CDS Central Counterparty Clearing Liquidation: Road to Recovery or Invitation to Predation?

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Abstract

Recent regulation, mandating the clearing of credit default swaps (CDS) by a Central Clearing Counterparties (CCP), has rendered it's possible failure a serious threat to global financial stability. This work investigates the potential failure of a CCP initiated by the default of a large dealer bank and the unwinding of its positions. The theoretical model examines variation margin exchange between dealer banks and the price impact of liquidation and predatory selling. It provides a measure of covariance between assets in banks' portfolios; price impact affects assets to varying degrees, based on their relative distance to defaulted assets. Key results show that liquidation lowers CCP profits, and how predation decreases the profits of all members, pushing banks to default. Furthermore, a hybrid CCP (vs. current) structure provides a natural disciplinary mechanism for predation. Also, it is more incentive compatible for the CCP, in expectation of a large loss. A multi-period, dynamic simulation, calibrated to OTC market data, provides parameter sensitivities concerning the magnitude of CCP and predatory bank gains/losses, specifically, the minimisation of those losses with a hybrid fund structure. Furthermore, regulatory implications concerning the timing of liquidity injection for a Lender of Last Resort (LoL) are determined for various liquidity scenarios; stable and decreasing market liquidity, as well as, a liquidity dry-up at the bottom of a financial crisis.

Keywords: Systemic Risk, CCP Recovery, CDS, CDS Spread Fire Sales, Liquidation, Predation, Price Impact, Contagion, Financial Network, Over the Counter Markets.

JEL Classification: G00, G01, G02, G14, G10, G18, G20, G23, G33

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1 Introduction

Regulatory reform and the introduction of CCPs for standardised CDS contracts has greatly lowered counterparty risk through compression of trades, better collateralisation, and more efficient risk-sharing. However, it has also increased the threat to the global financial system should a CCP fail. There is expressed concern that CCPs have become "too-big-to-fail", and that new dangers arise specifically for CCPs dealing with CDS. Thus, more rigorous investigation is needed into the way CDS function *within* a CCP; the mechanisms which can lead to default, its prevention and strategies for CCP recovery. This work focuses on this under-developed aspect of current research, which addresses tools/strategies which aid CCP recovery and provides guidance for interventions by a Lender-of-Last Resort. It is imperative to develop tools which allow the CCP to identify and recover from improbable, but possible scenarios which can destabilise the global financial system. By identifying dealer default and liquidation as a possible CCP default mechanism, and an alternate (hybrid) guarantee fund structure as a recovery tool, this work provides guidance for CCPs and policy-makers through theoretical and simulation results.

Building on previous frameworks, this paper provides a novel approach for analysing, simultaneously, the effect of both predation and price-feedback on the amplification of illiquidity cascades. Furthermore, it is the first to place these two mechanisms in the context of a CCP, and an asset liquidation. It identifies the tools available to the CCP to mitigate fire-sale contagion. By devising an explicit way to model the change in cds-spread, caused by the price impact of CCP liquidation and predatory behaviour, the paper establishes four very important results concerning financial, CCP, and member banks' stability. First, we find that the CCP always decreases its profits by using liquidation as a method to offload a defaulter's positions. Second, we find that predation will always lower the profits of, both, the CCP and the predatory banks. Importantly, we also identify the competing effects which determine whether a bank has the incentive to predate or not. Third, we find that the CCP already has a punitive mechanism in place, the initial margin, which can efficiently dis-incentivise predation, provided that the CCP employs the proper guarantee fund structure (hybrid structure). Fourth and finally, we find that the proposed *hybrid* structure for the guarantee fund, which allows direct netting of initial margin against liabilities, is more incentive compatible than the guarantee fund structure which is currently in place. Furthermore, when the CCP faces an extremely large defaulter and a large enough shortfall to wipe-out its equity, this hybrid structure leads to a smaller probability of failure and a higher likelihood of recovery. In this way, the paper defines the various mechanisms that a CCP could employ to increase its stability, which when extended over multiple CCP's could strengthen the stability of the global financial network.

