmo, 2020). As in the model of the incomplete contracting problem (Hart and Moore, 1998), the threat of liquidation plays a central role in determining the outcome of debt renegotiations. The borrower offers a take-it-or-leave-it offer to the bank lender, and the manager captures the full surplus.

Proposition 1 (Recovery Rate) The borrower makes a take-it-or-leave-it offer of recovery at default to the bank lender $\mathcal{R}_B^{(c11)}(\overline{\omega}_B) = \mathcal{R}_B^{(c7)}(\overline{\omega}_B)$.

Recovery rates at default of bank debt is

$$\varphi_B^{(c11)}(\overline{\omega}_B) = \frac{\mathcal{R}_B^{(c7)}(\overline{\omega}_B)}{q_B(\overline{\omega}_B)^{-1}b}$$

$$= \min\left\{1, \max\left\{\frac{s_{c7}k}{q_B(\overline{\omega}_B)^{-1}b}, 0\right\}\right\}$$
(19)

from Proposition 1 and equations (15) and (17). Full bargaining power is not only for simplifying computation and the theoretical model of asymmetric information but also consistent with recovery rates in the data.

2.11 Bondholders and Bank Lenders Problem

In this section, I will first explain the type score updating process and then the two types of debt markets.

Bayesian Inference Problem of Uninformed Investors. Type score s has two roles. First, bondholders use type score s to price their lending. Second, the Bayesian updating function of type score $g_s(s'|\omega_M, \Delta)$ changes continuation values in equations (8) and (13). I define the type score updating function such that $\psi_z(\omega_M, \Delta) : \Omega_M \times \{0, 1\} \times \mathcal{Z} \to [0, 1]$ is the *current* period posterior probability distribution of z conditional on firm's observable choices and states (ω_M and Δ). Bayes' rule yields the posterior density:

$$\underbrace{\psi_{z}(\omega_{M}, \Delta)}_{\text{Posterior}} = \underbrace{\frac{\sigma_{\omega_{B}}(\omega_{B})\sigma_{\Delta|\omega_{B}}(\omega_{B})\mathbb{1}_{\text{continuing}}}_{\hat{z}} s(z)}_{\text{Marginal likelihood}} s(\hat{z})$$
(20)

where $\hat{\omega}_B = (e, \hat{z}, s, b, \phi, e')$ and $\mathbb{1}_{\text{continuing}} \equiv (1 - \mathbb{1}_{\Delta=1, y=c7})$ an indicator function of continuing firm, the prior density s(z) = 1 - s (s(z) = s) if $z = z_L$ ($z = z_H$). $s(z) \equiv \Pr(z)$ is a subjective belief on the productivity level z. The likelihood depends on equilibrium decision rules in equations (6) and (7). Investors update their priors from new information of debt outstanding b, debt type ϕ , the next period internal finance e', and bankruptcy choice Δ . The firm's manager understands the Bayesian updating belief and their choice sends a signal

to uninformed lenders.

The next period posterior $\psi_{z'}(\omega_M, \Delta) : \Omega_M \times \{0, 1\} \times \mathcal{Z} \to [\underline{s}, \overline{s}]$ is an expectation of the current period posterior:

$$\psi_{z'}(\omega_M, \Delta) = \sum_{z \in \mathcal{Z}} g(z'|z)\psi_z(\omega_M, \Delta)$$
(21)

The transition probability of a symmetric two-state Markov process of productivity z gives the lower (upper) bound $\underline{s} = g_z(z_H|z_L)$ ($\overline{s} = g_z(z_H|z_H)$) (see Section D.1). This implies that the firm's type is not perfectly revealed since productivity shocks arrive after uninformed investors update their beliefs. For numerically solving the model, I discretize the codomain of the type score function $[\underline{s}, \overline{s}]$ to $\{s_1, s_2, \ldots, s_{N_s}\}$ where N_s is the number of grid points. $s_1 \equiv \underline{s}$ and $s_{N_s} \equiv \overline{s}$ are the lower and upper bounds of type score. To map between continuous values of the next period posterior $\psi_{z'}(\omega_M, \Delta)$ to discrete values of type score s, I apply a linear interpolation to compute the transition probability $g_s(s'|\omega_M, \Delta)$ in equations (8) and (13).²⁵

Market Debt Pricing. Bondholders provide defaultable one-period bonds to incumbent firms and new entrants. In the perfectively competitive corporate bond markets with free entry, bondholders earn expected zero profit,

$$\underbrace{(1 + \mu_{M})b}_{\text{Funding costs}} = \underbrace{q \left\{ (1 - \Lambda_{M}(\overline{\omega}_{M}))q_{M}(\overline{\omega}_{M})^{-1}b + \mathbb{E}_{(z,y)}[\mathcal{R}_{M}^{(y)}(\overline{\omega}_{B})|\Omega_{M}] \right\}}_{\text{Discounted return of market debt}}$$

$$\Leftrightarrow q_{M}(\overline{\omega}_{M}) = \underbrace{\frac{(1 - \Lambda_{M}(\overline{\omega}_{M}))b}{(1 - \Lambda_{M}(\overline{\omega}_{M}))b}}_{\text{Expected PD}}$$

$$(22)$$

where μ_M is intermediation costs of bondholders. The debt price depends on two factors: expected recovery at default in the denominator and expected Probability of Default (PD) in the numerator. Expected recovery at default is

²⁵To be more concrete, first, I find adjacent grid points $s_i \leq \psi_{z'}(\omega_M, \Delta) < s_{i+1}$ for each $z' \in \mathcal{Z}$. Second, assigned weights $\chi(\omega_M, \Delta, z')$ to s_i and $1 - \chi(\omega_M, \Delta, z')$ to s_{i+1} , where the weight function is $\chi(\omega_M, \Delta, z') = \frac{s_{i+1} - \psi_{z'}(\omega_M, \Delta)}{s_{i+1} - s_i}$.

$$\mathbb{E}_{(z,y)}[\mathcal{R}_{M}^{(y)}(\overline{\omega}_{B})|\Omega_{M}] \equiv \sum_{z \in \mathcal{Z}, y \in \{c11, c7\}} \sigma_{y}(\omega_{B})|_{\phi=M} \mathcal{R}_{M}^{(y)}(\overline{\omega}_{B}) s(z)$$

$$= \sum_{z \in \mathcal{Z}, y \in \{c11, c7\}} \sigma_{y}(\omega_{B})|_{\phi=M} \min\{q_{M}(\overline{\omega}_{M})^{-1}b, \max\{\Pi_{y}, 0\}\} s(z) (23)$$

where s(z) is a probability of type z defined in Section 2.11. The second line in equation (23) are derived from equations (15) and (18). The following proposition describes cross-subsidization from high to low productivity firms.

Proposition 2 (Cross-subsidization of Expected Recovery at Default) Expected recovery at default is lower (higher) than realized recovery at default for the high (low) productivity type.

This proposition is true because the high productivity type has a higher operating profit Π_{c11} than the low productivity type under Ch. 11 bankruptcy $(\Pi_{c11}(k, z_H) > \Pi_{c11}(k, z_L))$.

Expected Probability of Default (PD), the potential for cross-subsidization from the high to low productivity firm, is

$$\Lambda_M(\overline{\omega}_M) \equiv \sum_{z \in \mathcal{Z}} \sigma_{\Delta=1|\omega_B}(\omega_M)|_{\phi=M} s(z)$$
 (24)

Note that equation (22) has no closed-form solution for $q_M(\overline{\omega}_M)$ since the RHS's expected recovery at default also depends on $q_M(\overline{\omega}_M)$ (equation (23)). To avoid complicated numerical analysis to solve for $q_M(\overline{\omega}_M)$, Section C.1 introduces an approximation to derive a closed-form solution for $q_M(\overline{\omega}_M)$. The intuition is that credit spreads have a small impact on recovery at default.

Bank Debt Pricing. The price of bank debt is pinned down by the expected zero profit condition of the perfectly competitive FIs under the free entry:

$$\underbrace{(1 + \mu_B)b}_{\text{Funding costs}} = \underbrace{q\left\{(1 - \Lambda_B(\omega_B))q_B(\overline{\omega}_B)^{-1}b + \mathbb{E}_y[\mathcal{R}_B^{(y)}(\overline{\omega}_B)|\Omega_B]\right\}}_{\text{Discounted return of bank debt}}$$

$$\Leftrightarrow q_B(\overline{\omega}_B) = \underbrace{(1 - \overline{\Lambda_B(\overline{\omega}_B)})b}_{\text{Recovery at default}} \tag{25}$$

where μ_B is intermediation costs of bank lenders. In contrast to public debt pricing, private debt is contingent on the productivity level z. The expected repayment from the borrower to the informed lender is

$$\mathbb{E}_{y}[\mathcal{R}_{B}^{(y)}(\overline{\omega}_{B})|\Omega_{B}] \equiv \sum_{y \in \{c11,c7\}} \sigma_{y}(\omega_{B})|_{\phi=B} \mathcal{R}_{B}^{(y)}(\overline{\omega}_{B})$$

$$= \sum_{y \in \{c11,c7\}} \sigma_{y}(\omega_{B})|_{\phi=B} \min\{q_{B}(\overline{\omega}_{B})^{-1}b, \max\{\Pi_{y},0\}\}\}$$

$$= \Lambda_{B}(\overline{\omega}_{B}) \min\{q_{B}(\overline{\omega}_{B})^{-1}b, \max\{s_{c7}k,0\}\}$$

The third line shows recovery at default depends on the total assets k as a result of debt renegotiation from Proposition 1. Liquidation efficiency parameter s_{c7} under Ch. 7 bankruptcy is the key determinant of recovery rates at default. PD is $\Lambda_B(\overline{\omega}_B) \equiv \sum_{y \in \{c11, c7\}} \sigma_y(\omega_B)|_{\phi=B}$.

2.12 Household Problem

The problem of the representative household is standard: the representative household maximizes the utility subject to budget constraint with one-period risk-free bonds and shareholdings. Labor supply is inelastic since there is no disutility of labor. The representative household's risk aversion plays no role in pricing because there is no aggregate uncertainty in the economy. I describe the representative household's utility maximization problem in Appendix B.7.

2.13 Cross-sectional Firm Distribution

The cross-sectional distribution of firms characterizes the aggregate variables in the economy. The transition from the current period to the next period distribution is

$$\Gamma(\omega') = \sum_{(e,z,s)\in\{\mathcal{E},\mathcal{Z},\mathcal{S}\}} \sum_{(b,\phi)\in\{\mathcal{B}\times\Phi\}} \sum_{\Delta\in\{0,1\},y\in\{c11,c7\}} \times (1-\eta)g_z(z'|z)g_s(s'|\omega_M,\Delta)\sigma_{\omega_B}(\omega_B)\sigma_{\Delta|\omega_B}(\omega_B)\mathbb{1}_{\text{continuing}} \times (\Gamma(\omega) + \underbrace{\mathcal{M}_n\Gamma_n(\omega)}_{t,t,t})$$
(26)

The second and third lines show the density of firms continuing to the next period. $\Gamma_{\rm n}(\omega)$ in the third line is the distribution function of new entrants, which is normalized to one (i.e., $\sum_{\omega} \Gamma_{\rm n}(\omega) = 1$). Therefore, $\mathcal{M}_{\rm n}$ is the mass of entrants. New entrants are endowed with the lowest equity $\underline{e} = \min \mathcal{E}$ which corresponds to the fact that new entrants are typically small. They draw productivity from its ergodic distribution of productivity $\bar{g}(z)$ with no track record at the beginning (Diamond, 1989). The type score of the new entrants is consistent with the productivity distribution s.t. $s(z) = \bar{g}(z)$.

2.14 Stationary Recursive Competitive Equilibrium

I focus on a stationary recursive competitive equilibrium. The equilibrium debt prices are determined under an equilibrium.

Definition 1 (Stationary Recursive Competitive Equilibrium) A stationary recursive competitive equilibrium is a list of value function W, pricing functions and recovery rates $\{q, q_M, q_B, \varphi_B^{(c11)}\}$, the type score updating function ψ_z , decision rules $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$, the stationary distribution Γ, and mass of new entrants \mathcal{M}_n such that:

- 1. Given the price functions and recovery rates $\{q, q_M, q_B, \varphi_B^{(c11)}\}$, and the type score updating function ψ_z , the value function W is consistent with the firm's manager decision rules $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$ as a result of the optimization problem in equation (1).
- 2. Given the price functions $\{q, q_M, q_B\}$, recovery rates $\varphi_B^{(c11)}$ are determined by equation (19).
- 3. The equilibrium risky debt price schedules $\{q_M, q_B\}$ are determined by equations (22) and (25) where lenders earn zero profits in expectation on each contract. The risk-free debt price q is equal to a discount factor β (equation (44)).
- 4. The type scoring updating function ψ_z satisfies Bayesian learning equation (20), and $\psi_{z'}$ follows equation (21) given firm's manager policy functions in List 1.²⁶
- 5. The mass of new entrants \mathcal{M}_n is consistent with the evolution of the distribution equation (26) and the exogenous labor supply equation (27).

 $^{^{26}}$ The firm's manager is atomistic due to the continuum assumption such that the manager does not affect the type scoring updating function. This is the analog of "Big K, little k" trick in the macroeconomics literature.

6. Γ is an invariant distribution with respect to the transition of states in equation (26) given the type score updating function ψ_z , decision rules $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$, and the mass of new entrants \mathcal{M}_n .

Lists 1 and 3 are the optimality conditions of firms and lenders. List 4 explains that uninformed investors' beliefs are consistent with equilibrium outcomes of decision rules $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$. Theorem 3 in Appendix B.6 proves the existence of a stationary recursive competitive equilibrium.²⁷ The key insight is that manager's choices are subject to extreme value shocks to utility. Therefore, there is no need to take into account off-the-equilibrium-beliefs since all actions are taken with non-zero probability. Beliefs are consistent since all information sets are reached under the strategy profile $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$. Moreover, this helps the posterior of Bayesian inference to be finite (unless the denominator of the posterior becomes zero for some choice).

2.15 Aggregates

It is straightforward to characterize the aggregate quantities in the economy from the invariant distribution and the law of motion. Each firm demands one unit of labor. Since the representative household has no disutility of labor, aggregate labor supply is inelastic. Therefore, the aggregate labor demand is equal to the total mass of firms:

$$1 = \sum_{(e,z,s)\in\{\mathcal{E},\mathcal{Z},\mathcal{S}\}} \Gamma(\omega) \tag{27}$$

I normalize the total mass of households to one. Aggregate one-period market debt ($\phi = M$) and bank debt ($\phi = B$) are

$$Debt(\phi \in \{M, B\}) = \sum_{(e, z, s) \in \{\mathcal{E}, \mathcal{Z}, \mathcal{S}\}} \sum_{(b, e') \in \{\mathcal{B} \times \mathcal{E}\}} \sigma_{\omega_B}(\omega) \mathbb{1}_{\phi \in \{M, B\}} b\Gamma(\omega)$$

Aggregate bank debt ratio is $\operatorname{Debt}(\phi = B)/(\operatorname{Debt}(\phi = M) + \operatorname{Debt}(\phi = B))$. Aggregate equity is $\sum_{(e,z,s)\in\{\mathcal{E},\mathcal{Z},\mathcal{S}\}} e\Gamma(\omega)$. Aggregate capital is equal to the sum of aggregate debts and aggregate equity. Aggregate output is

 $^{^{27}}$ In order to eliminate off-the-equilibrium beliefs, preference shocks conditional on bankruptcy chapter choices need to be introduced in the theoretical model. The difficulty of separately identifying the scale parameter of these preference shocks and Ch. 11 bankruptcy costs f_{c11} , I assume that these extreme value shocks are infinitesimally small in the quantitative model. Then, the firm's decision collapses into a simple rule maximizing the value of Ch. 11 and Ch. 7 bankruptcies as in equation (3).

Output =
$$\sum_{\omega_B \in \Omega_B} \sum_{\Delta \in \{0,1\}} \sigma_{\omega_B}(\omega) \sigma_{\Delta|\omega_B}(\omega_B) \mathbb{1}_{\text{continuing}} F(b+e,z) \Gamma(\omega)$$
 (28)

From the representative household's budget constraint of equation (41) in Appendix B.7, the integration of equation (42), and stationarity of aggregate variables, aggregate consumption is

Consumption
$$= \sum_{\omega_{B} \in \Omega_{B}} \sum_{\Delta \in \{0,1\}} \sigma_{\omega_{B}}(\omega) \sigma_{\Delta|\omega_{B}}(\omega_{B})$$

$$\times \left[\underbrace{\left(1 - \mathbb{1}_{\Delta=1, y=c7}\right) \left(1 - \pi \mathbb{1}_{\Delta=1, y=c11}\right) \left\{ \left(1 - \eta\right) p + \eta \chi e' \right\} - \left\{ p - \left(d - \lambda(d)\right) \right\}}_{\text{net returns of equity}} + \underbrace{rb}_{\text{net returns of debt}} \right) \right] \Gamma(\omega)$$

where p is the cum-dividend price of equity per share (equation (43)). The household consumes the sum of net returns of equity and debt.

3 Model Estimation

The numerical algorithm to compute stationary recursive competitive equilibrium is explained in Appendix C.2.

To generate simulated data comparable to real-world data, I define the price of one-period risk-free debt, bonds, loans as $r \equiv q^{-1} - 1$, $r_M \equiv q_M^{-1} - 1$, and $r_B \equiv q_B^{-1} - 1$, where r, r_M , and r_B , are net returns of one-period risk-free assets, bonds, and loans. Definition of model variables (i.e., recovery rates at default, dividends/total assets, debt-to-assets ratio, and equity issuance/total assets) is discussed in Appendix B.8.