One of the most dangerous and likely risks for the CCP is fire-sale risk. Both, the structure of the CPP and the way CDS are traded, provide the perfect parameters for a fire-sale to take place. Yet, this work is the first to attempt a theoretical description and quantification of fire-sale risk in this context. In order to understand this risk, one must look at the composition of the market and how defaults play out in a CPP. The market for CDS is very concentrated, with approximately 14 large dealers dominating 80% of the gross notional value [Cont, 2010]. These dealers are also members of all the major global CCPs for CDS. The combination of large wrong-way risk for CDS,[1] the asymmetrical liabilities for CDS counterparties, and the recent example of AIG, makes the default of a large member a serious concern for the global stability. The default of such a large member becomes an extreme liability for the CCP, which is effectively left holding the exposures of that dealer. The CCP must unwind the positions of the defaulted member, while at the same time paying out any variation margin that is accruing on that dealer's account.[2]

There are further aspects which make a fire-sale contagion likely. The CCP holds very little equity,

[1]Wrong-way risk according to ISDA, is the risk that occurs with an exposure to a counterparty which is adversely correlated with the credit quality of that counterparty.

[2]This is especially problematic given that derivatives contracts are not subject to *automatic stay*[3], thus counterparties which are owed payments often call them in, while others owing payments often choose not to pay. [Fleming and Sarkar, 2014]

and cannot carry positions for an extended period of time. It collects margins, pooled into a guarantee fund, meant to cover variation payments over a 5-day liquidation window. Yet, margining calculations don't consider hidden illiquidity (caused by member failure in multiple CCPs), or factors which may further increase market illiquidity. Furthermore, these margins do not belong to the CCP, but to the contributing member. The (pure) guarantee fund, though seemingly large, cannot be indiscriminately applied to cover the variation payments of all defaulting members during liquidations. The default fund, which could be used for this purpose, is comparably small to the size of member positions, and insufficient to cover a large default during market stress. Certainly, the CCP will always liquidate while trying to balance market risk and liquidity risk, but in the face of its equity constraint, it's distressed selling can increase other members' distressed selling, as well as, predatory behaviour by unconstrained dealer members.

It is this last point which is especially worrying in terms of fire-sale contagion and is the focus of this work. Not only is the CCP constrained, but a rapid liquidation of a large position, given the market concentration, is a clear signal of the CCP's distress. Given that most transactions are dealer-to-dealer in a small group of large dealers, it is a reasonable assumption that each dealer has a rough idea of who has defaulted and some idea of their exposures. In addition, the CCP's liquidation time-line is common knowledge, due to the dealer-members' own initial margin requirement. The largest un-distressed members have an incentive to engage in predatory behaviour, accelerating contagion. In fact, this work is the first to address predation within a CCP and provides a unique solution to dis-incentivise this acceleration mechanism by exploiting the features of the hybrid guarantee fund. Finally, liquidation increases the covariance between different CDS, which can lead to spillover effects within the CCP, and throughout the network. We propose a unique, mathematical way to look at asset co-movement – and thus, price mediated contagion – as a *covariance map*; a mathematical description of the strength of the covariance effect that portfolios' assets (CDS) exert on each other in terms of relative distance. In short, this paper analyses a CCP liquidation scenario which is defined by marked contagion, amplification and spillover effects. These mechanisms arise and propagate by way of many channels; information leakage, predation, price-feedback effects, and illiquidity/insolvency cascades. Therefore, the main contribution lies in *simultaneously* addressing these channels through the lens of CDS covariance, which results in a change of the cds-spread, and thus, also in the variation margin exposure of the CCP. With the addition of a full numerical implementation, it not only provides theoretical insight on the destabilizing mechanisms involved, but also on the possible magnitude of loss caused by large liquidations and a CCP failure .

As proven in reality, large dealers have an incentive to engage in predation, given the belief that they will profit from the CCP liquidation. Being the first to introduce predation into a CCP framework, we find that due to the effects of CDS covariance and liquidation price feedback on variation margin, there is unanticipated spillover of varying degrees onto all assets held by the CCP; thus, to the predators assets. This work overwhelmingly illustrates that, as a result, predators can drive their own illiquidity and eventual insolvency, as well as that of fellow members. The more defaults which occur, the greater the increase in covariance between assets. This amplifies contagion and increasingly depletes the default fund of the CCP, driving it to failure. Importantly, any profit made by a single or colluding predators will optimally occur after the CCP liquidation is over. However, with increasing competition that optimal strategy is no longer viable; predators start buying back earlier, before the CCP liquidation is finished, which leads to decreased profits. The model results display that lower than expected profits, can further lead to the predators' illiquidity. This result, which is not immediately intuitive, though strongly illustrated by the simulation, ultimately destroys the belief that their is an benefit to predation.

Certainly, there may be some predators which have profited from the liquidation. Thus, this work isolates a clear tool – provided the CCP employs a hybrid guarantee fund structure – that both dis-incentives predation and aids CCP recovery. The risk-sharing structure of the hybrid fund, means that CCP liabilities can be met by the full guarantee fund. Thus, increased predation increases the likelihood that the predator's own guarantee fund contribution is depleted to meet other's liabilities. As contracted clearing privileges demand sufficient initial margin, the CCP can make multiple margin calls, demanding that predators refill their initial margins, and garnishing predator profits until liabilities are fully met. In this case, there is proof that there exists a dis-incentive for the predation motive, which the CCP can utilise both punitively and to aid in it's own recovery. This method employs a legal mechanism which the CCP already has in place, has been contracted on, and is a necessary condition to maintain CCP clearing privileges. Indeed, the model's numerical results highlight the power of this mechanism.

In order to obtain these results, one must first look closely at the mechanism of how variation margin is accrued and exchanged between counterparties. In this way, this paper is related to literature on financial networks which model the exchange of interbank liabilities. Papers such as [Cont et al., 2013b, Eisenberg and Noe, 2001] model a financial network of exogenous interbank liabilities, realisation of clearing payments and the proceeding insolvency cascade. However, unlike this work, they do not model liabilities cleared through a CCP. My work is also related to [Amini et al., 2015] in which the authors examine interbank liabilities in the context of a CCP, but they look at the network of bank liabilities with the CCP in order to identify the proper guarantee fund structure that is incentive compatible for member banks. In this work, we are interested in how network liabilities, cleared through the CCP, drive member's incentives to predate on a distressed CCP. Though they identify how their model is applicable to CDS, unlike my paper, they do not model these features explicitly. [Cont and Minca, 2014] address liabilities driven by CDS explicitly, and the change in the nominal value of positions, however, they do not model the drivers that cause the change in variational margin. To my knowledge, this the first paper which explicitly details the pricing mechanism that drives variation margin in this context.

The contribution of this paper to the existing literature is that it combines two strands of literature, financial networks and price impact with the feedback loop of predation. It is the first to explicitly model the price process in this context, and the mechanism of exchange of liabilities. Furthermore, it breaks down the common trading period structure to smaller time-steps allowing one to see the amplification mechanism in fire-sale contagion explicitly; it illustrates the underlying drivers of fire-sales and the cumulative effect over time. This paper is also the first to look at forced liquidation in a CCP, modelling particularities and complexities of liquidation of CDS positions. Finally, it is the first to look at the punitive possibilities for dis-incentivising the predatory behaviour that plagues all markets, and identifies novel (regulatory) tools to dissuade such behaviour; the initial margin (and a hybrid guarantee fund structure.)

This paper is related to literature on liquidations with price feedback effects. It looks at the effect of a large member's default and the CCPs mechanism of the subsequent liquidation mechanism of that member's position. In [Oehmke, 2014], the author models the price impact of distressed selling on constrained dealers, and the losses induced by crowded trades. However, unlike my work, it looks at large dealers liquidating illiquid collateral in the repo market rather than CDS. There is an important distinction in modelling the fundamental price changes between the two asset classes. In CDS, price changes are caused by various factors which drive changes in credit spread, which makes adequate modelling difficult. This is in contrast to standard asset classes, which can best be modelled as driven by brownian motion with a mean and volatility driven by information changes in the market. This paper also addresses the effect of predation by other member's during a CCP liquidation as in [Brunnermeier and Pederson, 2005]. In this paper, Brunnermeier looks at a group of large dealers, where the distressed dealers liquidate and the non-distressed dealers predate. He looks at how predators decide on their trading rate, and the timing of selling and buying amongst a single and competing predator. However, in his model dealers operate in a centralised market with standard asset classes, while in our model we have an opaque OTC market, trading a derivative class where both legs of the CDS contract can be sold off as 'separate' assets.[4]

This work is also related to work on the covariance between difference asset classes. CDS pose a

[4]Both the buy and sell sides of a CDS contract can be traded separately, or novated to another party. Recent reforms have instituted trade compression in order to reduce the amount of intermediaries and redundant positions which arise from this process.[Duffie et al., 2010]

real threat to the financial system precisely because they exhibit marked covariance with each other. Furthermore, in times of crisis they can vary with different asset classes. In [Cont and Wagalath, 2013, Cont and Wagalath, 2014] the authors model the effect of liquidation and distressed selling on the volatility of unrelated assets in different portfolios. This occurs through a realised covariance that arises from distressed selling amongst hedge funds that have cross-holdings. Similarly my model directly accounts for the covariance of different CDS in the various member's portfolios, with the defaulted asset. However, my approach is theoretical, applying to a financial network of derivatives assets in a CCP while the former is rather empirical, and focuses on assets such as index funds cross-held by hedge funds. There is a fundamental difference in the incentives and decisions that fuel liquidations.