3.1 Selected and Estimated Parameters

The parameters can be divided into two groups. The first group is a set of parameters selected outside the model (the first panel of Table 1). The parameters of the productivity process ρ and σ are matched to İmrohoroğlu and Tüzel (2014). These parameters are close to the average of parameters used in the literature (Appendix D.1). The capital elasticity of profits α_k is in line with Hennessy and Whited (2007). Following Carlstrom and Fuerst (1997), the

risk-free real interest rate r is 4\%. The exogenous exiting rate η is set to match the exiting rate of firms in the Compustat data, 0.8%. Firm exiting is measured from Compustat in a similar way to Corbae and D'Erasmo (2020). The market intermediation costs μ_M are set to 60 basis points (bps) which is the historical average of AAA Corporate bond spread. Defaultfree spreads $r_M - r_f = \mu_M q^{-1} \simeq \mu_M$ is approximated to intermediation costs. Schwert (2020) estimates the average loan premium relative to the cost of credit is implied by the corporate bond market after accounting for the seniority of bank debts, and reported the premium is 140 to 170 bps. Default-free spreads $r_M - r_B = (\mu_M - \mu_B)q^{-1} \simeq \mu_M - \mu_B$ are approximated to be the difference of intermediation costs. Liquidation efficiency is set to 38% from Bris et al. (2006) which analyzed cases from the Arizona and NY federal bankruptcy courts from 1995 to 2001. This liquidation value is close to Kermani and Ma (2020), which finds this value to be equal to 44% of book assets with cash holdings. I calibrate the loss rate of going concern value after Ch. 11 bankruptcy π to 30%. This loss rate π can be interpreted as the decline in the intrinsic firm value due to an impaired customer relationship and from being delisted from major stock exchange markets. Since the impact to firm value is hard to measure for these events, I use empirical evidence of the stock price after Ch. 11 bankruptcy as a proxy for the decline in the intrinsic firm value. Lang and Stulz (1992) find stock return drops by 28% after filing for bankruptcy between January 1970 and December 1989. Another interpretation is that the loss rate π captures the firm's manager turnover rate around Ch. 11 bankruptcy.

The second group is a set of parameters estimated inside the model via SMM (the second panel of Table 1). These internal parameters are chosen to minimize the distance between model generated moments and the corresponding moments from real-world data. The technicalities of the estimation procedure are deferred to Appendix C.5. Some parameters in the second panel of Table 1 can be interpreted literally. Linear external financing costs are 9.2% with a standard error 2.1%. Hennessy and Whited (2007) finds similar estimates (linear cost of equity issuance in their full sample is 9.1% with a standard error 2.7%).

I select targeted moments that are informative about structural parameters. I provide intuitions behind the identification of parameters (Appendix Table A6 summarizes local elasticities of moments w.r.t. parameters). Extreme value scale parameter α increases locally the variance of debt to assets. PD is decreasing in extreme value scale parameter α_{Δ} . The partial derivative of equation (7) w.r.t. α_{Δ} shows this relationship holds true if $\bar{v}(\omega_B) \equiv v_{\Delta=0}(\omega_B) - v_{\Delta=1}(\omega_B) \geq 0$ s.t. $\frac{\partial}{\partial \alpha_{\Delta}} \left(\sum_{\omega_B \in \Omega_B} \sigma_{\Delta|\omega_B} \right) = -\sum_{\omega_B \in \Omega_B} \bar{v}(\omega_B) e^{\alpha_{\Delta}\bar{v}(\omega_B)} / \left(e^{\alpha_{\Delta}\bar{v}(\omega_B)} + 1 \right)^2 \leq 0.28$ Hence, the likelihood of bankruptcy is informative to determine the size of α_{Δ} . Fixed costs

 $[\]overline{{}^{28}\bar{v}(\omega_B)} \geq 0$ implies default $(\Delta = 1)$ is not optimal in the limit of extreme values shocks $(\alpha_\Delta \to \infty)$.

for Ch. 11 bankruptcy f_{c11} pin down the ratio of Ch. 11 to Ch. 7 bankruptcy. Production fixed costs f are estimated from the relative choice of dividends ($d \geq 0$) and seasoned equity issuance (d < 0). The firm's manager pays fewer dividends when production costs are higher. Lastly, linear external financing costs λ_1 target the variance of dividends to assets. The upward shift of marginal costs of external equity financing lowers the variation of equity payouts.

Table 1: Calibrated and Estimated Parameters

Description	Notation	Value	S.E.	Target/Reference				
Panel A: Parameters Calibrated Outside the Model								
Capital elasticity of profits	α_k	0.650		Standard setting				
Depreciation rate	δ	0.150		Standard setting				
Persistence of productivity	ρ	0.700		İmrohoroğlu and Tüzel (2014)				
Std. dev. of productivity shock	σ	0.270		İmrohoroğlu and Tüzel (2014)				
Risk-free rate	r_f	0.040		T-Bill rate				
Exogenous exiting rate	$\eta^{'}$	0.008		Exiting rate				
Market intermediation costs	μ_M	0.006		AAA Corporate bond spread				
Bank intermediation costs	$\mu_B - \mu_M$	0.017		Schwert (2020)				
Liquidation efficiency (exiting)	χ	0.500		Crouzet (2017)				
Liquidation efficiency (Ch. 7)	s_{c7}	0.380		Bris et al. (2006)				
Reorganization efficiency	s_{c11}	0.869		Bris et al. (2006)				
Loss of continuation value	π	0.300		Lang and Stulz (1992)				
Panel B: Parameters Estimated Ins	Panel B: Parameters Estimated Inside the Model							
Extreme value scale parameter	α	2.251	(0.300)	Variance of debt to assets				
Extreme value scale parameter	α_{Λ}	0.102	(0.015)	Bankruptcy rate (Ch. 11+Ch. 7)				
Fixed costs for production	f	4.099	(0.298)	Equity issuance/assets				
Fixed costs for Ch. 11	f_{c11}	28.698	(4.468)	Bankruptcy rate (Ch. 11)				
Linear external financing costs	λ_1	0.092	(0.021)	Variance of dividends to assets				

Note: Calibrated and estimated parameters are reported in panel A and B. Internal parameters are determined via SMM. Standard errors are calculated from variance-covariance matrix of the data moments and are reported in parentheses. In panel B, the five moments estimated via SMM are reported in the column Target/Reference. These parameters are estimated to match the moments in Table 2.

4 Equilibrium Results

This section investigates the model performance. Section 4.1 looks into unconditional moments comparing the model and data. Section 4.2 states a static distributional property of the model and explores moments conditional on debt type. Section 4.3 studies the role of expectation in the model and data. Section 4.4 demonstrates firm dynamics. Finally, Section 4.5 investigates policy functions, the Bayesian updating function g_s , and stationary distribution.

4.1 Model Validations to Unconditional Moments

There are five structural parameters determined inside the model to match with the five data moments. Table 2 shows targeted moments in panel A. Construction of the sample from Compustat is explained in Appendix A.1. In both model and data, Ch. 7 bankruptcy (0.14% per annum) is a rare event compared to Ch.11 bankruptcy (0.72% per annum). However, Ch. 7 bankruptcy is necessary to model Ch. 11 bankruptcy since it gives the value of outside option under bank debt renegotiation. Besides this, the model does a good job matching observed variance of leverage, variance of dividends to assets ratio, and equity issuance to assets ratio with reasonable external equity financing costs parameter λ_1 .

Table 2 shows untargeted moments in panel B. In all, the model fits the moments I do not explicitly target well. First of all, debt structure is a key element in the model. Compustat does not provide the bank debt data. Hence, I rely on other data sources to compare model prediction. First, Crouzet and Mehrotra (2020) report the average bank debt ratios in the data as 0.28 for the upper 99.5th and 0.43 for 99-99.5th of size groups. The sample is manufacturing firms in the confidential Census data from 1977:Q1 to 2014:Q1. Compustat universe corresponds approximately to the upper values in total assets distribution. Second, the aggregate bank debt ratio in the data is 0.31. The sample is from Flow of Funds (Appendix A.1 explains the sample construction). These ratios in the data are close to my model outcomes.²⁹ What is essential to note here is that the trade-offs between market debt and bank debt (Assumptions 2 of monitoring, 3 of intermediation costs, and 4 of debt renegotiation) are overall consistent with real-world data. Even though bank debt is advantageous in terms of information and flexibility, more firms choose to issue market debt which has lower costs on average. In the last line in Table 2, Assumption 3 of intermediation costs helps to explain why bank debt spreads are close to the data reported in Strahan (1999). In turn, debt-to-EBITDA ratio is 2.45 in the model which is comparable to 1.77 in the data. Finally, the model generates dividends to assets ratio 0.09 which overshoots a dividends to assets ratio of 0.03 in real-world data. Fama and French (2001) provide an explanation about this discrepancy. They show that the average payout ratio in Compustat is biased downward by the entrance of small, high-growth firms that have zero dividends. Moreover, the fraction of firms that pay dividends has fallen dramatically over time. My model abstracts away from these mechanisms.

²⁹Another way to characterize market debt issuance between the model and data is through the timing of market debt issuance. The model has a median of 3 years, a mean of 3.7 years, and a standard deviation of 2.8 years computed from a simulated panel. On the other hand, the data have a median of 4 years, a mean of 5.6 years, and a standard deviation of 4.7 years.

Table 2: Targeted and Untargeted Moments from Model and Data

Description	Model	Data	Source				
Panel A: Targeted Moments							
Bankruptcy prob. (Ch. 11) (%)	0.72	0.72	Compustat				
Bankruptcy prob. (Ch. 7) (%)	0.14	0.14	Compustat				
Variance of debt-to-assets	0.06	0.07	Compustat				
Variance of dividends/total assets	0.01	0.02	Compustat				
Equity issuance /total assets	0.15	0.16	Compustat				
Panel B: Untargeted Moments (Financial Ratios)							
Debt-to-assets	0.39	0.24	Compustat				
Bank debt ratio	0.33	[0.28, 0.43]	CM (2020)				
Aggregate bank debt ratio	0.21	0.31	Flow of Funds				
Debt-to-EBITDA	2.45	1.77	Compustat				
Dividends/total assets	0.09	0.03	Compustat				
Spreads (Non-bankrupt) (bps)	174	n.a.	n.a.				
Spreads (Ch. 11) (bps)	378	n.a.	n.a.				
Spreads (Ch. 7) (bps)	227	n.a.	n.a.				
Spreads of bank debt (bps)	269	[251, 301]	Strahan (1999)				

Note: Panel A (B) reports model and data generated targeted (untargeted) moments. The aggregate bank debt ratio is a fraction of aggregate bank debt to aggregate debt. This ratio in the data is computed by the annual data from Flow of Funds as an average from 1960 through 2015. Bank debt ratio in the data is computed by Crouzet and Mehrotra (2020) (hereafter CM (2020)) with the sample of manufacturing firms in the confidential Census data from 1977:Q1 to 2014:Q1. The average spreads of bank debt in the data are reported in Strahan (1999). This paper computes term loans: drawn spread for small unrated borrowers is 301 bps and for large unrated borrowers is 251 bps. Small borrowers are those with total assets (debt plus market value of equity) above the median in the sample. The data source is Dealscan from 1988 to 1998 which is constructed by the Loan Pricing Corporation.

4.2 Model Validations to Conditional Moments

Table 3 summarizes untargeted moments of three main ingredients of risky debt pricing: leverage, PD, and recovery rates at default. Model simulations are compared to the data. Without being a calibration target, the model captures these cross-sectional profiles conditional on debt type. The relationship between the model and data proves the central assumptions of the model that bondholders and bank lenders differ in technologies of monitoring, intermediation costs, and debt renegotiation (Assumptions 2, 3, and 4).

Leverage. A large body of literature shows that rated firms, firms that have credit ratings assigned to individual debt instruments, are more levered than unrated firms. For example, Faulkender and Petersen (2006) find firms with access to public debt markets have 35% more debt after controlling for firm characteristics. My model generates consistent patterns with these observations. Panel A of Table 3 demonstrates that rated firms ($\phi = M$) have 10 percentage points higher leverage than unrated firms ($\phi = B$) in the model and 18 percentage points in the data. Assumption 3 of intermediation costs is crucial to replicate the pattern

of highly levered rated firms.

PD. Next, I break down the likelihood of bankruptcy to the type of debt in panel B of Table 3. This middle panel shows that both unrated and rated firms file for Ch. 11 and Ch. 7 bankruptcies in the model and data. Ch. 11 bankruptcy is a viable option for these firms because the firm's manager receives the continuation value. However, Ch. 11 bankruptcy is not necessarily an optimal choice since it incurs high fixed costs. Ch. 7 bankruptcy is more common among firms issue bank debt (25bps in the model) compared to market debt (8bps in the model). This happens because Ch. 7 bankruptcy is less attractive for the borrower when Ch. 11 bankruptcy repays less than Ch. 7 bankruptcy ($\mathcal{R}_M^{(c11)} - \mathcal{R}_M^{(c7)} \leq 0$). Appendix B.5 discusses the trade-offs in detail using a stylized version of the model.

RR at Default. Realized recovery rates at default in the model and data are compared in panel C of Table 3. The data are from Moody's Ultimate Recovery Database (statistics are reported in Altman and Kalotay (2014), hereafter MURD).³⁰ In the data, (i) bank debt has higher average recovery rates, (ii) both market debt and bank debt have high standard deviations of 30-40% and interquartile ranges over 50 percentage points. The main finding is that the model is close to the data despite the fact that recovery rates at default are not targets of estimation. Although fixed costs for Ch. 11 bankruptcy are primarily identified by the fraction of Ch. 11 to Ch. 7 bankruptcy, the model agrees with low recovery rates of market debt in the data. Moreover, the model replicates the large variation in recovery rates of market debt. This attributes to Assumption 4 of debt renegotiation as a result of the free-rider and coordination problem of dispersed bondholders and the calibrated parameter of productivity σ_z which governs the dispersion of cash flows.

In turn, panel D in Table 3 reports expected recovery rates at default conditional on the lowest and highest type scores. These conditional moments are unobservable in the data. It is not surprising to find that expected recovery rates at default differ substantially by type scores. Expected recovery rates at default are

Expected RR at Default =
$$\sum_{z \in \mathcal{Z}, y \in \{c11, c7\}} \sigma_{y|\omega_B}(\omega_B) \frac{\mathcal{R}_M^{(y)}(\overline{\omega}_B)}{q_M(\overline{\omega}_M)^{-1} b} s(z)$$
 (29)

where $\mathcal{R}_{M}^{(y)}(\overline{\omega}_{B})$ is market debt repayment under Ch. 11 and Ch. 7 bankruptcies defined by equations (14) and (18). The difference of expected recovery rates at default between the

³⁰MURD contains recovery rates of bonds and loans from the time of resolution to default for large U.S. corporations. Default includes not only Ch. 11 bankruptcy but also covenant violation. However, MURD usually measures recovery rates at the time of emergence from Ch. 11 bankruptcy proceedings (See "Special Comment Moody's Ultimate Recovery Database" in April 2007).

lowest and highest types scores is substantially large (74 percentage points). Therefore, type score has a quantitative impact on expected recovery rates at default and corporate bond pricing.

Table 3: Untargeted Cross-Sectional Moments from Model and Data

	Market Debt		Bank Debt			
Description	Model	Data	Model	Data	Source	
Panel A: Leverage						
Debt-to-assets	0.42	0.39	0.32	0.21	Compustat	
Panel B: Bankruptcy Probabilities						
Chapter 11 Reorganization (%) Chapter 7 Liquidation (%) Fraction of Chapter 11	0.76 0.08 0.90	0.61 0.08 0.88	0.64 0.25 0.72	0.74 0.15 0.83	Compustat Compustat Compustat	
Panel C: Realized Recovery Rates a	t Default					
Mean Standard deviation Interquartile range 10th percentile 90th percentile	0.32 0.37 0.69 0.00 0.88	0.45 0.38 0.73 0.00 1.00	0.64 0.24 0.43 0.38 1.00	0.75 0.33 0.51 0.20 1.00	AK (2014) AK (2014) AK (2014) AK (2014) AK (2014)	
Panel D: Expected Recovery Rates at Default						
Mean (lowest type score) Mean (highest type score)	0.12 0.86	n.a. n.a.	n.a. n.a.	n.a. n.a.		

Note: Panels A, B, C, and D report untargeted moments in the model and data. Panels A and B use policy functions and distributions to compute moments. Rated (unrated) firms are $\phi = M$ ($\phi = B$). Bankruptcy rates are reported in percentages. Bankruptcy rate in the data computes the frequency of bankruptcy divided by the total number conditional on the debt type ϕ . Statistics of recovery rates are computed from MURD reported in Altman and Kalotay (2014) (hereafter AK (2014)). The interquartile range is the difference between the 75th percentile and 25th percentile. Panels C and D use a simulated panel to compute moments. I generate 10,000 firms for 50 years from the benchmark model to compute model generated realized and expected recovery rates at default. I drop entrants from the simulated panel. This is a parsimonious way to correct the bias toward entrants compared to the stationary recursive competitive equilibrium.

4.3 Model Validations to Expectations

Unfortunately, type score s —the assessment of unrecognized productivity —is hard to observe. However, credit ratings and recovery ratings allow me to examine expectations of uninformed investors in the data though the lens of the model.

My model provides a theory of credit ratings and recovery ratings. The goal of this section is to establish a mapping from expected PD and expected RR at default in the model to credit ratings and recovery ratings in the data. According to "S&P Global Ratings Definitions", long-term issue credit ratings are defined as *ordinal* measures of credit risks which are the likelihood of payment in the event of financial distress. However, credit ratings

do not imply a particular level of PD. Therefore, I measure the historical PD to infer the long-run average. On the other hand, recovery ratings focus specifically on a numerical range of expected RR at default (see assigned ratings in Appendix Table A1). Mappings to credit rating and recovery rating are: Credit Ratings = F(Expected PD) and Recovery Ratings = G(Expected RR) at Default) where credit rating (recovery rating) matching function $F: \mathbb{R} \to \{n|n \in \mathbb{Z}, 1 \leq n \leq 6\}$ ($G: \mathbb{R} \to \{n|n \in \mathbb{Z}, 1 \leq n \leq 6\}$) is an increasing (a decreasing) function of PD (RR at default). The assigned score takes an integer value between 1 denotes safest to 6 denotes the riskiest rating. Details about assigned ratings are explained in Appendix A.1. I use S&P ratings for credit ratings and recovery ratings (Appendix A.2.1 shows S&P holds the largest share in the industry).

The construction of credit rating matching function $F(\cdot)$ includes three steps. First, I sort expected PD by firms issuing market debt in the model from low to high. Equation (24) computes expected PD as a weighted sum of PD. Next, I calculate credit rating scale thresholds of PD to match the average shares of credit ratings in the data. Panel A in Table 4 shows these average shares of credit ratings in the model and data. Historically speaking, the average shares of credit ratings have been relatively stable since the 2000's, albeit I find a small change in the share of high credit quality (AAA and AA) (see Appendix Figure A3). Finally, I assign a credit rating scale for each firm based on these thresholds for PD. In contrast, the recovery rating matching function $G(\cdot)$ is simpler because recovery ratings are cardinal measures of expected RR at default. In the model, equation (29) computes expected RR at default as a weighted sum of RR.