Finally, this paper is related to work on movements in credit spreads. In [Pierre Collin-Dufresne, 2001] and [Pu and Zhao, 2012], the former identifies that variables which should explain credit spread changes only have a fraction of explanatory power, while much of the change is driven by a common systematic component. In the latter, the authors show that the common systemic component seems to have a time dimension which completely explains co-movements in credit default spreads. This paper is the first to explicitly model this unexplained component through the higher order effects of price impact and predation, and model the correlation between CDS. [Tang and Yan, 2013] empirically identifies the determinants of the fundamental portion of cds-spreads and the effects of excess demand and supply. This paper expands theoretically on these insights, creating a novel way to introduce both the fundamental cds-spread and the insights on excess demand in the unexplained portion.

2 The Model

2.1 Background

This section presents the basics of CDS contracts and provides an overview of the CCP functions. CDS contracts and the market possess special features, different from other assets, with unique risks and benefits. CDS are derivatives traded in an OTC-market rather than a centralised exchange. They come in standardised form or in more complicated bespoke forms, though only standardised forms are currently cleared by CCPs.

This paper examines the intricacies of CDS, specifically the mechanism of variation margin and its potential for large fluctuations. CDS are traded as single-name or as an index. The standard, most liquid contracts have a term of 5 years, with less liquid 10 year or off-the-run issues also available. CDS act effectively as an insurance contract which one party can buy from another, on a third-party reference entity. They can be used to hedge a position in a bond (issued by the underlying reference entity), or they can be bought speculatively (naked). The number of CDS contracts purchased can far outweigh the number of physical bonds available for a reference-entity, requiring special procedures at the time of default. Each CDS contract consists of a buy-leg and a sell-leg; the buyer is long protection and the seller is short protection. Each part of the contract can be sold off autonomously. In fact, CDS are often not held to term, but rather each leg is bought and sold as needed. In recent years, regulation has introduced *trade compression* in order to reduce redundant positions and the chain of intermediaries – between direct counterparties – resulting from multiple sales.

The introduction of the CCP into this market is supposed to enhance its stability. The CCP's role is as an important intermediary to every CDS transaction; it acts as the buyer to every seller, and the seller to every buyer. This function allows it to *compress*[5] trades efficiently, and is a bilateral netting benefit. Since the highly concentrated CDS market is dominated by 14 large dealer members, these members tend to be member's of all large, global, systemically important CCPs. Therefore, though the CCP offers risk-sharing benefits, it can also pose a global threat through the concentration of counterparty risk. Critics of the CCP have also cited the loss of multilateral netting benefits[6], as another problem with

[5] Trade compression is the action of removing redundant trades or multiple intermediaries between two counterparties which increase counterparty risk, but not market risk.

[6] The benefit of netting across trades in multiple asset classes with the same counterparty

the institution. However, [Augustin et al., 2014] which references Cont and Kokholm [2014], points out that multilateral netting through a CCP outweighs "the losses of bilateral netting if sic(they) account for correlations and heterogenous risk characteristics of cleared assets." This is interesting as CDS, in particular, show marked covariance with other asset classes such as equities and bonds (underlying the CDS) which can further destabilise the global financial network.

The CCP, in order to minimise counterparty risk, demands that each member contributes an initial margin determined by the size and risk profile of it's portfolio (in terms of netted positions). In practice, each of these contributions is proprietary, and by law, can only be applied to the contributing member bank, itself. In total, these contributions make up the guarantee fund[7]. In addition, each member must also contribute to the default fund, which can be fully applied to any member's shortfall. However, the default fund is estimated, by various parties, to be small in comparison to the guarantee fund, and considered to be insufficient to offset the risk exposure of most positions. Though the CCP has the incentive to maintain a sufficient collateral buffer to prevent the use of it's own equity[8], problems still exist in the way it assesses risk and determines margin contributions. The CCP sets large initial margins on highly unilateral positions, however, it ignores the risk associated with large netted positions; in default, positions which are highly risky, but fully hedged and seemingly safe, are often woefully under-collateralized. Furthermore, the largest dealers are all members of multiple global CCPs and have the ability to spread their positions; small exposures in each, result in low collateral demand overall[9]. Since their are no significant multi-lateral CCP agreements, should such a member default[10], each CCP may face a significant shortfall as a large-scale liquidation ensues.

This work directly addresses the liquidations in a CCP and their amplification into a large scale fire-sale through the mechanisms of price impact and predation. When a member bank defaults for any reason, the CCP is left holding the positions. The CCP is not capable, financially or legally, of holding on to these positions for a significant time. It is constrained to offload them during a pre-set window, a period which the initial margin is pre-supposed to cover. In this model, it is supposed that the CCP liquidates by a search-and-quote liquidation strategy. This is a reasonable assumption as, in this over-the-counter (OTC) cds market, this is the strategy dealers employ with each other and commonly employed in literature concerning dealers in OTC markets. In reality, the CCP may use an alternate method, but due to this information's private nature, this method cannot be determined with reasonable certainty. Nevertheless, for modelling it is not of marked concern whether this liquidation is a direct OTC sale or a blind auction (where only the CCP can see the bids). In each case, the CCP and the members can act in the same manner; the CCP signalling the default, and predatory member banks selling in the same direction and depressing the price in order to buy back at a profit, through market quotes or submitted bids. The premise is the same in both cases; liquidation is an inefficient method for offloading positions, lowering the recovery value to the CCP, increasing the number of defaults in the network, and decreasing the profits of surviving members.

In order to enter a standardised CDS contract, one of the parties offers a buy or sell quote based on the credit default spread (cds-spread) for that reference entity. The amount of buy and sell quotes in the market signals demand and is one of the factors that moves cds-spread. The CDS contract commences when one party makes a up-front premium payment to the other. At this time, the buyer and the seller agree upon the notional amount to be insured, the price of the contract and the standard

[7]The current guidelines (SEC, CFTC, ESMA) require that the guarantee fund is of at least the size that captures the simultaneous default of 2 clearing members and 3 additional references entities. The reference entities are selected based on largest uncollateralised losses under extreme but plausible scenarios. [Ivanov and Underwood, 2011]

[8]The CCP has a *Waterfall* procedure it follows in the event of shortfall; it first uses the initial margin and default fund contributions of the member, its own small equity tranche, the full default fund, then it's full equity.

[9]We consider initial margin on, both, uni-direction long/short portfolio and well-balanced long short portfolios, representing different risks to the CCP. Well-balanced portfolios have the lowest margin levels, this can lead to the largest loss to the CCP. Also, members can spread risky positions between CCP's, paying a margin insufficient to compensate for the risk profile. [Cont, 2010, Cont, 2015]

[10]Using Lehman for a model to estimate dealer size and reach, it represented 5 percent of the global derivatives transactions, and was a member of at least 8 CCP's. [Fleming and Sarkar, 2014]

premium payment of 100 or 500 bps (on the notional) to be made quarterly, for the lifetime of the contract. The contract and payments are structured such that, at commencement, each leg of the contract has zero value. Thus, the upfront payment is the difference between the quoted price (bps) and the premium on the notional, over the lifetime of the contract. If the quoted price is higher than the premium, the buyer pays the seller; this supplements the shortfall from the *smaller* coupon payments accumulated over the lifetime of the contract. Cds-spreads fluctuate daily, and can be very volatile. Thus CDS positions are marked-to-market and a daily variation margin call is made to the party owing a payable, which that party pays to the CCP. The CCP then pays out the receivable to the counterparty on the other leg of the contract. The CCP, therefore, takes on the counterparty risk in the contract. If a counterparty on one side of the contract defaults, the CCP, must replace/sell that position or be liable for any variation margin calls.

Daily movements in the cds-spread determine the variation margin, which is paid when the spread moves above or below the agreed-upon premium. If the cds-spread moves higher than the premium, the buyer must pay the difference over the notional to the seller. This increase in the spread reflects the increased demand for insurance on that reference entity, the increased risk to the seller of having to deliver on the contract, or the decreased willingness of sellers to supply insurance. Similarly, if the cds-spread drops, the seller must pay variation margin to the buyer. This reflects the decreased interest/demand for insurance on the reference entity, increased supply in the market due to the selling *buy* positions, or increased supply due to selling *sell* positions (reflecting sellers' wish to offload their positions).