Panel B summarizes expected bankruptcy rates from a simulated panel and the data. Expected bankruptcy rates in the model are close to historical annual bankruptcy rates with 3 years of the time horizon. Panel C compares expected RR at default in the model and data. Expected RR at default in the data is from S&P's Recovery Rating, which is issued for corporate debentures, and is only available for a speculative grade. The mean and standard deviation of expected RR at default credit ratings are more or less similar in the model and data.

Table 4: Expected Bankruptcy and Recovery of Investment Grade and Speculative Grade in Model and Data

		S&P Credit	Rating					
		Investment Grade		Speculative Grade				
		AAA/AA	A	BBB	BB	В	CCC/C	
Panel A: Sha	re (%)							
Model		4.00	15.00	24.00	27.00	27.00	3.00	
Data		3.97	14.32	23.75	27.26	27.27	3.43	
Panel B: Ban	kruptcy and Default of M	arket Debt						
Expected b	ankruptcy rates (%)							
Model		0.08	0.15	0.33	0.83	2.07	5.81	
Historical	annual bankruptcy rates (%	6)						
Data	1 year	0.00	0.00	0.07	0.12	0.57	14.13	
	3 years	0.05	0.03	0.13	0.53	1.32	7.35	
Panel C: Exp	Panel C: Expected Recovery Rates at Default of Market Debt							
Model	Mean	1.00	0.98	0.85	0.39	0.25	0.26	
	Std. Dev.	0.01	0.03	0.19	0.36	0.29	0.23	
Data	Mean	n.a.	n.a.	n.a.	0.43	0.38	0.38	
	Std. Dev.	n.a.	n.a.	n.a.	0.26	0.30	0.32	
	Number of observations	n.a.	n.a.	n.a.	1150	728	248	

Note: The credit rating data are provided from S&P Global. The time horizon of default is one year in the model. The bankruptcy rate in the model is computed as an expected likelihood of bankruptcy within one year from a simulated panel. Shares and bankruptcy rates in the data are historical averages in the U.S. from 1981 to 2017. I report "AA" historical annual bankruptcy rates in the column of "AAA/AA". Expected recovery rates in the data are computed from S&P's Recovery Rating issued for corporate debentures. S&P's Recovery Rating is in 5% increments. The data were downloaded on June 2nd, 2020. I drop rating data from 2020 to avoid the concern of COVID-19.

Finally, I claim that expected PD and expected RR at default explain the majority of credit risks in Appendix Table A7. The correlation between default and RR has a smaller impact on credit risks.

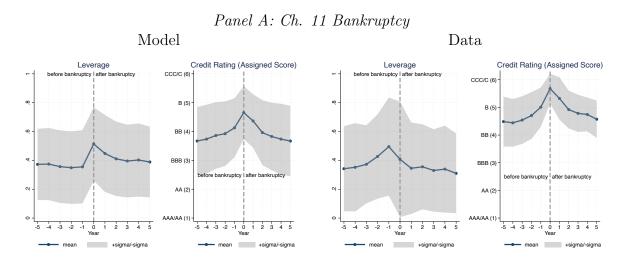
4.4 Model Validations to Dynamics

What are firm dynamics leading to bankruptcy events in the model and data? Figure 2 depicts evolutions of the average and +/- one standard deviation of leverage and credit rating between ± 5 years around bankruptcy filing (upper panels for Ch. 11 bankruptcy and lower panels for Ch. 7 bankruptcy). I define the credit rating scale, which ranges from 1 to 6, based on ranking of expected PD (described in detail in Section 4.3 and Appendix Table A1). 1 is the highest quality and 6 is the lowest quality. I find both in the model and data (a) the averages of leverage and credit rating increase before and after Ch. 11 bankruptcy (t = -1, 0, +1) and mean revert to the previous levels, and (b) the dispersion of leverage

 $+1~\sigma$ is large and persistent among Ch. 11 and Ch. 7 bankruptcy.³¹ In the model, the productivity process captures the mean reversions of leverage and credit ratings. Moreover, the costs of external equity finance imply slow adjustment to a change in equity. Figure 2 confirms that model parameters of the productivity process and external equity financing costs are consistent with dynamics in the data. Additionally, heterogeneity across firms in the model are close to that in the data near default events, which is not targeted in the estimation procedure.

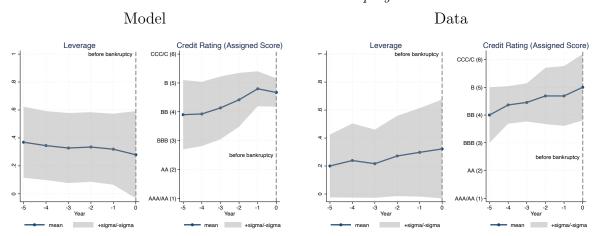
Leverage and credit rating follow similar dynamic patterns. Appendix Figure A4 finds a positive relationship between leverage and credit rating which drives the comovement between these financial variables.

Figure 2: Event Analysis of Leverage and Credit Rating at Ch. 11 Reorganization and Ch. 7 Liquidation: Model versus Data



³¹The difference in peak leverage between the model and data is coming from the timing assumption. The model assumes Ch. 11 reorganization happens right after the firm filed bankruptcy. However, bankruptcy does not occur immediately. The hand collected data from the Public Access to Court Electronic Records show that Ch. 11 bankruptcy takes on average 2 years to resolve (Bris et al. (2006)).

Panel B: Ch. 7 Bankruptcy



Note: The x-axis represents the time before and after the bankruptcy. The y-axis is the mean of the debt-to-asset ratio or credit rating. The range of credit rating is between 1 an 6. The assigned scores are reported in Table A1. The left two panels are the model generated data. I generate 10,000 firms for 50 years from the benchmark model. I drop the initial period to exclude entrants. The right two panels are the data from Compustat. In both the simulated and real-world data, events are constructed using firms that go through only one Ch. 11 bankruptcy or Ch. 7 bankruptcy during the duration of the event analysis. Area graph show +/- one standard deviation for each period.

4.5 Understanding Model

Choice of Debt Outstanding b. I compute average policy functions (equation (30) in the notes of Figure 3) and report them in Figure 3 (solid lines and blue and red fan charts).³² The four panels in this figure illustrate debt outstanding by productivity z and type score s (highest and lowest). On the horizontal axis is internal finance e and on the vertical axis is debt outstanding b. The frictionless model corresponds to Modigliani–Miller's frictionless markets (equation (33) in Appendix B.3, blue and red dotted 45 degrees downward slope lines). The irrelevancy of capital structure implies that total assets are fixed among each productivity level. High productivity firms borrow more because their MPK is higher. In the benchmark model, policy functions of debt are distributed around the modal choice. The size of dispersion of debt conditional on state variables e, z, and s depends on the size of the standard deviation of preference shocks which varies proportionally with α^{-1} . Inference of the firm's type after observing the choice of debt outstanding becomes relatively easier when α^{-1} is small. This is because the policy functions of the high and low productivity firms do not overlap, and uninformed lenders are almost certain about the firm's type. In the figure, the right two panels show that high productivity firms with small internal finance are financially constrained to issue debt in the presence of costs of bankruptcy inefficiency.

 $^{^{32}}$ Average policy functions are useful for visualization but cannot capture the correlation between debt type choice ϕ and next period internal funding choice e'.

Financial constraints are tighter for firms with lower reputations for two reasons. In short, firms are either paying costs of intermediation of bank lenders or information rents in corporate bond markets. More broadly, firm prefers to borrow from bank lenders over bondholders because interest rates are lower but largely offset by costly bank financial intermediation. Second, firms that issue corporate bonds face higher interest rates of market debt for lower reputations. (Appendix E.4 examines the effect of borrowing costs of corporate bonds on the exogenous change in type score). Appendix Figure A5 depicts policy functions of debt outstanding conditional on market debt. Similar to unconditional policy functions in Figure 3, conditional policy functions exhibit lower borrowing for firms with lower reputations.

Figure 3: Policy Functions of Debt Outstanding

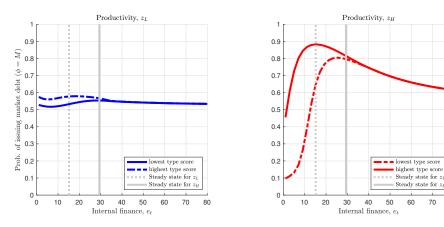
Note: The figures above report policy functions of debt outstanding b_t for low (z_L) and high (z_H) technology firms, and the lowest (\underline{s}) and highest (\overline{s}) type scores. The two panels on the left (right) side are figures of low (high) productivity z_L (z_H) . The x-axis is the amount of internal finance e. Solid black lines are the median of the probability of debt outstanding. The y-axis shows the equilibrium average policy function of debt type which is computed by summing up choice variables debt type ϕ and next period equity e' (equation (30)). Fan charts show 5-95th, 10-90th, and 25-75th percentiles. Dotted lines are policy functions of debt from the frictionless model in Appendix B.3. The model parameters of the frictionless model are matched to the model parameters in the benchmark model.

Policy Functions of Debt Outstanding
$$(e, z, s, b) = \sum_{(\phi, e') \in \{\Phi \times \mathcal{E}\}} \sigma_{\omega_B}(\omega_B) / \sum_{(b, \phi, e') \in \{\mathcal{B} \times \Phi \times \mathcal{E}\}} \sigma_{\omega_B}(\omega_B)$$
 (30)

Choice of Debt Type ϕ . Figure 4 plots equilibrium policy functions of debt type $\phi = M$ (equation (31)). The y-axis shows the probability of market debt issuance. For high productivity firms having small internal finance (between the size of internal finance by entrants and that of the stochastic steady state z_L), a highest type score firm is approximately 30 percentage points more likely to issue market debt than a lowest type score firm. When the high productivity firm has lowest type score and small internal finance, the firm prefers to borrow from bank lenders, because recovery rates at default is higher for bank debt on average for small firms. This resembles to the finding that in earlier stages of the firm's life cycle (i.e., start-up firms and risky ventures), bank debt is the main funding source (Petersen and Rajan, 1994, 1995). While a high productivity firm reaches to stochastic steady state

of low productivity, the firm reduces leverage and becomes less risky. It takes time to build up internal finance in the presence of external equity financing costs. Market debt becomes more attractive since recovery rates at default become higher and interest rates are lower when the firm size is large enough to cover fixed costs for Ch. 11 bankruptcy from operating profit. Then, the benefit of market debt outweighs the relatively expensive costs of bank intermediation (Assumption 3). When a firm reaches a size (as measured by equity) around steady states or beyond, then the firm is indifferent between choosing market debt and bank debt. This is because the firm's manager is subject to action specific preference shocks on the choice of debt type. While outstanding debt is relatively low, the difference in financing costs between market debt and bank debt are almost negligible in the nearly bankruptcy-free region.

Figure 4: CHOICE OF DEBT TYPE AMONG PRODUCTIVITY AND TYPE SCORE



Note: The x-axis is the internal finance. The y-axis is the probability of market debt issuance. Right (Left) panel red (blue) line corresponds to z_H (z_L). Dotted lines correspond to lowest (highest) type score s (\bar{s}) for high productivity z_H (low productivity z_L). The equilibrium average policy function of debt type is computed by summing up choice variables for debt b and next period internal finance e' (equation (31)). The vertical solid (dotted) grey lines are stochastic steady states for high (low) productivity. Stochastic steady states are defined as the mode of the stationary distribution of internal finance by productivity (see Figure A7).

Policy Functions of Debt Type
$$(e, z, s, \phi = M) = \sum_{(b, e') \in \{\mathcal{B} \times \mathcal{E}\}} \sigma_{\omega_B}(\omega_B)|_{\phi = M} / \sum_{(b, \phi, e') \in \{\mathcal{B} \times \Phi \times \mathcal{E}\}} \sigma_{\omega_B}(\omega_B)$$
 (31)

70

Type Score and Borrowing Rates. I find that the firm benefits from a high type score in terms of market debt funding costs. The central mechanism is that type score changes expected PD and expected recovery at default in market debt pricing. Appendix Figure E.4 examines the impact of an exogenous change in type score on market debt returns. Interest rates of market debt are lower (higher) for higher (lower) type score.

Type Score Updating. This part studies the determinants of the type score updating function. The model's distinct feature is the Bayesian inference problem for updating the prior of firm's productivity type. The Bayesian updating function g_s provides the signaling intensity, the amount by which an investor's belief shifts due to signaling, and therefore the essential element of the information problem in the model.

I perform a simple OLS regression to predict the current period type score from the previous period firm's characteristics. The Bayesian updating function g_s (equation (20)) is a nonlinear transformation of multiple observables (Section 2.11). I approximate this function by a linear combination of explanatory variables: leverage; log of internal finance; Ch. 11 bankruptcy; market funding ratio; firm age; and lagged type score. The benchmark specification (4) is:

$$s_{i,t} = \alpha_i + \beta_0 + \beta_1 \text{Leverage}_{i,t-1} + \beta_2 \ln(\text{Internal finance}_{i,t-1}) + \beta_3 \text{Bankruptcy}_{i,t-1} + \beta_4 \text{Market funding ratio}_{i,t-1} + \beta_5 \ln(\text{Firm age}_{i,t-1}) + \beta_6 s_{i,t-1} + \varepsilon_{i,t}$$

where i represents the identity of the firm in a simulated panel and $\varepsilon_{i,t}$ is the error term. β_i is the coefficient of interest where $i \in \{1, 2, 3, 4, 5\}$. α_i is fixed effects and β_0 is a constant. The independent variable "Bankruptcy" is an indicator function of Ch. 11 bankruptcy (non-bankruptcy and Ch. 7 bankruptcy correspond to zero), motivated from the theoretical model of Diamond (1989, 1991). "Market funding ratio" is also known as a signaling device of the firm's type (Houston and James, 1996). "Internal finance" and "Firm age" are proxy variables of reputation widely used in the literature (e.g., Datta et al. (1999)). I generate the simulated panel of 10,000 firms for 50 years from the benchmark model to perform this exercise.

Table 5 summarizes regression results. The signs of the coefficients are reasonable. One standard deviation in leverage explains 20 percentage points (= 0.81×0.25) of type score.³³ The second largest contribution comes from firm size measured by internal finance —one standard deviation in the logarithm of internal finance explains 11 percentage points(= 0.19×0.60) of type score.³⁴ In contrast, Ch. 11 bankruptcy has a less significant signaling effect thereby the economic magnitude is small compared to that of leverage and internal finance. Specifically, Ch. 11 bankruptcy increases the probability of being a high type by only 3 percentage points. Ch. 11 bankruptcy is a noisy signal in the model since preference shocks ε_{Δ} have a large variance. Moreover, the magnitude of market funding ratio and firm age are economically insignificant.

³³The standard deviation of leverage inside the model is 0.25.

³⁴The standard deviation of the logarithm of internal finance is 0.60.

Table 5: Simulated Regressions for the Determinants of the Next Period Type Score from Simulated Panel

	Dependent variable: Type score s_t					
	(1)	(2)	(3)	(4)		
Leverage t-1	0.739***		0.943***	0.806***		
	(493.43)		(881.76)	(789.83)		
$ln(Internal\ finance_{t-1})$		0.212***	0.306***	0.191***		
		(325.37)	(716.11)	(386.23)		
Chapter 11 bankruptcy _{t-1}			0.0283***	0.0314***		
			(13.07)	(16.44)		
Market funding ratio _{t-1}			0.00853***	0.0000488		
			(15.95)	(0.10)		
ln(Firm age _{t-1})			-0.00424***	-0.0000297		
			(-15.38)	(-0.12)		
Type score s_{t-1}				0.346***		
				(360.18)		
Number of observations	475568	475568	475568	475568		
R^2	0.339	0.182	0.696	0.762		
Fixed effects	No	No	Yes	Yes		

Note: This table presents simulated regression results for the determinants of the next period type score. I simulate 10,000 firms over 50 years and exclude entrants and firms filing Ch. 7 bankruptcy or exiting from the simulated sample. The numbers in parentheses denote standard errors. *, ***, and **** indicate 10%, 5%, and 1% significance levels.

5 Counterfactual Experiments

This section runs two sets of counterfactuals. The first experiment measures the long-run impact on asymmetric information. The second experiment introduces the corporate taxation of COD income under Ch. 11 bankruptcy to restore efficient allocation in the economy without changing the information structure.

5.1 Monitoring Technology in Corporate Bond Markets

First, I explain three changes to monitoring technology in counterfactual economies. Then, I present the results from the experiments. Partial monitoring technology and strong debt enforcement technology are explained in Appendix F.2.

Change in Full Monitoring Tech. The bank lender has an informational advantage in the benchmark model. In order to measure the inefficiency associated with information asymmetry in corporate bond markets, I introduce corporate bond pricing by charging different

prices based on productivity. The corporate bond pricing function is:

$$q_{M}(\overline{\omega}_{B}) = \frac{(1 - \sigma_{\Delta=1|\omega_{B}}(\omega_{B})|_{\phi=M})b}{(1 + \mu_{M})q^{-1}b - \sum_{y \in \{c11, c7\}} \sigma_{y|\omega_{B}}(\omega_{B})\mathcal{R}_{M}^{(y)}(\overline{\omega}_{B})}$$
(32)

which effectively alters Assumption 2 of monitoring. The difference between equation (22) is that expected PD and expected recovery are replaced without expectation by type score. Firm's productivity level z is observable for bondholders but still preference shocks to the manager are private information. This hypothetical environment is implemented by introducing a representative agent mimicking the bank lender's monitoring technology.³⁵ I assume monitoring is costless for bondholders. I will revisit this issue later.

Results. I investigate the aggregate implications of the long-run welfare measured by consumption of adopting full and partial monitoring technology of skills of bondholders. This hypothetical environment is implemented by introducing a representative agent mimicking the bank lender's monitoring technology. Table 6 compares the benchmark model to the counterfactual model (ii) with symmetric information for bondholders by employing full monitoring technology (equation (32)). It leads to higher consumption (1.4%), substitutes monitored bank loans to corporate bonds by 6 percentage points of total debt, increases measured TFP by 29bps, and shifts capital structure from equity to debt. Measured TFP is K^{α_k}/Y where K(Y) stands for aggregate capital (output). Other allocation efficiency measures such as Olley and Pakes (1996) decomposition and variance of the logarithm of the MPK (mpk) show improvement in the counterfactual.³⁶ The decrease in variance of the mpk is explained by the increase in covariance between productivity and total assets.³⁷ This evi-

$$\hat{z} = \sum_{\omega_B \in \Omega_B} \sum_{\Delta \in \{0,1\}} \sigma_{\omega_B}(\omega) \sigma_{\Delta|\omega_B}(\omega_B) e^z (1 - \mathbb{1}_{\Delta=1,y=c7}) F(b+e,z) \Gamma(\omega) / \text{Output}$$

where the aggregate output follows equation (28). The average output-weighted productivity is decomposed into two factors. The first factor is the average (unweighted) productivity:

$$\bar{z} = \sum_{\omega_B \in \Omega_B} \sum_{\Delta \in \{0,1\}} \sigma_{\omega_B}(\omega) \sigma_{\Delta|\omega_B}(\omega_B) e^z (1 - \mathbb{1}_{\Delta=1,y=c7}) \Gamma(\omega)$$

The second factor is the covariance of productivity output weights.

³⁵A similar idea is proposed by Amihud et al. (1999). In their paper, a representative agent mimicking the bank lender's monitoring technology is named "supertrustee".

³⁶Olley and Pakes decomposition is a standard measure of capital allocation. The average output-weighted productivity is measured as:

³⁷Table 6 shows the decomposition of the logarithm of the MPK: $Var(\log MPK) = Var(z) + (\alpha_k - 1)^2 Var(\ln k) + (\alpha_k - 1)Cov(z, \ln k)$. The last term is negative when $\alpha_k \in (0, 1)$ and $Cov(z, \ln k) \ge 0$.

dence proves capital allocation is more efficient in the counterfactual. Consumption increases but output and capital are reduced in the counterfactual model (ii). In the benchmark model (i), information asymmetry induces the low productivity firm to overinvest because the low type wants to send a good signal by accumulating more internal finance and issuing more debt, while the marginal benefits of reputation building vanish in the counterfactual model (ii) (Appendix B.4). Output goes down, but the household savings decline even further because TFP improves. Finally, I check the sensitivity of these results. The benchmark model has a lower restoration effect from reputation building when persistence of productivity ρ and the standard deviation of productivity shocks σ are lower (Appendix F.1). Asymmetric information becomes more severe in these parameter settings with a weaker effect of reputation building.

To deeply understand channels of asymmetric information in corporate bond pricing, counterfactual economies (iii) and (iv) in Table 6 implement partial monitoring technologies in equations (51-52) of Appendix F.2. Monitoring on PD (column (iii)) has a larger improvement in consumption than monitoring on recovery at default (column (iv)). However, the effect of monitoring on recovery at default is not small (consumption increases by 0.7%). This numerical evidence suggests that recovery at default has an important role in cross-subsidization.

Table 6: Comparison of Simulated Moments Between Benchmark Model and Models with Full and Partial Monitoring

	Benchmark	C	ounterfactual	
		Perfect Monitoring	Partial Mon	nitoring
	(i)	(ii)	(iii)	(iv)
Panel A: Techonology				
Monitoring on PD		\checkmark	\checkmark	
Monitoring on recovery at default		\checkmark		\checkmark
Panel B: Capital Structure and Welfare				
Debt	20.80	22.74	22.38	21.13
Debt (zL)	3.22	3.22	3.22	3.19
Debt (zH)	17.58	19.52	19.16	17.95
Equity	24.24	21.86	22.36	23.66
Equity (zL)	9.52	8.57	8.78	9.32
Equity (zH)	14.72	13.28	13.59	14.34
Aggregate bank debt ratio	0.21	0.15	0.16	0.19
Consumption	1.380	1.398	1.404	1.389
Change in % compared to benchmark	n.a.	1.35	1.80	0.65
Output	12.81	12.77	12.79	12.78
Capital	45.03	44.60	44.74	44.79
Change in % compared to benchmark	n.a.	-0.97	-0.64	-0.54
Capital (zL)	12.74	11.80	12.00	12.50
Capital (zH)	32.30	32.80	32.74	32.29
Panel C: Allocation Efficiency				
TFP	1.079	1.082	1.081	1.079
Change in % compared to benchmark	n.a.	0.29	0.24	0.05
Avrg. output-weighted productivity	1.179	1.185	1.184	1.181
Avrg. productivity	1.037	1.037	1.037	1.037
Cov (productivity, output weights)	0.143	0.149	0.148	0.144
Variance of mpk×100	2.87	2.52	2.58	2.79
Variance of productivity	7.28	7.28	7.28	7.28
Variance of log capital	4.76	5.37	5.23	4.83
Cov (z,capital)	-9.18	-10.13	-9.93	-9.32
Panel D: Bankruptcy				
Bankruptcy prob. (Ch. 11) (%)	0.72	0.85	0.82	0.73
Bankruptcy prob. (Ch. 7) (%)	0.14	0.12	0.13	0.14

Note: Column (i) in the table reports the benchmark model results. The second column reports the counterfactual model results. The counterfactual model (ii) assumes that bondholders possess monitoring technology without any additional costs. Models (iii) and (iv) implement the partial monitoring technologies in equations (51-52). TFP is measured by Y/K^{α_k} where Y(K) is the aggregate output (capital). Variance of the logarithm of the MPK (mpk) measures the variance of $\ln (e^z \alpha_k k^{\alpha_k - 1})$.

Additional counterfactuals illuminate the interaction between asymmetric information and recovery at default. I claim that Assumption 4 of debt enforcement technology is essential for obtaining a correct quantitative implication in the benchmark model. This assumption is crucial not only to introduce cross-subsidization but also to match observed recovery rates

at default in the data. I run counterfactuals with strong debt enforcement in corporate bond markets (equation (53) of Appendix F.2). Results are reported in Table A9. Counterfactual economics (iii) and (iv) assume strong debt enforcement in corporate bond markets. Thus, the recovery at default in counterfactual economies (iii) and (iv) depend on the liquidation value of asset. The difference between (iii) and (iv) is monitoring technology. (iv) implements full monitoring technology. Therefore, cross-subsidization in recovery at default is absent in these economies. The improvement of consumption in counterfactual models (iii) to (iv) is smaller in contrast to the models without coordination among bondholders in renegotiation. The welfare loss related to information asymmetry is smaller because the low type has less of an incentive to overinvest. As a result, TFP only increases by 5bps. Note that crosssubsidization does not completely vanish in the model (iii) because PD depends on private information. Why is asymmetric information less severe? In the benchmark model, type score affects corporate bond pricing since recovery rates at default depend on productivity, which is unobserved by bondholders. This is not the case for counterfactual economies (iii) and (iv). Which models are closer to the data? Counterfactual economies (iii) and (iv) are not consistent with the data. The interquartile range of corporate bond recovery rates is 27% in the model (iii) with liquidation threat compared to 69% in the model without liquidation threat and 73% in the data. Dispersion of cash flow explains the wide variation in recovery rates at default. Therefore, this suggests that Assumption 4 of debt renegotiation technology is indispensable.

The model generates a unique prediction about expected and realized recovery rates at default. I check that the predictive power in forecasting actual recovery is consistent with the model and data. First, I document that the predictive power is low in forecasting recovery rates at default in the data. Second, I argue that the asymmetric information model (the benchmark model (i) in Table 6) exhibits similar predictive power to the data, but the symmetric information model (the counterfactual model (ii) in Table 6) fails to capture this reality. Expected and realized Loss Given Default (LGD) —one minus recovery rate at default —is collected by Moody's. A numerical scale of Moody's LGD assessment runs from 1 to 6 (assigned scores are explained in Table A1) and provides an opinion about the expected LGD. Figure 5 depicts the accuracy of predicting LGD in the model and data. LGD assessment is on the horizontal axis and the vertical axis shows the percentage of firms within 10 percentage points of the LGD bucket. The data shows significantly fewer realized LGDs within 10 percentage points of the LGD bucket. The model with asymmetric information is closer to this data feature.³⁸ Idiosyncratic productivity shocks and asymmetric information

³⁸The data exhibit larger errors than the benchmark model. This uncertainty in recovery rates can be attributed to the aftermath of the financial crisis, which is not included in the model.

lead to predictive errors. In turn, the perfect information model predicts recovery rates at default without errors.

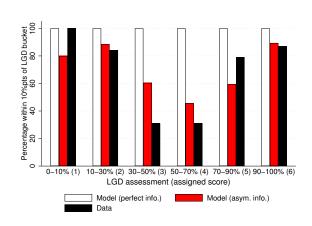


Figure 5: REALIZED AND EXPECTED LGD

Note: The data are from the Moody's article "Building the Better LGD Mousetrap" with a sample period from 2008 to 2010. The data are based on 185 defaulted debt security classes. LGD assessment is at the period of origination and is only available for the speculative grade. LGD 0-10% at origination has only one observation. For simulated data, I report realized and expected LGD of market debt in the model. Simulated data exclude the investment grade rated market debt.

So far I assume monitoring is costless in the counterfactual, but monitoring costs consist of labor costs of hiring experts and costs of equipment in reality. However, there is limited empirical evidence focusing on costs of monitoring and related activities. Since monitoring costs are hard to observe from publicly available accounting information, the model is used to find the upper bound of additional spreads which generates welfare improvement. The left panel of Appendix Figure A10 calculates a set of counterfactuals for different market debt intermediation costs μ_M on the x-axis without information asymmetry in public debt markets (equation (22)). I compare the level of consumption on the y-axis between counterfactual models (solid blue line) and the benchmark model (dotted black line). The contraction of credit supply by bondholder reduces output which leads to a downward sloping curve for consumption. The break-even point is +7bps of baseline calibration $\mu_M = 60$ bps in Table 1 (12% of intermediation costs). For comparison, S&P charges up to 7bps as credit rating fees to compensate for the costs of information processing according to "S&P Global Ratings U.S. Ratings Fees Disclosure". As evident in the right panel of Appendix Figure A10, I find that the market debt ratio decreases by 75bps at the break-even point.

The model has a rich set of financing options: two types of debt and external equity. What happens to quantitative implications if I shut down debt substitution or costly equity financing? Adding imperfect substitutions among debt-debt and debt-equity is essential to drive quantitative results. The first set of alternative models consists of single debt mar-

kets, which are equivalent to setting infinite intermediation costs for bank debt $(\mu_B \to \infty)$. Consumption modestly increases by 0.4% in the counterfactual economy. The full model in Table 6 has an effect that is roughly three times larger. An economy with only unmonitored debt overemphasizes the effect of reputation building because a good reputation becomes a central issue of debt financing when there are no bank loans. In the alternative benchmark economy, reputation building adds a strong incentive to invest in capital. Overall, this creates a positive effect on output and consumption even though capital misallocation lowers TFP. The second set of alternative models includes models with zero external equity financing costs ($\lambda_1 = 0$). Capital misallocation in the alternative benchmark model is small since debt and equity are close to perfect substitutes. This is especially true for high productivity firms who are less constrained and therefore can issue external equity without paying premiums. As a result, TFP slightly increases by 18bps in the counterfactual model, but inefficient Ch. 11 bankruptcy increases by 7bps and capital are further reduced. And therefore, consumption decreases by 0.7%. This counterfactual suggests that the interaction between asymmetric information and financial frictions alters not only quantitative but also qualitative implications.

Table 7: NO BANK DEBT AND ZERO EQUITY FINANCIAL EFFICIENT ECONOMY FOR BONDHOLDERS

	No ba	nk debt		mal equity
	Alternative benchmark	Counterfactual	Alternative benchmark	Counterfactual
Panel A: Techonology				
Monitoring by bondholders		\checkmark		\checkmark
Panel B: Capital Structure and Welfare				
Debt	21.76	23.65	19.48	20.80
Debt (zL)	2.75	2.68	3.54	3.52
Debt (zH)	19.01	20.98	15.94	17.28
Equity	24.78	21.69	28.57	26.73
Equity (zL)	10.06	8.71	10.30	9.74
Equity (zH)	14.72	12.98	18.26	16.99
Consumption	1.475	1.482	1.857	1.843
Change in % compared to full info	n.a.	0.49	n.a.	-0.72
Output	13.11	12.94	13.32	13.25
Capital	46.54	45.34	48.05	47.53
Change in % compared to full info	n.a.	-2.58	n.a.	-1.09
Capital (zL)	12.81	11.38	13.85	13.26
Capital (zH)	33.73	33.96	34.21	34.27
Panel C: Allocation Efficiency				
TFP	1.080	1.084	1.075	1.077
Change in % compared to full info	n.a.	0.36	n.a.	0.18
Avrg. output-weighted productivity	1.182	1.190	1.179	1.182
Avrg. productivity	1.036	1.036	1.037	1.037
Cov (z,output weights)	0.146	0.154	0.142	0.145
Variance of mpk×100	2.78	2.36	3.43	3.24
Panel D: Bankruptcy				
Bankruptcy prob. (Ch. 11) (%)	0.98	0.98	0.70	0.77
Bankruptcy prob. (Ch. 7) (%)	0.23	0.21	0.01	0.00

Note: No bank debt sets infinite intermediation costs for bank debt ($\mu_B \to \infty$). Zero external equity financing costs set external equity financing costs to zero ($\lambda_1 = 0$). Other parameters are unchanged.

5.2 Corporate Taxation of Cancellation of Debt Income Under Ch.11 Bankruptcy

This section explores the effect of the corporate taxation of COD income in order to correct the inefficient capital allocation. The capital misallocation happens because low productivity types borrow in order to reduce a substantial amount of debt under Ch. 11 bankruptcy. This debt reduction creates information rents, and high productivity firms underinvest in capital. I propose the taxation of COD to reduce the incentive for overborrowing by low productivity

firms.

The U.S. tax law recognizes COD as taxable income unless (i) cancellation occurs in bankruptcy or (ii) the firm is legally insolvent. Insolvency never happens in the model since debt is strictly less than total assets. In the previous sections, firms do not pay tax for COD. This section runs a counterfactual experiment modifying the tax code to penalize COD. In the model, COD is defined as: COD $\equiv q_{\phi}^{-1}b - \mathcal{R}_{\phi}^{(c11)}$ where $\phi \in \{M, B\}$. I set the tax rate τ_{ϕ} to 10%. Tax revenue is transferred to a representative household. I allow COD to be recognizable in any circumstance. This tax incentive minimizes distortions while it disincentivizes the low productivity firm from receiving the benefit from COD under Ch. 11 bankruptcy. Since COD creates welfare loss from cross-subsidization of asymmetric information, the taxation of COD reduces information rents and inefficient bankruptcy. And therefore, it decreases the variance of the logarithm of the MPK and increases welfare by 1.3% in models with information asymmetry (the first set of columns in Table 8). Financial constraints are relaxed in response to the taxation of COD. Capital of the high productivity firm increases in the counterfactual. Moreover, I find the capital structure in counterfactual economies is closer to the efficient debt market economy (the last column in Table 8). As a result, corporate bond markets become more efficient by introducing the taxation of COD.

The taxation of COD reduces inefficient bankruptcy. However, the impact on welfare is small. I find that welfare improvement is substantially dampened to 0.2% in the alternative benchmark and the counterfactual has no asymmetric information about productivity in corporate bond markets. This experiment shows the contribution of the taxation of COD without cross-subsidization. The economy with asymmetric information leads to an effect seven times larger due to amplification by adverse selection. Finally, the taxation of cancellation of bank debt does not alternate the qualitative results (the second set of columns in Table 8). In summary, asymmetric information is critical to evaluate tax code changes.

Table 8: Corporate Taxation of Cancellation of Debt Under Ch. 11 Bankruptcy

	w/ asymmetric information			w/o asymm	etric info	mation	Efficient debt market		
	Benchmark	Counterfactual		Alternative benchmark	Counterfactual				
Panel A: Techonology									
Monitoring by bondholders				\checkmark	\checkmark	\checkmark	\checkmark		
Tax rate of COD (market debt)	0%	10%	10%	0%	10%	10%	0%		
Tax rate of COD (bank debt)	0%	0%	10%	0%	0%	10%	0%		
Panel B: Capital Structure and Welfa	ire								
Debt	20.80	21.29	21.35	22.74	23.09	23.12	27.16		
Debt (zL)	3.22	3.28	3.28	3.22	3.30	3.30	4.73		
Debt (zH)	17.58	18.01	18.07	19.52	19.79	19.81	22.43		
Equity	24.24	23.76	23.66	21.86	21.39	21.35	16.74		
Equity (zL)	9.52	9.33	9.29	8.57	8.37	8.36	6.67		
Equity (zH)	14.72	14.42	14.37	13.28	13.02	12.99	10.07		
Consumption	1.380	1.397	1.399	1.398	1.401	1.403	1.500		
Change in % to benchmark	n.a.	1.25	1.44	n.a.	0.17	0.33	n.a.		
Output	12.81	12.82	12.82	12.77	12.75	12.75	12.65		
Capital	45.03	45.04	45.02	44.60	44.48	44.47	43.90		
Change in % to benchmark	n.a.	0.02	-0.04	n.a.	-0.27	-0.29	n.a.		
Capital (zL)	12.74	12.61	12.57	11.80	11.67	11.66	11.40		
Capital (zH)	32.30	32.43	32.45	32.80	32.81	32.81	32.50		
Panel C: Allocation Efficiency									
TFP	1.079	1.079	1.079	1.082	1.082	1.082	1.082		
Change in % to benchmark	n.a.	0.06	0.07	n.a.	0.02	0.03	n.a.		
Avrg. output-weighted productivity	1.179	1.180	1.181	1.185	1.186	1.186	1.187		
Variance of mpk×100	2.87	2.79	2.78	2.52	2.50	2.50	2.47		
Panel D: Bankruptcy			_	_		-			
Bankruptcy prob. (Ch. 11) (%)	0.72	0.69	0.67	0.85	0.80	0.79	n.a.		
Bankruptcy prob. (Ch. 7) (%)	0.14	0.14	0.14	0.12	0.12	0.13	n.a.		

Note: Efficient debt market is an economy that eliminated bankruptcy by setting $\alpha_{\Delta} \to \infty$ and $f_{c11} \to \infty$. TFP is measured by Y/K^{α_k} where Y(K) is the aggregate output (capital).