The cds-spread has fundamental determinants, which have been, in part, empirically identified by [Tang and Yan, 2013]. These authors have identified the factors which drive about 40% of the explainable determinants of cds-spread movements. Cds-spread on a reference identity changes with information on firm or industry fundamentals, leverage being one important driver. Interestingly, cds-spread doesn't respond greatly to changes in the VIX, reflecting it's independence from investor sentiment. The effect of illiquidity is pronounced; it is common for only one or two trades daily on any one reference entity, causing volatile fluctuations in the cds-spread in response to market information.[Cont, 2010] Finally, excess demand and supply move cds-spreads considerably. The effect is usually temporary and tends to die out after 3 trades, as cds-spreads are mean-reverting. However, those effects which last up to 5 days will permanently drive the cds-spread upward. However, excess demand tends to have an asymmetrically larger effect than excess supply.

A counterparty to a CDS contract can default due to, either, a margin call or the default event of the underlying reference entity. This risk is asymmetric and far larger for the seller of the contract. The buyer or seller is subjected to an initial margin call if there are large cds-spread changes, and the variation margin owed is larger than their initial margin. If they cannot meet this margin call – perhaps by liquidating other assets – they are considered *in default*, and the CCP takes on the liabilities of their positions. The CCP faces a serious risk that cds-spreads will exhibit marked covariance: the variation margin demanded on multiple assets, held by a defaulted counterparty, can be so large and volatile as to deplete the CCP guarantee and default funds over the closeout period[11]. Furthermore, if any reference entities underlying a CDS actually default, the payment required is substantially larger. The combination of variation margin and insurance payment may mean the depletion of the default fund before the end liquidation period. This very plausible risk of CCP failure motivates this work. We seek to provide novel recovery tools for the CCP, to be used as soon as a serious threat to CCP viability is identified (ie. possible depletion of the default fund.)

Finally, CDS are traded in a concentrated market of dealers. Dealer trades are proprietary or made on the account of their clients, thus, transactions are dealer-to-dealer (DD) or dealer-to-customer (DC).

[11][Duffie, 2010] discuss the replacement cost of a portfolio owed to a counterparty, when one dealer defaults. The defaulting cost, inherent in the bid-ask spread, is effectively transferred to the CCP and other members in situations when the defaulting member can't pay. This termination loss on the derivatives portfolio would be above and beyond any loss associated with the fair market value of the portfolio (which is about halfway between the bid value and the offer value)."

The buy-side clients of dealers tend to be hedge funds or institutional investors; CDS are too technical for the average retail investor and opening a margin account with a dealer requires a prohibitively large amount of collateral. The inter-dealer market is dominated by 14 large dealers, holding 80% of the notional value of the global CDS market. These large global dealers are active members of all the large CDS CCPs.[12] When these dealers join a CCP, they are expected to post initial margin based on the size, position direction, and liquidity of the CDS in their portfolio. The CCP applies a proprietary margin calculation model to the each member's portfolio. The is based on the market risk associated with a market value decrease in the 0.99th percentile, between the time of default and that needed to close out the position. The initial margin calculation, often, also misses the risk associated with liquidating very large, but well-hedge positions.

2.2 Model Setup

This model considers the CCP structure as a star-shaped financial network; the CCP is in the centre, denoted $i = 0$, and connected to dealer banks $i = \{1, .., m\}$. Each dealer clears CDS contracts through the CCP, thus, the CCP acts as the counterparty to each contract. The dealer banks can trade in standard CDS contracts written on reference entities $k = \{1, ..., K\}$.

The side of the CDS contract position is represented by the nominal value (X) of the position with a positive buy position (B) and negative sell position (S).

$$X^p \quad \text{where } p \in \{B, S\}, \quad X^B = +X \quad \text{and} \quad X^S = -X$$

Furthermore, each contract is held between two counterparties, bank i and bank j. Thus, if bank i is holding a buy position, then bank j must be holding a sell position. X_{ij}^k is the matrix of CDS positions between bank i and bank j, for contracts on one reference entity k, while X_{ij} is that for all k reference entities. The portfolio of bank i for CDS on k reference entities has value,

$$V_i^k = \sum_{k=1}^{K} X_i^k \triangle S^k(t_\ell)$$

where V is the current market value of the position, X is the nominal value and $\triangle$S is the difference between the cds-spread and the premium coupon at time t_ℓ. Importantly, the change in cds-spread $\triangle S$ will start at zero and move up, taking a positive value ($\triangle S > 0$), or it will move down, taking a negative value ($\triangle S < 0$).

Each time period, t, is subdivided into ℓ different trading periods, τ. The length of the trading period is determined by the CCP liquidation window, such that $t_\ell \in [\ell\tau, T\tau]$. For example, the CCP can trade on a daily intervals of τ= 1 day for T=5 days[13] to meet its liquidation period deadline.

The net value of bank i's position is determined by the difference between its liabilities and receivables. The amount of variation margin on CDS k which bank i owes to other banks j is the *payable*,

$$L_i^k = \sum_{j=1}^{m} L_{ij}^k \tag{1}$$

Thus, bank i's net position at any time is the exposure,

$$\Lambda_i = \sum_{j=1}^{m} L_{ji}^k - \sum_{j=1}^{m} L_{ij}^k \tag{2}$$

[12]In past years, dealers have acted as net-sellers, but in recent years they have been net-buyers. Interestingly insurance companies have historically been net-buyers of CDS.[Augustin et al., 2014, Cont and Minca, 2014]

[13]The CCP can also trade on an interval of τ= 1 minute for T=5x24x60 minutes. We assume the CCP can institute continuous trading in dire situations.

Bank i incurs a liability on its account only in the case that its variation margin is negative, and obtains a receivable only when it is positive. Thus, the following holds,

$$[L_{ji}]^+ = \max\,[0, L_{ji}]$$
$$[L_{ij}]^- = \min\,[0, -L_{ij}]$$

giving a positive final amount in both cases.

The value of the net exposure, whether a net liability or net receivable, is determined by the notional and the change in cds-spread,

$$\Lambda_i^{k,B}(\ell\tau) = X_{ji}^{k,B}(\ell\tau)\,\triangle S^k(\ell\tau) - X_{ij}^{k,B}(\ell\tau)\,\triangle S^k(\ell\tau) \tag{3}$$

Finally, there is some slight probability that the reference entity k can experience a default event, this is addressed in a future section. Currently, it is assumed that no underlying reference entity for a CDS defaults[14], so that the effects of variation margin can be isolated from that of asset default events.[15]

2.3 Pricing Structure & Variation Margin

It is the cds-spread that determines bank i's its variation margin payments and drives the changes in the net value of its position. The cds-spread is determined by known and unknown components, with features which are complex to model. An attempt to model an exact mathematical mode of the mean-reverting cds-spread is beyond the scope of this analysis, which instead focuses on approximate patterns of behaviour.[16] Thus, for each period, one takes the fundamental portion of the cds-spread as given and determined by common market information about firm and industry fundamentals. This fundamentals cause permanent price impact. Then, the effects of the other explanatory determinants are modeled; those which can move cds-spreads temporarily, such as, liquidity, and excess market demand.

The cds-spread on a CDS written on k, moves due to changes in fundamentals which have permanent price impact.[17] These are most likely market information on firm and industry-level characteristics. This portion of the cds-spread each period is given by,

$$\triangle S^k(\ell\tau) = f\Big(\triangle S^k\big((\ell-1)\tau\big)\Big) \tag{4}$$

Definition 1: The fundamental cds-spread prices the fundamental value of bank i's portfolio for each period,

$$X_{ij}^{k,p}(\ell\tau)\,\triangle S^k(\ell\tau) \;=\; X_{ij}^{k,p}\big((\ell-1)\tau\big)\,f\Big(\triangle S^k\big((\ell-1)\tau\big)\Big) \;=\; \Big[X_{ij}^{k,p}\big((\ell-1)\tau\big)\,\triangle S^k\big((\ell-1)\tau\big)\Big]^+$$

and its action does not depend on the the side of the contract (buy/sell).

This is the benchmark value of the liability of the contract absent any covariance effects from liquidation or predation. It is assumed that the fundamental value takes into account any covariance from firm or industry shocks.