6 Conclusion

This paper presents an equilibrium model of reputation building with financing via public debt, private debt, retained earnings, and external equity issuance. I model the information problem faced by lenders and assess quantitative implications of the degree of information asymmetry. I focus on two financial frictions in this study. First, bondholders learn about the borrower's type and bank lenders are special *monitors* of the borrower. Second, bank lenders have the ability to participate in the bargaining process with credible threats of liquidation, but dispersed bondholders do not due to a free-rider problem. These financial frictions

interact with each other and create cross-subsidization. Alternatively, the way to prevent information costs from signaling is levering up or increasing internal finance. However, the manager dissuades signaling since bankruptcy and external equity issuance are costly.

In turn, I analyze the effects of monitored bondholders as a counterfactual experiment. I find capital allocation is more efficient and welfare improves.

Then, I apply the model to study the corporate taxation of COD income. Since this taxation disincentivizes the low productivity firm from receiving the benefit from COD under Ch. 11 bankruptcy, the high productivity type pays fewer information rents to the low productivity type. This substantially improves the efficiency of the economy.

In summary, my model takes an important first step in estimating a dynamic corporate financing model to quantitatively analyze the effect of borrowers and lenders being subject to adverse selection. This paper's framework demonstrates a dynamic process and an endogenous distribution of beliefs unequivocally. Long-lived information is a challenging extension to the dynamic corporate financing model. This paper successfully applies the idea of Diamond (1991) and runs deep "tests" of theory. An important avenue for future research is to use this methodology involving dynamic adverse selection to investigate extensive questions in corporate finance: debt maturity choice; dividend smoothing; external equity issuance and share repurchase; and relationship banking in Small and Medium-sized enterprises. Alternatively, relaxing assumptions in the model is also an interesting extension. ³⁹

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³⁹I leave comments for future investigation in Appendix B.9.

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Appendix

A Appendix: Data

A.1 Sample Construction

Compustat. The sample is non-financial firms from 1986 to 2015. I exclude utility firms (SIC codes 4000-4999), financial firms (SIC codes 6000-6999), and public administration (SIC codes 9100-9729) from the sample. All variables are in terms of real value in 1986 deflated by annual CPI. I drop firms from the sample who report zero or negative total assets, capital expenditure, or sales. I trim the top 1% and bottom 1% of the variables total assets, EBITDA/total assets, debt/total assets, and net investment ratio/total assets. The Compustat variable for total assets is "at" (Book Value of Assets). Debt/total assets is a ratio between the sum of "dltt" (Long-term Debt) and "dlc" (Debt in Current Liabilities) and total assets. Net equity issuance is computed from the difference between dividends and "sstk" (Sale of Common and Preferred Stock). Dividends are the total cash distributions calculated as the sum of "dvp" (Dividends - Preferred/Preference), "dvc" (Dividends Common/Ordinary), and "prstke" (Purchase of Common and Preferred Stock).

S&P (Capital IQ) and Moody's. Credit rating provides a forward-looking opinion about the likelihood of payment. I collect S&P Domestic Long Term Issuer Credit Rating. A long-term issue credit rating is assigned to debt instruments with an original maturity of greater than 365 days. S&P's recovery ratings and Moody's LGD assessments provide a specific opinion about the expected recovery. Recovery ratings and LGD assessments are in 5% increments. I categorize credit ratings, recovery ratings, and LGD assessments into a numerical scale that runs from 1 to 6 ("1" is the safest debt and "6" is the riskiest debt). Corresponding scores and credit ratings (recovery ratings and LGD assessments) are reported in panel A (B) of Table A1.

⁴⁰I use the procedure by McKeon (2013) to address the concern pointed out by several authors (Begenau and Salomao (2018)) that this variable includes the exercise of stock options or warrants for employee compensation. These compensations are not the mechanism taken into account in the model. Therefore, I use an approximate method to exclude these compensations without using other data sources.

Table A1: RATINGS AND ASSIGNED SCORES

Panel A: Credit Ratings

Credit rating	Assigned score	Grade
AAA, AA+, AA, AA-	1	
$\mathrm{A+,\ A,\ A-}$	2	Investment grade
BBB+, BBB, BBB-	3	
BB+, BB, BB-	4	Speculative grade; High
$\mathrm{B+,B,B-}$	5	yield bond; Junk bond
CCC+, $CCC-$, $CCC-$, D , SD	6	yleid bolid, Julik bolid

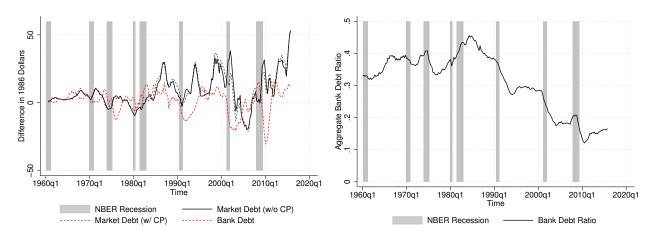
Panel B: Recovery Ratings (S&P) and LGD Assessments (Moody's)

Expected recovery rate at default (%)	Expected LGD (%)	Assigned score
90-100	0-10	1
70-90	10-30	2
50-70	30-50	3
30-50	50-70	4
10-30	70-90	5
0-10	90-100	6

Note: The first column in panel A shows S&P credit ratings. Investment grades are credit ratings above BBB. Letter category "SD" or "D" are considered to be a default on one or more of its financial obligations.

Flow of Funds. Table L. 103 of the Flow of Funds is the data source for the aggregate bank debt ratio. The denominator of the ratio is the sum of depository institution loans to non-financial corporate business (series FL103168005.Q). The numerator is the sum of depository institution loans to non-financial corporate business and the sum of corporate bonds (series FL103163003.Q). The historical average of the aggregate bank debt ratio is 30.99% from 1960 to 2015. I start to calculate the ratio from 1960 for two reasons. First, the data of credit rating in Compustat - CapitalIQ are only available from 1986. Second, the model cannot incorporate the declining trend from 1986 to 2015 since the model solves for the stationary equilibrium.

Figure A1: AGGREGATE MARKET DEBT AND BANK DEBT



Note: The left panel presents the difference in aggregate market debt and bank debt from 1960 to 2015 in 1986 dollars. The nominal values are deflated by CPI. Each series corresponds to a four quarter rolling-window difference in depository institution loans to non-financial corporate business and corporate bonds. Market debt (bank debt) corresponds to corporate bonds (loans). The right panel presents aggregate bank debt ratio from 1960 to 2015. The aggregate bank debt ratio is the ratio of the total sum of bank debt to total debt of the non-financial corporate sector. Shaded areas are the NBER based recession indicators for the U.S.. Both panels are compiled from the quarterly U.S. Flow of Funds Accounts data.

A.2 U. S. Credit Rating Agency

A.2.1 Industry

In this section, I document that S&P is a representative rating agency in the U.S.. Table A2 demonstrates the market structure of credit rating agency industries in the U.S.. The data are collected from "Annual Report on Nationally Recognized Statistical Rating Organizations". Most of the share in credit rating agency industries in terms of the number of corporate issuers and credit analysts are dominated by BIG3: Fitch, Moody's, and S&P. S&P holds the largest share in the industry which is 40% of the total number of credit rating issue in the market. Firms issue multiple credit ratings from BIG3. Therefore, S&P covers almost all rated firms in the Compustat Universe.

Table A2: NATIONALLY RECOGNIZED STATISTICAL RATING ORGANIZATIONS

Number of Corporate Issuers					Number of Credit Analysts			Revenue	
	(thousands) (%)					(%)	(billions)		
Year	BIG3	Others	Total	S&P/Total	BIG3	Others	Total	S&P/Total	Total
2007	70	5	75	38.5	n.a.	n.a.	n.a.	n.a.	n.a.
2008	73	14	87	31.1	n.a.	n.a.	n.a.	n.a.	3.8
2009	85	11	96	43.3	3,262	300	3,562	30.3	3.7
2010	88	11	99	44.8	3,482	363	3,845	35.0	4.2
2011	90	8	99	46.0	3,636	330	3,966	35.7	4.2
2012	95	8	103	46.1	3,651	371	4,022	35.7	5.1
2013	105	10	115	43.4	3,811	407	4,218	34.7	5.4
2014	110	17	127	41.6	4,012	516	4,528	30.3	5.9
2015	111	14	124	41.1	4,154	609	4,763	30.5	5.9

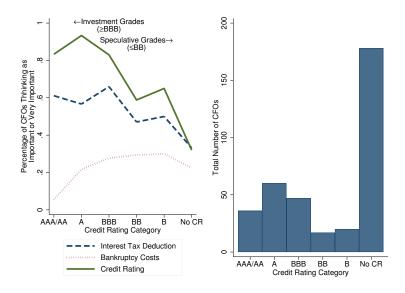
Note: The data source is "Annual Report on Nationally Recognized Statistical Rating Organizations" published from 2008 to 2016. "BIG3" include Fitch, Moody's, and S&P. "Others" include A. M. Best, DBRS, EJR, HR Ratings, JCR, LACE, KBRA, R&I, Realpoint.

A.2.2 What CFO Thinks About Determinants of Debt Financing

Attempts to gain a sense of the magnitude to reputation building using reduce-form regressions suffers from an endogeneity problem and a measurement problem of unobservable beliefs. For a qualitative study, surveys help researchers to understand the practice of debt financing. In this section, I revisit the survey data which asks the firm's CFO about determinants of debt financing conducted by Graham and Harvey (2001). I show evidence that credit ratings, an evaluation of creditworthiness based on reputation building, seem to be one of the most crucial factors of debt financing. This finding also holds conditional on firms with or without credit ratings.

Figure A2 presents new evidence for debt policy factors conditional on credit rating categories and firms without credit ratings. The data are constructed from surveys and total 392 CFOs in 1999. Nearly half of CFOs have no credit rating in the sample (right hand in Figure A2). The fraction of CFOs who consider credit ratings as "Important" or "Very Important" drop from around 80% to 60% when credit ratings are lower, and 32% of CFOs think credit ratings are significant even if their firm does not have a credit rating. In addition, these CFOs consider credit ratings to be as important as traditional trade-off theory factors ("Interest Tax Deduction" and "Bankruptcy Costs"). This evidence motivated me to write down a model in which credit ratings have a real effect and to explore determinants of debt financing through the lens of the model.

Figure A2: Survey Responses to Question of Rating Firms Classified into Credit Rating Category and Unrated Firms: "What factors affect how you choose the appropriate amount of debt for your firm?"



Note: The figure shows the survey evidence on some of the factors that affect the decision to issue debt. Surveys are conducted in 1999 (http://faculty.fuqua.duke.edu/~charvey/Research/GHSurvey/GH_JFE2001.XLS). A total of 392 CFOs responded to the survey. The left hand panel asks "What factors affect how you choose the appropriate amount of debt for your firm?". The response is on a scale from 0 to 4 (0 meaning not important, 4 meaning very important). The right hand panel asks "What is the credit rating for your firm's debt? Write NONE if debt not rated ______ (e.g., AA-, B+)". Each firm reports credit ratings in different ways. Most firms report S&P long-term. A few firms report Moody's long-term. Some firms report both or other ratings. I consolidate other ratings into a single category to make an easier comparison. EU Credit Rating Agency Regulation (CRAR) provides a Table to compare credit ratings issued by different CRAs. If the firm reports two or more credit ratings, I use S&P credit rating. If the firm does not report a credit rating issued by S&P, I use the CRAR Table to find a corresponding credit rating. The firm that responds "NONE" or empty is classified into an unrated firm. There are no CCC/C reported in the sample.

A.3 Additional Data Evidence

Exaction of Credit Patings Letter (%)

1985

1990

1995

2000

2005

2010

2015

CCC/C

B

BB

BB

AAA/AA

Figure A3: Fraction of Credit Rating Scale from 1986 to 2015

Note: All ratings are S&P long-term. "AAA/AA" includes both AAA and AA ratings. The y-axis is the fraction of credit rating scale in percentage. The data source is Compustat. Sample period is 1986 to 2015.

B Appendix: Theory

B.1 Definition of Type Scores s and Binomial Productivity

This section defines the type score function s(z) in general and introduces binomial productivity in a quantitative model for state space reduction. The type score function is mapping $s(z): \mathcal{Z} \to [0,1]$. In words, the type score function s(z) is a probability given the level of productivity z. Type scores \mathbf{s} are a vector of the type score function $\mathbf{s} = (s(z_1), s(z_2), \dots, s(z_{N_z}))$. Type scores are defined on $\mathbf{s} \in \mathcal{S} \equiv [0,1]^{N_z}$ with $\sum_{s \in \mathcal{S}} s = 1$. The second equation explains that the sum of probability of type is normalized to one. I redefine $\mathbf{s} \equiv (s(z_2), \dots, s(z_{N_z})) = [0,1]^{N_z-1}$ to reduce the state space. In practice, the size of the state space expands very quickly since it depends exponentially on the number of productivity levels. In this paper, I estimate a model of two levels of productivity $(N_z = 2)$. Accordingly, the space of type score is defined on the real line $s(z_H) \in \mathcal{S} = [0,1]$. Given two types of productivity levels, the probability of the firm becoming low productivity z_L is $s(z_L) = 1 - s(z_H)$.

B.2 Generalized Type-I Extreme Value Distribution

Stochastic shocks ε ($\varepsilon_{b,\phi,e'}$ and ε_{Δ} in the main text) are draw from Generalized Type-I Extreme Value distribution (Gumbel distribution). CDF is $F(x) = e^{-e^{-z(x)}}$. PDF is $f(x) = \frac{1}{\beta_{\varepsilon}}e^{-z(x)}e^{-e^{-z(x)}}$ where $z(x) = \frac{x-\mu_{\varepsilon}}{\beta_{\varepsilon}}$. The mean of x is $\mu_{\varepsilon} + \beta_{\varepsilon}\gamma_{E}$ where $\gamma_{E} \simeq 0.577$ is Euler-Mascheroni constant. I set $1/\beta_{\varepsilon} = \alpha$ and $\mu_{\varepsilon}/\beta_{\varepsilon} = -\gamma_{E}$ where α is a scale parameter. Hence, the mean and the variance of x are equal to zero and $\frac{\beta_{\varepsilon}^{2}\pi^{2}}{6} = \frac{\pi^{2}}{6\alpha^{2}}$.

B.3 Frictionless Model

To simplify the exposition, this section assumes a frictionless model. The frictionless model has no information asymmetry in public debt markets, no external financing costs $(\lambda_1 = 0)$, no private benefits to manager and no bankruptcy $(\alpha, \alpha_{\Delta} \to \infty)$, and $f_{c11} \to \infty$, no market debt intermediation costs $(\mu_M = 0)$, and no bank debt $(\mu_B \to \infty)$. This world corresponds to Modigliani–Miller's frictionless markets. Thus, the source of funding is irrelevant to the firm's value. Debt financing and equity financing are perfect substitutes. The firm's manager simply chooses debt and next period internal finance given current period internal finance. $W(e, z) = \max_{b \ge 0, e'} d(e, z, b, e') + \eta \chi e' + (1 - \eta) \beta \mathbb{E}_{z'|z}[W(e', z')]$ subject to equity payout $d(e, z, b, e') = \exp(z)(b + e)^{\alpha_k} - f + (1 - \delta)(b + e) - (1 + r)b - e'$ and accounting identity k = b + e. The policy function of debt is

$$b(e,z) = \max\left\{ \left(\frac{\alpha_k \exp(z)}{r+\delta} \right)^{\frac{1}{1-\alpha_k}} - e, 0 \right\}$$
 (33)

Hence, the optimal capital level is $k(z) = \left(\frac{\alpha_k \exp(z)}{r+\delta}\right)^{\frac{1}{1-\alpha_k}}$.

B.4 Marginal Benefits and Costs of Internal Finance e

This section investigates the benefits and costs of the choice of internal finance e for the continuing firm. The marginal value of conditional value functions w.r.t. the next period internal finance e' is:

⁴¹Standard Gumbel distribution is the parametrization of $\mu_{\varepsilon} = 0$ and $\beta_{\varepsilon} = 1$. Another common parameterization is Chatterjee et al. (2020) parametrization which is similar to standard Gumbel distribution s.t. $\mu_{\varepsilon} = 0$ and $1/\beta_{\varepsilon} = \alpha$.

$$\frac{\partial v_{\Delta=0}}{\partial e'} = (1 + \lambda_1 \mathbb{1}_{d_{\Delta=0}<0}) \left(\underbrace{-\frac{\partial q_{\phi}^{-1}}{\partial e'} b}_{\text{MB/MC of debt}} - \underbrace{1}_{\text{MC of equity}} \right) + \underbrace{\eta \chi}_{\text{MB of exiting}} + (1 - \eta) q \sum_{(z',s')\in\{\mathcal{Z}\times\mathcal{S}\}} g_z \left(\underbrace{\frac{\partial g_s}{\partial e'} W(e',z',s')}_{\text{MB of reputation (next)}} + \underbrace{g_s \frac{\partial W(e',z',s')}{\partial e'}}_{\text{MB of contituuation}} \right) \tag{34}$$

Condition (34) is the intertemporal Euler equation, which equates the marginal benefits and costs. The contribution of information asymmetry comes from the fourth term in equation (34) of the marginal benefit of reputation building. A similar relationship in equation (34) holds for the marginal value for Ch. 11 bankruptcy firm $(\frac{\partial v_{c11}}{\partial e'})$. The fifth term in equation (34) finds the marginal value of continuation is:

$$\frac{\partial W(e, z, s)}{\partial e} = \sum_{(b, \phi, e') \in \{\mathcal{B} \times \Phi \times \mathcal{E}\}} w_{b, \phi, e'} \sum_{\Delta \in \{0, 1\}} w_{\Delta} \frac{\partial v_{\Delta}}{\partial e}$$

where weights $w_{b,\phi,e'} \equiv e^{\alpha V} / \sum_{(b,\phi,e') \in \{\mathcal{B} \times \Phi \times \mathcal{E}\}} e^{\alpha V}$ and $w_{\Delta} \equiv e^{\alpha_{\Delta} v_{\Delta}} / \sum_{\hat{\Delta} \in \{0,1\}} e^{\alpha_{\Delta} v_{\hat{\Delta}}}$ are derived from equations (4-5). The marginal value of conditional value functions:

$$\frac{\partial v_{\Delta=0}}{\partial e} = (1 + \lambda_1 \mathbb{1}_{d_{\Delta=0}<0}) \left(\underbrace{e^z \alpha_k k^{\alpha_k - 1}}_{\text{MPK}} + (1 - \delta) \underbrace{-\frac{\partial q_{\phi}^{-1}}{\partial e} b}_{\text{MB/MC of debt}} \right) \\
+ (1 - \eta) q \sum_{(z', s') \in \{\mathcal{Z} \times \mathcal{S}\}} g_z \underbrace{\frac{\partial g_s}{\partial e} W(e', z', s')}_{\text{MB of reputation (after next)}} \\
\frac{\partial v_{c7}}{\partial e} = (1 + \lambda_1 \mathbb{1}_{d_{c11}<0}) \left(\underbrace{s_{c7}}_{\text{MB of liquidation}} \underbrace{-\frac{\partial \mathcal{R}_{\phi}^{(c7)}(\overline{\omega}_B)}{\partial e} b}_{\text{MB of debt}} \right)$$

Again, a similar relationship to the continuing firm's marginal value $(\frac{\partial v_{\Delta=0}}{\partial e})$ in equation (35) holds for the Ch. 11 bankruptcy firm's marginal value $(\frac{\partial v_{c11}}{\partial e})$. In summary, the marginal benefits of internal finance consist of (i) debt pricing, (ii) exogenous exiting, (iii) reputation building of next and following periods, (iv) production, and (v) liquidation. Reputation building enters from two distinct periods since the type score updating function g_s depends

on both the current and next periods' internal finance e and e'. On the other hand, marginal costs of internal finance in equation (34) are 1 for retained earning and $1 + \lambda_1$ for seasoned equity issuance.

This analysis predicts that adopting full monitoring technology of skills of bondholders decreases internal finance from two channels: MPK and reputation building. First, MPK decreases (increases) if market debt becomes less (more) costly for high (low) productivity firms. Second, the marginal benefits of reputation building vanish in the counterfactual economy. And these effects shift the marginal benefit curve downward. Whether the high productivity firm is more or less responsive than the low productivity firm depends on the size of these shifts in the marginal benefit curve.

B.5 Decisions of Bankruptcy Filing: Ch. 11 versus Ch. 7

What is the trade-off between Ch. 11 and Ch. 7 bankruptcy? Specifically, why do firms that are small in terms of internal finance and total assets, issue bank debt, and have low productivity file Ch. 7 bankruptcy more often in the model? (Table 3 and A8) I investigate the case in which a firm's manager prefers Ch. 7 bankruptcy among Ch. 11 bankruptcy: $v_{c7} \geq v_{c11}$ to answer these questions. To proceed in the analytical argument, let me assume (i) no external equity financing costs s.t. $\lambda_1 = 0$, (ii) recoveries $\mathcal{R}_{\phi}^{(c11)}$ and $\mathcal{R}_{\phi}^{(c7)}$ are constant values, (iii) a going-concern value of Ch. 11 bankruptcy only depends on e and e (but independent of type score e), and (iv) the going-concern value is increasing in e and e. Substituting equations (13) and (16) into the condition e0 to the case in which Ch. 7 bankruptcy is the optimal choice, I find

$$s_{c7}k \ge e^z k^{\alpha_k} - f + s_{c11}(1-\delta)k - f_{c11} - \underbrace{\left(\mathcal{R}_{\phi}^{(c11)} - \mathcal{R}_{\phi}^{(c7)}\right)}_{\text{Surplus of Lenders}} + \text{going-concern value}(e, z)$$
 (36)

Define $k_{\phi}^*(e,z) \geq 0$ as the capital threshold. This threshold keeps Ch. 11 and Ch. 7 bankruptcy indifferent (above formula holds with equality). In equilibrium under my parameterization, the firm's manager chooses Ch. 7 bankruptcy when total assets are below the threshold $(k \leq k_{\phi}^*(e,z))$ conditional on e and z. This shows large firms in terms of total assets above the threshold are not optimal to file Ch. 7 bankruptcy.

Moreover, I find that the thresholds are higher for bank debt such that $k_M^*(e,z) \leq k_B^*(e,z)$

⁴²The marginal benefit curve is downward sloping since the MPK is decreasing in internal finance and debt.

if $\mathcal{R}_{M}^{(c11)} - \mathcal{R}_{M}^{(c7)} \geq 0$ and $\mathcal{R}_{B}^{(c11)} - \mathcal{R}_{B}^{(c7)} = 0$. Remember that the renegotiation surplus of the bank lender is zero according to Proposition 1. In the benchmark model, $\mathcal{R}_{M}^{(c11)} - \mathcal{R}_{M}^{(c7)}$ can be positive or negative.

Alternatively, $k_{\phi}^{*}(e, z)$ does not exist in equilibrium if the going-concern value is sufficiently high. This is the case in which the high productivity firm always chooses Ch. 11 bankruptcy.

Finally, I show that $k_{\phi}^{*}(e, z)$ is decreasing in e. This shows large firms in terms of internal finance are not optimal to file Ch. 7 bankruptcy. Equation (36) gives a simple illustration of bankruptcy decisions between Ch. 11 bankruptcy and Ch. 7 bankruptcy. Assume that internal finance e and total assets k are continuous variables. Define:

$$F_{\phi}(e, z, k) = s_{c7}k - e^{z}k^{\alpha_k} - s_{c11}(1 - \delta)k - f_{c11} + \mathcal{R}_{\phi}^{(c11)} - \mathcal{R}_{\phi}^{(c7)} - \text{going-concern value}(e, z)$$

where the capital threshold is $F_{\phi}(e,z,k^*)=0$. The implicit function theorem gives

$$\frac{\partial k^*}{\partial e} = -\frac{\frac{\partial F_{\phi}(e,z,k^*)}{\partial e}}{\frac{\partial e}{\partial k^*}}$$

$$= \frac{\frac{\partial going\text{-concern value}(e,z)}{\partial e}}{s_{c7} - e^z \alpha_k k^{\alpha_k - 1} - s_{c11}(1 - \delta)}$$

$$< 0$$

In the second line, the numerator is positive since $\frac{\partial \text{going-concern value}(e,z)}{\partial e} \geq 0$ in equilibrium. The denominator in the second line is strictly negative under the parameterization in Table 1 since $s_{c7} - s_{c11}(1-\delta) < 0$. This means the marginal value of bankruptcy w.r.t. total assets is larger for Ch. 11 than Ch. 7. Hence, the larger firm in terms of internal finance is less likely to file Ch. 7 bankruptcy. In summary, firms with higher internal finance e firm are less likely to choose Ch. 7 bankruptcy because Ch. 11 bankruptcy is more favorable in terms of a larger continuation value and a higher value of assets.

B.6 Proof of Theorem, Lemma, and Proposition

B.6.1 Existence of Equilibrium

Theorem 1 (Uniqueness) Given policy functions and price functions, there exists a unique solution of value function W to the manger's decision problem in equations (4), (5), (8), (13), and (16). Moreover, W(f) is continuous in f where f is a vector of policy and price functions.

Proof. Define an operator $T_f(W): \mathbb{R}^{\Omega} \to \mathbb{R}^{\Omega}$ where f is a vector of policy and price functions. The map takes in a \mathbb{R}^{Ω} vector of W to \mathbb{R}^{Ω} vector of W in equations (4), (5), (8), (13), and (16). The mapping T_f satisfies the Blackwell's sufficient condition for a contraction mapping. Lemma 1 shows T_f meets the Blackwell's sufficient condition. Lemma 2 shows that T_f is a contraction mapping.

Next, I show T_f is continuous in f as a first step to prove that W(f) is continuous in f. Suppose $f_n \in \mathbb{R}^{K+M}$ is a sequence converging to \bar{f} . Then given (i) the continuity of f with respect to $d - \lambda(d)$ since $\lambda(d)$ is a linear function of d (no fixed external financing costs) with the continuity of q_{ϕ} where $\phi \in \{M, B\}$ and (ii) the continuity of g_s respect to ψ , $\lim_{n\to\infty} T_{f_n} = T_{\bar{f}}$. Furthermore, since \mathbb{R}^{K+M} is a Banach space, we may apply Theorem 4.3.6 in Hutson et al. (2005) to conclude that W(f) is continuous in f.

Lemma 1 The mapping T_f satisfies Blackwell's sufficient condition for a contraction mapping.

Proof. (Monotonicity) If $W(\omega) \leq W'(\omega)$ for all $\omega \in \Omega$, then $T_fW(\omega) \leq T_fW'(\omega)$ for all $\omega \in \Omega$ where: (i) $v_{\Delta=0}$ and $v_{\Delta=1}$ are increasing in W; and (ii) α and α_{Δ} are positive. The second condition ensures that equations (4) and (5) are increasing in $v_{\Delta=0}$ and $v_{\Delta=1}$.

Proof. (Discounting) Suppose $a \geq 0$ is a scalar constant. For all $\omega \in \Omega$, $v_{\Delta=0}(\omega_B; W(\omega) + a) = v_{\Delta=0}(\omega_B; W(\omega)) + (1 - \eta)qa$ and

$$v_{\Delta=1}(\omega_B; W(\omega) + a) = \max\{v_{c11}(\omega_B; W(\omega) + a), v_{c7}(\omega_B; W(\omega) + a)\}$$

$$\leq v_{\Delta=1}(\omega_B; W(\omega)) + (1 - \eta)qa$$

Hence, $T_f(W(\omega) + a) \le T_fW(\omega) + (1 - \eta)qa$ where $(1 - \eta)q \in (0, 1)$ is an effective discount factor.

Lemma 2 (Contraction Mapping Theorem) T_f has a unique fixed point W in Ω .

Proof. Suppose a complete metric space s.t. The Banach space where \mathbb{R}^{Ω} is a set and $d(W, W') = \max_{\omega \in \Omega} |W(\omega) - W'(\omega)|$ is a sup metric. According to the Contraction Mapping Theorem, $T_f: \Omega \to \Omega$ is a contraction mapping with modulus $(1 - \eta)q$.

Theorem 2 There exists a unique invariant distribution Γ .

Proof. Suppose Ω is a set with $M = |\Omega|$. T_{Γ} is a transition matrix of a distribution from period t to period t+1. There exists $n \geq 1$ s.t. the transition matrix T_{Γ} has a strictly positive element $\pi_{\Gamma}^{(n)}$. The exogenous transition of z has a full support since g_z is a 1st-order Markov transition function. Moreover, g_s has a full support since the Bayesian inference assigns a nonzero probability to each firm's type score s after a finite number of periods. Finally, for any f, a transition matrix of e' has a full support because the extreme value shocks assign a nonzero probability to every feasible choice. Therefore, any state of e is reached after a finite number of periods. Stokey (1989) Theorem 11.2 ensures that there exists a unique invariant distribution Γ .

Theorem 3 (Existence) There exists a stationary recursive competitive equilibrium.

Proof. Assume f is the vector of policy and price functions s.t. $q = (q_B, q_M) \in [0, 1]^K$ and $\sigma \in [0, 1]^M$ where $K = K_B + K_M$, $K_B = |\Omega_B|$, $K_M = |\Omega_M|$, and $M = |\mathcal{G}|$, s.t. \mathcal{G} is a set of choice. Therefore, $f \in [0, 1]^{K+M}$. Let $W(f) : [0, 1]^{K+M} \to \mathbb{R}^{\Omega}$. This is a mapping from policy and price functions to the value function established in Theorem 1. Given the value function W, equations (6-7) compute decision rules σ and $\sigma_{\Delta|\omega_B}$. This is a mapping $J_{\sigma}(W) : \mathbb{R}^{\Omega} \to (0, 1)^M$. The mapping from policy functions to updated price functions and posteriors are $J_{q_B}(\sigma) : (0, 1)^{K_B} \to [0, 1]^{K_B}$, $J_{q_M}(\sigma) : (0, 1)^{K_M} \to [0, 1]^{K_M}$, and $J_{\psi}(\sigma) : (0, 1)^M \to [0, 1]^M$. The mapping of $J_{q_B}(\sigma)$ does not depend explicitly on W since the borrower gives a take-it-or-leave-it offer to the lender. Hence, the joint mapping is $J_f(\sigma) : (0, 1)^{K+M} \to [0, 1]^{K+M}$ where $f = (q, \psi)$. Finally, the composite mapping $J_f(W) = J_f(\sigma) \circ J_{\sigma}(W) : \mathbb{R}^{\Omega} \to [0, 1]^{K+M}$. Theorem 1 gives a continuous function of $J_W(f)$. $J_f(\sigma)$ and $J_{\sigma}(W)$ are both continuous functions, therefore $J(f) = J_f(W) \circ J_W(f)$ is a continuous selfmapping. Since $[0, 1]^K$ is a compact set and convex subset of \mathbb{R}^K , the Brouwer's Contraction Mapping Theorem guarantees the existence of a fixed point $f^* = J(f^*)$. Given the fixed point f^* , Theorem 2 produces a unique invariant distribution Γ^* .

B.6.2 Consistency

Proposition 3 (Consistency of Distribution and Belief) The cross-sectional firm distribution satisfies:

$$s(z) = \frac{\Gamma(e, z, s)}{\sum_{\hat{z}} \Gamma(e, \hat{z}, s)}$$
(37)

Proof. Mathematical induction can be used to prove this proposition. The proof proceeds in two steps.

Step 1: I need to show that if $\Gamma(\omega) = s(z) \sum_{\hat{z}} \Gamma(\hat{\omega})$ for incumbents in the current period and $\Gamma_n(\omega) = s(z) \sum_{\hat{z}} \Gamma_n(\hat{\omega})$ for entrants in any period are both true, $\Gamma(\omega') = s'(z') \sum_{\hat{z}' \in \mathcal{Z}} \Gamma(\hat{\omega}')$ must also be true in the next period where $\hat{\omega} = (e, \hat{z}, s)$. Substitute equation (37) for equation (26),

$$s'(z') \sum_{z' \in \mathcal{Z}} \Gamma(\hat{\omega}') = \sum_{(e,z,s) \in \{\mathcal{E},\mathcal{Z},\mathcal{S}\}} \sum_{(b,\phi) \in \{\mathcal{B} \times \Phi\}} \sum_{\Delta \in \{0,1\}, y \in \{c11,c7\}} \times (1 - \eta) g_z(z'|z) g_s(s'|\omega_M, \Delta) \sigma_{\omega_B}(\omega_B) \sigma_{\Delta|\omega_B}(\omega_B) \mathbb{1}_{\text{continuing}} \times s(z) \sum_{\hat{z} \in \mathcal{Z}} (\Gamma(\hat{\omega}) + \mathcal{M}_n \Gamma_n(\hat{\omega}))$$
(38)

since equations (20) and (21) give the rational belief updating function

$$s'(z') = \frac{\sum_{z \in \mathcal{Z}} g_z(z'|z) \sigma_{\omega_B}(\omega_B) \sigma_{\Delta|\omega_B}(\omega_B) \mathbb{1}_{\text{continuing}} s(z)}{\sum_{z \in \mathcal{Z}} \sigma_{\omega_B}(\omega_B) \sigma_{\Delta|\omega_B}(\omega_B) \mathbb{1}_{\text{continuing}} s(z)}$$

Comparing left hand sides of equations (26) and (38), I verify the relationship holds for $\Gamma(\omega') = s'(z') \sum_{\hat{z}' \in \mathcal{Z}} \Gamma(\hat{\omega}')$.

Step 2: I need to show that entrants satisfy

$$s(z) = \frac{\Gamma_n(e, z, s)}{\sum_{\hat{z} \in \mathcal{Z}} \Gamma_n(e, \hat{z}, s)}$$
(39)

This is straightforward. I assume that the productivity of entrants matches with it's ergodic distribution of productivity (see Section 2.13). This implies $\bar{g}(z) = \Gamma_n(e,z,s)/\sum_{\hat{z}\in\mathcal{Z}}\Gamma_n(e,\hat{z},s)$. I also assume that entrant's belief is consistent with it's ergodic distribution of productivity (see Section 2.13). This implies $s(z) = \bar{g}(z)$. Therefore, equation (39) for entrants holds true.

Finally, I redefine n+1 th and n th generations in equation (26) such that

$$\Gamma^{(n+1)}(\omega') = \sum_{(e,z,s)\in\{\mathcal{E},\mathcal{Z},\mathcal{S}\}} \sum_{(b,\phi)\in\{\mathcal{B}\times\Phi\}} \sum_{\Delta\in\{0,1\},y\in\{c11,c7\}} \times (1-\eta)g_z(z'|z)g_s(s'|\omega_M,\Delta)\sigma_{\omega_B}(\omega_B)\sigma_{\Delta|\omega_B}(\omega_B)\mathbb{1}_{\text{continuing}} \times (\Gamma^{(n)}(\omega)+\mathcal{M}_n\Gamma_n(\omega))$$

using notations $\Gamma^{(n+1)}(\omega')$ and $\Gamma^{(n)}(\omega)$ for distributions in n th and n+1 th periods. Suppose there is no incumbent in the initial period (i.e., $\Gamma^{(0)}(\omega)=0$). Steps 1 and 2 establish the claim that $s(z)=\Gamma^{(1)}(\omega)/\sum_{\hat{z}\in\mathcal{Z}}\Gamma^{(1)}(\hat{\omega})$. Following this argument, mathematical induction proves that $s(z)=\Gamma^{(n)}(\omega)/\sum_{\hat{z}\in\mathcal{Z}}\Gamma^{(n)}(\hat{\omega})$ for all $n\geq 0$. Since the distribution converges to the stationary distribution (i.e., $\lim_{n\to\infty}\Gamma^{(n)}(\omega)=\Gamma(\omega)$), this establishes $s(z)=\Gamma(\omega)/\sum_{\hat{z}\in\mathcal{Z}}\Gamma(\hat{\omega})$.

B.7 Representative Household Problem

Since a representative household does not affect the decision making of the firm's manager, the household sector only plays a role for pricing equity and risk-free debt. The representative household chooses intertemporal consumption and investment in financial assets such as stocks and one-period risk-free bonds. Since households are not monitoring specialists, they also rely on Bayesian inference to price the shareholdings. The idea of information asymmetry between corporate insiders and outsiders is rooted in Myers and Majluf (1984)'s pecking order model. The representative household prices the stock under the expectation in the same information set that bondholders use for pricing market debt. Therefore, the firm's internal finance e and type score e are observables, but the level of productivity e is unobservable. The household takes into account the firm manager's policy function and Bayesian updates the next period type score. The household solves

$$\max_{\{C_t \ge 0, S_{jt}, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) | \Omega_M \right]$$

$$\tag{40}$$

subject to

$$C_{t} + \int (p_{jt} - d_{jt} + \lambda(d_{jt})) S_{jt+1} dj + B_{t+1}$$

$$= \int (1 - \mathbb{1}_{\Delta=1, y=c7, jt}) (1 - \pi \mathbb{1}_{\Delta=1, y=c11, jt}) \{ (1 - \eta) p_{jt} + \eta \chi e_{jt} \} S_{jt} dj + q_{t}^{-1} B_{t}$$
(41)

where $\beta \in (0,1)$ is the discount factor, $U(C_t)$ is a per-period utility function of consumption: $\mathcal{R}^{++} \to \mathcal{R}$, d_t is a dividend payment $(d_{jt} = \mathbb{1}_{\Delta=0,t}d_{\Delta=0,jt} + \mathbb{1}_{\Delta=1,y=c11,jt}d_{c11,jt} + \mathbb{1}_{\Delta=1,y=c7,jt}d_{c7,jt})$, and S_{jt} is the share of firm j's ownership. Integration is defined as

$$\int \cdots dj \equiv \sum_{z \in \mathcal{Z}} \sum_{\omega_M \in \Omega_M} \sum_{\Delta \in \{0,1\}} \sigma_{\omega_B}(\omega) \sigma_{\Delta|\omega_B}(\omega_B) \cdots s(z) \bar{\Gamma}(e,s)$$

where $\bar{\Gamma}(e, s) \equiv \sum_{z \in \mathcal{Z}} \Gamma(e, z, s)$. It adds up all firms across internal finance and type score, and calculates the expectation using the probability weight of type score. This expression is simplified to

$$\int \cdots dj = \sum_{\omega_B \in \Omega_B} \sum_{\Delta \in \{0,1\}} \sigma_{\omega_B}(\omega) \sigma_{\Delta|\omega_B}(\omega_B) \cdots \Gamma(\omega)$$
(42)

using the relationship $\Gamma(e,z,s) = s(z)\bar{\Gamma}(e,s)$ which is the result of Proposition 3. Interpretation is simple. In equilibrium, type scores should be consistent with distribution of type z conditional on equity e and type score s (i.e., $s(z) = \frac{\Gamma(e,z,s)}{\sum_{\hat{z}\in\mathcal{Z}}\Gamma(e,\hat{z},s)}$). The first order conditions are:

$$[B_{t+1}]: q_t U'(C_t) = \beta \mathbb{E}_t [U'(C_{t+1})|\Omega_M]$$

$$[S_{jt+1}]: (p_{jt} - d_{jt} + \lambda(d_{jt}))U'(C_t) = \beta \mathbb{E}_t \Big[U'(C_{t+1})(1 - \mathbb{1}_{\Delta=1, y=c7, jt+1})(1 - \pi \mathbb{1}_{\Delta=1, y=c11, jt+1})$$

$$\times \{(1 - \eta)p_{jt+1} + \eta \chi e_{jt+1}\} |\Omega_M\Big]$$

In a steady state equilibrium, consumption is constant $(C_t = C_{t+1})$ because there is no aggregate uncertainty in the economy. The price of equity is:

$$p_{jt} = d_{jt} - \lambda(d_{jt}) + \beta \mathbb{E}_t \Big[(1 - \mathbb{1}_{\Delta=1, y=c7, jt+1}) (1 - \pi \mathbb{1}_{\Delta=1, y=c11, jt+1}) \\ \times \{ (1 - \eta) p_{jt+1} + \eta \chi e_{jt+1} \} |\Omega_M \Big]$$
(43)

The price of risk-free debt is:

$$q = \beta \tag{44}$$

 p_{jt} is the cum-dividend price of equity per share. The firm's value is an infinite sum of the future dividend stream discounted by risk-free rate. There are two market clearing conditions. First, the equity market must clear (shareholdings sum up to 1) $S_j^d = 1.43$ Second, the total of one-period bank and market debt must clear.

B.8 Definition of Model Variables

Recovery rates at default are the ratio of principal and interest on bankruptcy that can be recovered divided by the face value of debt. The definition is close to the one used by Moody's.

Table A3: Definition and Equation of Model Variables

Description	Definition and equation
Recovery rates at default of debt type $\phi \in \{M, B\}$	$rac{R_{\phi}}{q_{\phi_{a}}^{-1}b}$
$Dividends/total\ assets$	$\frac{d}{b+e}$, conditional on firms $d \ge 0$
Debt-to-assets ratio	$\frac{b}{b+e}$
Equity issuance/total assets	$\frac{\frac{b}{b+e}}{\frac{d}{b+e}}$, conditional on firms $d < 0$

B.9 Discussions of Model Assumption

This section discusses modeling assumptions in this paper.

Mixture of Debt Type. The firm's manager chooses the extensive margin of debt type but not the intensive margin. Since bank debt is secured or senior to market debt, bank debt has higher recovery rates when the firm mixes two types of debts. This might improve the fit of bank debt recovery rates at default in the model (Table 3).

Private Information Collected by CRAs. Credit ratings are assessments of the firm's underlying credit risk that are certified by rating agencies. My model assumes that credit ratings contain no information on firm's credit quality beyond other publicly available information. Thus, I assume that CRAs are specialized financial intermediaries in gathering public information. In fact, I find no evidence of monitoring in market debt to improve forecasting LGD (Figure 5).

Instrumental Difference in Bonds and Loans. This paper focuses on monitoring technology, intermediation costs, and debt restructuring. However, there are other differences.

⁴³Since the CEO ownership is very small compared to the trading and holding of other investors, I ignore the trading and holding of shares by the CEO.

For example, I abstract away that maturity of bonds is longer than loans and that bonds are callable but loans are not (except for high yield leverage loans).

C Appendix: Computation

C.1 Numerical Approximation for Debt Pricing

Structural estimation requires a number of iterations solving the model for different parameters. Using the Interest Expense Exemption Approximation (IEEA) can reduce the computation for debt pricing for one loop. This approximation allows the derivation of closed-form solutions to debt price equations for market (22) and bank (25) debt to be

$$q_{M}(\overline{\omega}_{M}) \simeq \frac{(1 - \Lambda_{M}(\overline{\omega}_{M}))b}{\tilde{q}_{M}^{-1}b - \sum_{z \in \mathcal{Z}} \sum_{y \in \{c11, c7\}} \sigma_{y}(\omega_{B})|_{\phi = M} \underbrace{\min\{\tilde{q}_{M}^{-1}b, \max\{\Pi_{y}, 0\}\}}_{\text{approximated recovery}} s(z)}, \quad (45)$$

$$q_B(\overline{\omega}_B) \simeq \frac{(1 - \Lambda_B(\overline{\omega}_B))b}{\tilde{q}_B^{-1}b - \Lambda_B(\overline{\omega}_B)|_{\phi=B} \underbrace{\min\{\tilde{q}_B^{-1}b, \max\{\Pi_{c7}(k), 0\}\}}_{\text{approximated recovery}}$$

$$(46)$$

where $\tilde{q}_M^{-1} \equiv (1+\mu_M)q^{-1}$ and $\tilde{q}_B^{-1} \equiv (1+\mu_B)q^{-1}$ are bankruptcy-free debt prices characterized in Proposition 4.

Proposition 4 (Bankruptcy-free Debt Pricing) Assuming that bankruptcy rates are zero, then the prices of market debt and bank debt are \tilde{q}_M^{-1} and \tilde{q}_B^{-1} .

The second term of the denominators in equations (45-46) is the recovery of debt. IEEA replaces the maximum recovery at default of market debt (bank debt) $q_M^{-1}b$ ($q_B^{-1}b$) to recovery at default based on bankruptcy-free debt pricing $\tilde{q}_M^{-1}b$ (\tilde{q}_B^{-1}). Hence, IEEA is a numerical approximation such that interest expenses are negligible in terms of debt pricing.⁴⁴ The full solution changes market debt spreads only by 2bps. The low average recovery rates at default of market debt explain most firms do not repay interest expenses (Table 3).

 $^{^{44}}$ Another way to understand IEEA is that financial intermediaries take into account interest rate reduction under debt renegotiation.

Table A4: Evaluation of Interest Expense Exemption Approximation

Description	Approximated Solution	Full Solution
Debt-to-assets	0.39	0.40
Variance of debt-to-assets	0.06	0.06
Fraction of rated firms	0.67	0.67
Bank debt ratio	0.33	0.33
EBITDA/total assets	0.15	0.15
EBITDA/total assets (non-bankrupt)	0.15	0.15
Variance of dividends/total assets	0.02	0.02
Equity issuance /total assets	0.16	0.17
Spreads of bank debt (bps)	269	281
Spreads of market debt (bps)	130	132

Note: The approximated solution uses the price formula in equations (45-46). The full solution finds the interior solution of the price function.

C.2 Numerical Algorithm to Compute Stationary Recursive Competitive Equilibrium

- 1. The representative household problem pins down risk-free debt price $q = \beta$.
- 2. Guess the value function $W^{(0)}$ and decision rules $\{\sigma_{\omega_B}^{(0)}, \sigma_{\Delta|\omega_B}^{(0)}\}$.
- 3. Solve the value function iteration of equation (1). The solution gives the updated value function $W^{(1)}$ and the decision rules $\{\sigma_{\omega_B}^{(1)}, \sigma_{\Delta|\omega_B}^{(1)}\}$.
- 4. Update the guess of the value function $W^{(0)} = W^{(1)}$ and the guess of decision rules $\{\sigma_{\omega_B}^{(0)}, \sigma_{\Delta|\omega_B}^{(0)}\} = \{\sigma_{\omega_B}^{(1)}, \sigma_{\Delta|\omega_B}^{(1)}\}$. Repeat steps 2 and 3 until the value function and decision rules converge to the initial guess. The equilibrium decision rules are $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$.
- 5. Solve for the stationary distribution Γ using mappings $\{\sigma_{\omega_B}, \sigma_{\Delta|\omega_B}\}$. Find the mass of new entrants \mathcal{M}_n which is consistent with the exogenous labor supply equation (27).

C.3 Numerical Procedure of Computing Value Function W

Equations (4) and (5) are infeasible to calculate directly on a computer because the sum of exponents becomes too large. This section provides a way to avoid this problem. Instead of calculating the exponents, I break the process into two parts. First, I search for

a maximum $V(\omega_B)$ and $v_{\Delta}(\omega_B)$ s.t. $V_{\max}(\omega) = \max_{(b,\phi,e')\in\{\mathcal{B}\times\Phi\times\mathcal{E}\}} V(\omega_B)$ and $v_{\max}(\omega_B) = \max_{\hat{\Delta}\in\{0,1\}} v_{\hat{\Delta}}(\omega_B)$. This optimization problem matches with the best interest of a representative household. Then, I compute exponentials of the difference of conditional value function and the maximum.

$$W(\omega) = \frac{1}{\alpha} \ln \left(\sum_{(b,\phi,e')\in\{\mathcal{B}\times\Phi\times\mathcal{E}\}} \exp\left(\alpha \left\{V(\omega_B) - V_{\max}(\omega)\right\}\right) \right) + V_{\max}(\omega)$$

$$V(\omega_B) = \frac{1}{\alpha_\Delta} \ln \left(\sum_{\hat{\Delta}\in\{0,1\}} \exp\left(\alpha_\Delta \left\{v_{\hat{\Delta}}(\omega_B) - v_{\max}(\omega_B)\right\}\right) \right) + v_{\max}(\omega_B)$$

Each exponential inside the log function are less than equal to 1 since $V(\omega_B) - V_{\text{max}}(\omega) \le 0$ and $v_{\Delta}(\omega_B) - v_{\text{max}}(\omega) \le 0$.

C.4 Simulation Settings of Discrete Choice

The number of grid points is large enough that the simulation results are robust against a change in size. The upper bounds of internal finance e and debt b are set to a value close to the optimal level of total assets in the frictionless model (Appendix B.3). Given parameters in the model (Table 1), the optimal level of total assets for the high productivity firm is $k(z) = \left(\frac{\alpha_k \exp(z)}{r+\delta}\right)^{\frac{1}{1-\alpha_k}} = \left(\frac{0.65 \times \exp(0.27)}{0.04+0.15}\right)^{\frac{1}{1-0.65}} = 72.64$. Therefore, I set the maximum of internal finance and debt to 80. Discrete grid points are a linearly spaced vector.

Table A5: Number of Grid Points of Discrete Choices

Notation	Description	Number of grid point	Range
$N_e N_h$	Internal finance Debt	40 40	[1,80] [0,80]
- 10	2000	10	[0,00]

C.5 Estimation via SMM

I use SMM to estimate the model to the U.S. data. The estimation is just-identified —it uses the same number of parameters and moments. A combination of global and local search algorithms is used to find a minimum of the SMM criterion function $J(\Theta, W_0) \equiv (X^{\text{data}} - X^{\text{simulation}}(\Theta))' W_0 (X^{\text{data}} - X^{\text{simulation}}(\Theta))$ where X^{data} is a vector of the data moments and $X^{\text{simulation}}(\Theta)$ is a vector of simulated moments. Θ is a set of model parameters

and W_o is a positive semi-definite weighting matrix. I apply a uniform weighting matrix in the first stage such that this criterion function simplifies down to the sum of squared deviations of the model moments. I use Sobol sequence and particle swarm for the global search. In turn, Nelder-Mead algorithm works well for the local search. My model exhibits a smooth response of moments to a change in Θ . This is due to the discrete choice model having smooth policy functions. To compute the standard errors of my estimates, I use a set of estimated parameters Θ in the first stage, replace weighting matrix Θ with the covariance matrix of the moment conditions from real-world data, and compute the numerical partial derivative of the model moments $X^{\text{data}} - X^{\text{simulation}}(\Theta)$ with respect to parameters Θ . Then the covariance matrix of estimated parameters is given by:

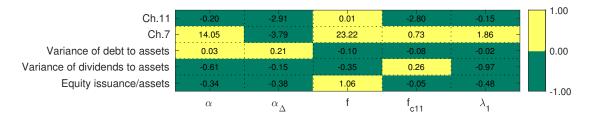
$$(1 + S^{-1}) \left[\left(\frac{\partial (X^{\text{data}} - X^{\text{simulation}}(\Theta))}{\partial \Theta} \right)' W \left(\frac{\partial (X^{\text{data}} - X^{\text{simulation}}(\Theta))}{\partial \Theta} \right) \right]^{-1}$$

while S is the ratio of the number of observations in a simulated data set to the number of observations in a real-world data set. In particular, a simulated panel is computed from the stationary distribution $(S \to \infty)$.⁴⁵

C.6 Analysis of Identification

I compute the local elasticity of moments w.r.t. five parameters $(\alpha, \alpha_{\Delta}, f, f_{c11}, \lambda_1)$ estimated inside the model.

Table A6: Elasticity of Moments W.R.T. Parameters



Note: Columns are parameters. Rows are targeted moments. Local elasticity is computed from $\frac{\Theta_j}{X_i^{\text{data}}} \frac{\partial (X_i^{\text{data}} - X_j^{\text{simulation}}(\Theta))}{\partial \Theta_j}$ where $i,j \in \{1,\ldots,5\}$ corresponds to rows and columns.

⁴⁵Most of the dynamic structural corporate finance papers use a finite sample of the simulated panel. This is a convenient way to estimate via SMM when the stochastic process is an exogenous Markov process. The same random draw must be used for every iteration of the minimization procedure. However, it is impossible to use the same random draw for dynamic discrete choice model because the endogenous choice is randomly drawn.

D Appendix: Calibration

D.1 Symmetric 2-state Markov Chain of Productivity

I calibrate a 1st-order Markov process with transition $g_z(z'|z)$ to AR(1) process: $z' = \rho z + \varepsilon_z$. $\varepsilon_z \sim N(0, \sigma^2)$ is an idiosyncratic i.i.d. shock to the productivity process with the standard deviation σ . I discretize AR(1) process to a symmetric 2-state Markov chain. The benchmark model uses parameters ρ and σ are calibrated to İmrohoroğlu and Tüzel (2014). The parameter of persistence ρ (the standard deviation σ) is 0.70 (0.27). I compute the mean and standard deviation of ρ and σ in other literature for comparison.⁴⁶ The mean of ρ (σ) is 0.69 (0.20) and standard deviation is 0.06 (0.11).

Productivity follows the 1st order Markov process with transition $g_z(z'|z)$. I use the Rouwenhorst method to discretize AR(1) process to the symmetric 2-state Markov chain s.t. $N_z = 2$ and p = q where $p \equiv g_z(z'_H|z_H)$ and $q \equiv g_z(z'_L|z_L)$. The Rouwenhorst method has a simple analytical expression (See Kopecky and Suen (2010)) for the transition matrix:

$$g_z = \begin{pmatrix} g_z(z_H'|z_H) & g_z(z_L'|z_H) \\ g_z(z_H'|z_L) & g_z(z_L'|z_L) \end{pmatrix} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$$
(47)

where $p=q=\frac{1+\rho}{2}=0.85$ and the productivity vector ($z_H z_L$)' = ($\sigma -\sigma$)' = (0.27 -0.27)' after substituting in calibrated parameters from Table 1. Therefore, the speed of transition comes from the persistence of productivity ρ and the type difference comes from the standard deviation σ . Each year 15% of the firms change his/her state upwards or downwards. Note that the lower and the upper bounds of type score are $\underline{s} = g_z(z_H'|z_L) = 0.15$ and $\bar{s} = g_z(z_H'|z_H) = 0.85$.

E Appendix: Model Solution

E.1 Decomposition of Expected Recovery

To illustrate the link between default and recovery, I decompose expected recovery into expectation and covariance terms:

 $^{^{46}}$ Gomes (2001); Hennessy and Whited (2007); Khan and Thomas (2013); Corbae and D'Erasmo (2020); Crouzet (2017)

Expected recovery = {Expected PD×Expected RR at default + cov(PD, RR at default)}
$$\times \underbrace{q_M^{-1}b}_{\text{EAD}}$$

Expected recovery consists of four factors: expected PD; expected RR at Default; Covariance of PD and RR at default; and Exposure At Default (EAD). Although the covariance term is not easily observed in real-world data, the model is able to simulate an economy to gauge the contribution from the covariance term. Table A7 explains the decomposition of expected recovery divided by EAD conditional on credit rating category. Expected recovery divided by EAD represents recovery per dollar of lending including interest rates. The covariance term is negligible within B or higher credit ratings. In turn, the covariance term accounts for 30% expected recovery within CCC/C credit rating. Moreover, I find PD and RR at default have a positive correlation. RR at default is higher for the high productivity type and because it borrows more than the low productivity type, the likelihood of bankruptcy is higher as well.

Table A7: Decomposition of Expected Recovery Divided by EAD

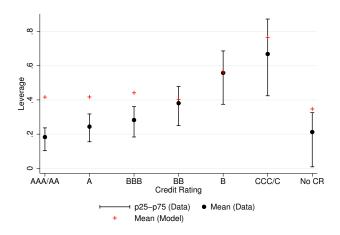
	Investment Grade		Speculative Grade			
S&P Credit Rating	AAA/AA	A	BBB	BB	В	CCC/C
Expected recovery divided by exposure at default (%)	0.08	0.15	0.29	0.34	0.63	2.61
Expected PD×Expected recovery rates at default (%)	0.08	0.15	0.29	0.29	0.50	1.85
Covariance (%)	0.00	0.00	0.00	0.05	0.13	0.76

Note: Expected PD and recovery rates at default are computed in equations (24) and (29).

E.2 Leverage and Credit Ratings

Figure A4 depicts leverage by credit ratings. The model captures riskier firms in speculative grades have higher leverage in the data.

Figure A4: How Are Credit Rating and Leverage Related?

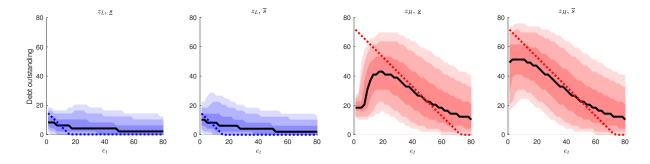


Note: The plus (circle) markers represent the model (data). The upper (lower) hinge of the whiskers is 75th (25th) percentile in the data. The x-axis is the credit rating category. "AAA/AA" ("CCC/C") includes "AAA" ("CCC") and "AA" ("C"). Rating is issued by S&P. The y-axis is leverage measured by debt-to-assets. The data source is Compustat. Sample period is 1986 to 2015.

E.3 Policy Functions

Choice of Debt Conditional on Market Debt.

Figure A5: Policy Functions of Debt Outstanding Conditional on Market Debt



Note: See the note in Figure 3. The y-axis shows the equilibrium average policy function conditional on market debt is computed by summing up next period equity e' (equation (48)).

Policy Functions of Debt Outstanding
$$(e, z, s, b) = \sum_{e' \in \mathcal{E}} \sigma_{\omega_B}(\omega_B)|_{\phi = M} / \sum_{(b, e') \in \{\mathcal{B} \times \mathcal{E}\}} \sigma_{\omega_B}(\omega_B)|_{\phi = M}$$
 (48)

Choice of Bankruptcy. Table A8 shows the equilibrium choice of bankruptcy by chapter. I compute the conditional likelihood of bankruptcy from a simulated panel. There are two key findings from the estimated model. First, small firms (measured by internal finance or

total assets) with high productivity are most likely to file bankruptcy. The reason is that high productivity firms are more levered conditional on small internal finance. Second, only low productivity firms with small size (measured by internal finance or total assets) use Ch. 7 bankruptcy (panels A and B). This is because low productivity firms do not benefit from filing Ch. 11 bankruptcy to gain a going-concern value by continuing business (see Appendix B.5 for an analytical discussion).

Table A8: Choice of Bankruptcy among Ch. 11 Reorganization and Ch. 7 Liquidation By Size

	Pro	Fraction of Ch. 7				
_	Ch. 11 (%)		Ch. 7	(%)	(%)	
Size Percentile	z_L	z_H	z_L	z_H	z_L	z_H
Panel A: Internal Finance				_		
<25%	1.11	2.20	0.57	0.00	33.83	0.00
25%-50%	0.89	0.72	0.12	0.00	12.31	0.00
50-75%	0.68	0.26	0.01	0.00	1.12	0.00
>75%	0.45	0.15	0.00	0.00	1.00	0.00
Panel B: Total Assets						
<25%	0.65	0.51	0.56	0.00	46.61	0.00
25%-50%	1.21	0.83	0.01	0.00	0.73	0.00
50-75%	0.93	0.65	0.00	0.00	0.49	0.00
>75%	2.25	0.41	0.00	0.00	0.00	0.00

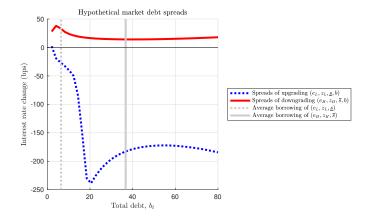
Note: I generate 10,000 firms for 50 years from the benchmark model. The size percentile is a distribution among internal finance and total assets. Fraction of Ch. 7 is equal to the ratio of the likelihood of Ch. 11 bankruptcy over total bankruptcy (Ch. 11 bankruptcy plus Ch. 7 bankruptcy).

E.4 Benefits of Type Score

I study the effects of debt pricing from monitoring. A direct measure of the benefit of having a good reputation is the price of one-period public debt q_M . I compare price reactions to the exogenous change in type score. In Figure A6, I plot a change in interest rates q_M^{-1} on the vertical axis with respect to type score by controlling for the size of debt b on the horizontal axis (equations (49-50)). I consider two scenarios. First, the low productivity firm z_L with the lowest type score \underline{s} receives an offer as if the firm has the highest type score \overline{s} (dotted blue line). Second, the high productivity z_H firm with the highest type score \overline{s} receives an offer as if the firm has the lowest type score \underline{s} (solid red line). The high (low) productivity firm experiences roughly 14 bps (26 bps) increase (decrease) in market debt funding rates when the firm's reputation downgrades (upgrades). Interestingly, the magnitude of information

revelation fits the literature.⁴⁷

Figure A6: Impact of Information on Conditional Market Debt Returns by Total Debt



Note: In the figure, solid red (dotted blue) lines are firms with high (low) productivity whose types are almost truthfully revealed. The panel plots return spreads of upgrading (dotted blue line) in equation (49) and downgrading (solid red line) in equation (50). The vertical solid (dotted) line is the stochastic steady states for high (low) productivity. Hypothetical market debt spreads are calculated using formulas:

Spreads of upgrading
$$(e_L, z_L, s, b) = \sum_{e' \in \mathcal{E}} \sigma_{b,\phi}(e_L, z_L, \bar{s}, b, \phi) \underbrace{\left(q_M(\overline{\omega}_M)^{-1}|_{s=\bar{s}} - q_M(\overline{\omega}_M)^{-1}|_{s=\underline{s}}\right)}_{\text{Decrease in borrowing costs}(<0)} |_{e=e_L, z=z_L}$$
 (49)

Spreads of downgrading $(e_H, z_H, \bar{s}, b) = \sum_{e' \in \mathcal{E}} \sigma_{b,\phi}(e_H, z_H, \bar{s}, b, \phi) \underbrace{\left(q_M(\overline{\omega}_M)^{-1}|_{s=\underline{s}} - q_M(\overline{\omega}_M)^{-1}|_{s=\bar{s}}\right)}_{\text{Increase in borrowing costs}(>0)} |_{e=e_H, z=z_H}$ (50)

where the average policy function is $\sigma_{b,\phi}(e,z,s,b,\phi) \equiv \sigma(e,z,s,b,\phi,e')/\sum_{\tilde{e}'\in\mathcal{E}}\sigma(\tilde{e},z,s,b,\phi,\tilde{e}')$. Internal finance e_H (e_L) is the mode of the stationary distribution conditional on high (low) productivity z_H (z_L) and highest (lowest) type score \bar{s} (\underline{s}) firms. Average borrowings are defined as the average debt outstanding conditional on the average internal finance, productivity, and type score.

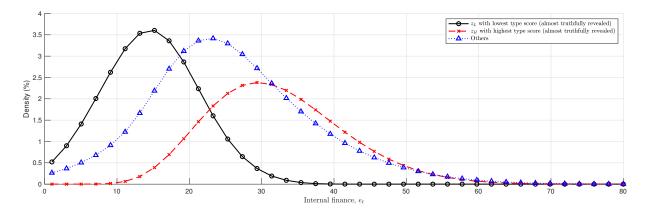
E.5 Stationary Distribution Γ

Figure A7 depicts the stationary distribution in the equilibrium. I show three different groups of distribution among internal finance. First and second groups are the types revealed almost perfectly (solid black line and dashed red line). The high productivity firm accumulates more internal finance because the MPK is higher than that of the low productivity firm. The third group is comprised of those firms whose types are not almost perfectly revealed (blue dotted

⁴⁷Tang (2009) is one of the papers that investigates the real effect of credit market information asymmetry. He finds that credit market information asymmetry significantly affects firms' real outcomes. In addition, he points out that the potential importance of precise information revealed by Moody's rating refinement changes yield spreads by 15-30 bps on average.

lines). This group includes the new entrants with median type score and the smallest internal finance as well as the low productivity firms with the lowest type score who experience an upward shift in productivity (the "rising stars").

Figure A7: Stationary Distribution of Internal Finance By Productivity and Type Score



Note: The stationary distribution of internal finance $\Gamma(\omega)$ is computed from the estimated model. The lowest and highest type scores are s and \overline{s} . Others (blue dotted line) are the sum of firms that excludes firms with the lowest or highest type scores.

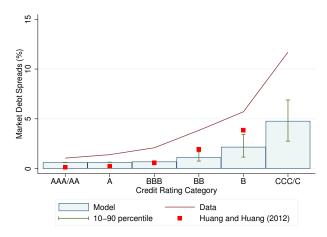
E.6 Credit Spreads and Credit Ratings

Figure A8 shows credit spreads for each bin of credit ratings from the model. As most models of the structural approach to credit risks are subject to the credit spread puzzle (model estimates from a base case in Huang and Huang (2012) are compared to my model in this Figure), my research fails to explain the actual level of credit spreads.^{48,49} Although my model does not target historical default rates, recovery rates, and equity premiums, the model predictions are close to other models in the literature.

⁴⁸This is a comparison between traditional option pricing models. My model is differing from the Merton model (Merton (1974)), since it endogenies firm size, leverage, debt type, and bankruptcy. Moreover, the total economy is characterized by endogenous distribution. All of these mechanisms are absent in the Merton model.

⁴⁹See Huang and Huang (2012) for an extensive literature review.

Figure A8: Credit Spreads of Market Debt By Credit Rating: Model versus Data



Note: The figure compares the model and data generated market debt spreads among credit ratings. I generate 10,000 firms for 50 years from the benchmark model. Firms are assigned to 6 credit rating categories in Table 4. Market debt spreads in the data are the historical average of ICE BofAML option-adjusted returns by credit ratings. Option-adjusted credit spreads are calculated spreads between Option-adjusted Spread Index of all U.S. corporate bonds in a rating category and a spot Treasury curve. The sample period is 1986 to 2015. Market debt spreads in the data "AAA/AA" corresponds to AA option-adjusted returns (the share of AAA bonds are negligibly small). Model estimates from Huang and Huang (2012) are reported in the figure. The estimates are from the base case results (column 7) in Table 2. "AAA/AA" corresponds to "Aa" in their paper.

F Appendix: Additional Counterfactual

F.1 Parameter Choice of Productivity

Figure A9 demonstrates the change in consumption from the benchmark model to counterfactual models with full monitoring technology. Equation (32) describes the market debt pricing function in the counterfactual. I solve models with different parameters of the productivity process. Around parameterizations of benchmark model ($\pm 10\%$ range) without reestimating other parameters, the increase in consumption is more pronounced if the economy has lower persistence of productivity ρ and the standard deviation of productivity shock σ . A low ρ implies types are changing more frequently while a low σ corresponds to a small type difference. In these cases, adverse selection becomes more severe. Types are harder to predict because types change too frequently before lenders learn the borrower's type. Therefore, asymmetric information creates larger inefficiency. However, the effect is non-monotonic. When both ρ and σ are small enough, the inefficiency associated with asymmetric information becomes smaller (or even negative).

Baseline

1
0.5
0.5
0.29
0.62
0.64
0.66
0.68
0.7
0.72
0.74
0.76
0.78
0.24

Persistency of productivity, ρ
Std. dev. of productivity shock, σ

Figure A9: Parameters of Productivity to Welfare Change

Note: Baseline models (a model with asymmetric information and a model with full monitoring technology) use the parameters persistence of productivity $\rho=0.7$ and the standard deviation of productivity shock $\sigma=0.27$. Simulation exercises exhibit changes in ρ and σ . Other model parameters are fixed. The z-axis shows the change in consumption in percentages from the benchmark model to the counterfactual model with full monitoring technology.

F.2 Change in Technology

Partial Monitoring Tech. Asymmetric information affects corporate bond pricing through expectations of PD and recovery at default. I decompose these contributions by solving counterfactual models with partial monitoring technologies. The first type of partial monitoring technology on PD is:

$$q_{M}(\overline{\omega}_{B}) = \frac{(1 - \overbrace{\sigma_{\Delta=1|\omega_{B}}(\omega_{B})|_{\phi=M}}^{\text{PD}})b}{(1 + \mu_{M})q^{-1}b - \sum_{\substack{y \in \{c11,c7\}}} \sigma_{y|\omega_{B}}(e, z, s, b, \phi, e') \sum_{\hat{z} \in \mathcal{Z}} \mathcal{R}_{M}^{(y)}(e, \hat{z}, s, b, e')s(\hat{z})}$$

$$(51)$$
PD×Expected recovery at default

This technology has an ability to screen PD but still relies on the expectation of recovery at default by type score. The second type of partial monitoring technology on recovery at default is:

$$q_{M}(\overline{\omega}_{B}) = \frac{(1 - \overline{\Lambda_{M}(\overline{\omega}_{M})})b}{(1 + \mu_{M})q^{-1}b - \sum_{\underline{y} \in \{c11, c7\}} \mathcal{R}_{M}^{(y)}(e, z, s, b, e') \sum_{\hat{z} \in \mathcal{Z}} \sigma_{y|\omega_{B}}(e, \hat{z}, s, b, \phi, e')s(\hat{z})}$$
Recovery at default×Expected PD

Bondholders screen recovery at default and expected PD is subject to information asymmetry.

Strong Debt Enforcement Tech. Another comparison to the benchmark model is to allow for a collective action to renegotiate on corporate bonds. This turns cash flow-based debt into asset-based debt. In the model, I replace recovery at default in equation (14) to

$$\mathcal{R}_{M}^{(c11)}(\overline{\omega}_{B}; \varphi_{M}(\overline{\omega}_{B})) = \mathcal{R}_{M}^{(c7)}(\overline{\omega}_{B})$$
(53)

which effectively alters Assumption 4 of flexibility. This resembles a renegotiation technology for bank lenders with liquidation threat (equation (19)).

F.3 Results

Table A9 alters the model assumption about market debt renegotiation. In alternative models (iii) and (iv), I allow the borrower and bondholders enter to renegotiation stage. Therefore, market debt is asset-based debt in these alternative models.

Table A9: Comparison of Simulated Moments Between Benchmark Model and Model with Symmetric Information Under Flexibility

	Data	Benchmark	Counterfactual	Alternative benchmark	Counterfactual	
		(i)	(ii)	(iii)	(iv)	
Panel A: Techonology						
Monitoring by bondholders			\checkmark		\checkmark	
Bond flexibility under Ch. 11				\checkmark	\checkmark	
Panel B: Capital Structure and Welfare						
Debt	n.a.	20.80	22.74	18.04	18.36	
Debt (zL)	n.a.	3.22	3.22	3.66	3.66	
Debt (zH)	n.a.	17.58	19.52	14.37	14.69	
Equity	n.a.	24.24	21.86	24.36	24.01	
Equity (zL)	n.a.	9.52	8.57	8.98	8.85	
Equity (zH)	n.a.	14.72	13.28	15.38	15.16	
Aggregate bank debt ratio	0.31	0.21	0.15	0.24	0.22	
Consumption	n.a.	1.380	1.398	1.281	1.283	
Change in % compared to full info	n.a.	n.a.	1.35	n.a.	0.14	
Output	n.a.	12.81	12.77	12.29	12.29	
Capital	n.a.	45.03	44.60	42.40	42.37	
Change in % compared to full info	n.a.	n.a.	-0.97	n.a.	-0.08	
Capital (zL)	n.a.	12.74	11.80	12.65	12.51	
Capital (zH)	n.a.	32.30	32.80	29.75	29.86	
TFP	n.a.	1.079	1.082	1.076	1.077	
Change in % compared to full info	n.a.	n.a.	0.29	n.a.	0.05	
Panel C: Bankruptcy						
Bankruptcy prob. (Ch. 11) (%)	0.72	0.72	0.85	0.72	0.76	
Bankruptcy prob. (Ch. 7) (%)	0.14	0.14	0.12	0.19	0.18	
Panel D: Market Debt Recovery Rates						
Mean	0.45	0.32	0.36	0.62	0.61	
Standard deviation	0.38	0.37	0.34	0.20	0.20	
Interquartile range	0.73	0.69	0.66	0.27	0.24	
10th percentile	0.00	0.00	0.00	0.41	0.41	
90th percentile	1.00	0.88	0.81	1.00	0.98	

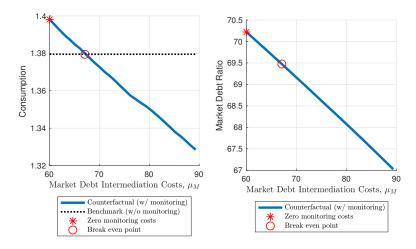
Note: The first column in the table reports the benchmark model results. The counterfactual model (ii) and (iv) assumes that bondholders possess monitoring technology without any additional costs. Change in % compared to full info in column (ii) (column (iv)) computes consumption changes between benchmark (i) (alternative benchmark (iii)).

F.4 Monitoring Costs

I solve counterfactual models with full monitoring technology and different market debt intermediation costs μ_M . In the benchmark model, μ_M is set to 60bps. I report the level of consumption (market debt ratio) in the left (right) panel of Figure A10. The horizontal line in the left panel is the level of consumption in the benchmark model with asymmetric

information in corporate bond markets. The intersection of these two lines is the break-even point of welfare improvement.

Figure A10: Monitoring Costs of Market Debt And Break-even Point



Note: The solid blue lines show the counterfactuals under different parameterizations of market debt intermediation costs μ_M ranges from 60 to 90 bps. The y-axis exhibits consumption (market debt ratio) in the left (right) panel.