[14] As put forth in [Cont, 2015], I assumes that default of a large dealer member occurs due to difficult macroeconomic conditions and market volatility rather than default of the underlying on a CDS they hold. "These members often turn out to be large broker-dealer banks, whose default is very likely to be associated with a high level of market volatility (Duffie 2010) and/or widened bid-ask spreads."

[15] If bank i is the seller, he will not receive or pay margin, instead he will have to pay out a proportion of the notional $\gamma \in (0, 1]$ to bank j (depending on value of bond at time of default and whether contract settled through physical delivery of the bond to seller). If the seller is already unable to meet the obligation, the CCP must meet the obligation.

[16] There is much fine literature in this area, for example, [Cont and Kan, 2011] or [Brendan O'Donoghue, 2014]

[17] For a thorough overview of cds-spread determinants see [Tang and Yan, 2013].

2.3.1 The Price Impact of Liquidation & Predation

When a member defaults on its liabilities, the CCP must unwind that members positions over $t_\ell = T\tau$ periods – the time period used for estimation of the initial margin contribution. However, in liquidating these positions, it creates a price impact. This price impact serves as an additional volatility term which temporarily moves the cds-spread away from it's long-term value; this is the effect of excess demand or supply on the market. Since CDS exhibit covariance, one can assume a volatility-like structure, using a formulation common to empirical investigations,

$$X_{ij}^{k,p}\, \Sigma_{ij}\, X_{ij}^{k,p}$$

where Σ is the covariance matrix between assets and X is the portfolio of CDS contracts bank i holds with various banks j. Specialising to a linear price impact formulation, so as to incoporate price impact, gives the form,

$$X_{ij}^{k,p}\, F(X_{ij}^{k,p}) \quad \text{with} \quad F(X_{ij}^{k,p}) = \triangle S^k(\ell\tau)\left(\frac{X_{ij}^{k,-p}}{D_k}\right)$$

where $F(X)$ is the is the *change* in the closing market value of the CDS in a banks portfolio. Thus, S is the cds-spread and $\triangle S^k(\ell\tau) = S^k\Big(\ell\tau\Big) - S^k\Big((\ell-1)\tau\Big)$ is the change in the cds-spread.

It reflects the effect on the portfolio of bank i, of liquidating a CDS position, where the liquidation of a defaulted bank j for asset k is given by,

$$\triangle S^k(t_\ell) = \triangle S^k(t_{\ell-1})\left(1 - \frac{1}{D_k}\sum_{j\in\mathcal{D}} X_j^k\right)$$

where D_k is the vector of market depths[18], bounded from below, for CDS assets of type k. This means that the price of a cds (written on k) moves 1% when the net supply is equal to $\frac{D_k}{100}$. Furthermore, since $\triangle S$ moves in both positive and negative directions, the price impact does not always drag this value downwards. The portfolio market value of bank i is then altered by,

$$V_i^k = X_i^k\, \triangle S^k(t_\ell) = X_i^k\, \triangle S^k(t_{\ell-1}) - \frac{1}{D_k}\sum_{j\in\mathcal{D}} X_i^k\, \triangle S^k(t_{\ell-1})\, X_j^k$$

This price impact structure embeds the covariance which occurs between assets; the second term reflects how cds X_j^k, held by bank j, exerts an effect on cds X_i^k held by bank i. This method permits quantification of the covariance relationships of every asset bank i holds with those of the other defaulted bank(s). Some of these relationships are as direct counterparties, and some relationships are based on weaker ties between assets. As opposed to using an empirical *covariance matrix* between all assets, one can isolate the covariance effect with the defaulted assets alone, since these are the ones which will be liquidated and cause multiple price impacts. This is best illustrated spacialy by figure 1 below[19].

As assets interact in multiple ways, with varying strength, there must be multiple price impacts. The primary price impact results from those assets that i holds as a direct counterparty with those being liquidated. The primary price impact term extracts the effects due to the CCP's liquidation, banks'

[18] The model must have a parameter which adjusts for the illiquidity of the market. This illiquidity is not be caused by the default of an underlying reference entity on a CDS held by a dealer, but by the default of the large dealer itself. [Fleming and Sarkar, 2014]

[19] The figure gives the covariance relationships between banks. Filled *red* dots are banks holding the defaulted asset. *Unfilled* dots don't hold that asset, but both have a relationship through another asset. Banks are affected by holding defaulted asset as a counterparty, holding without being a counterparty, holding asset with other assets in portfolio, not holding the asset, but having a counterparty who holds the asset.

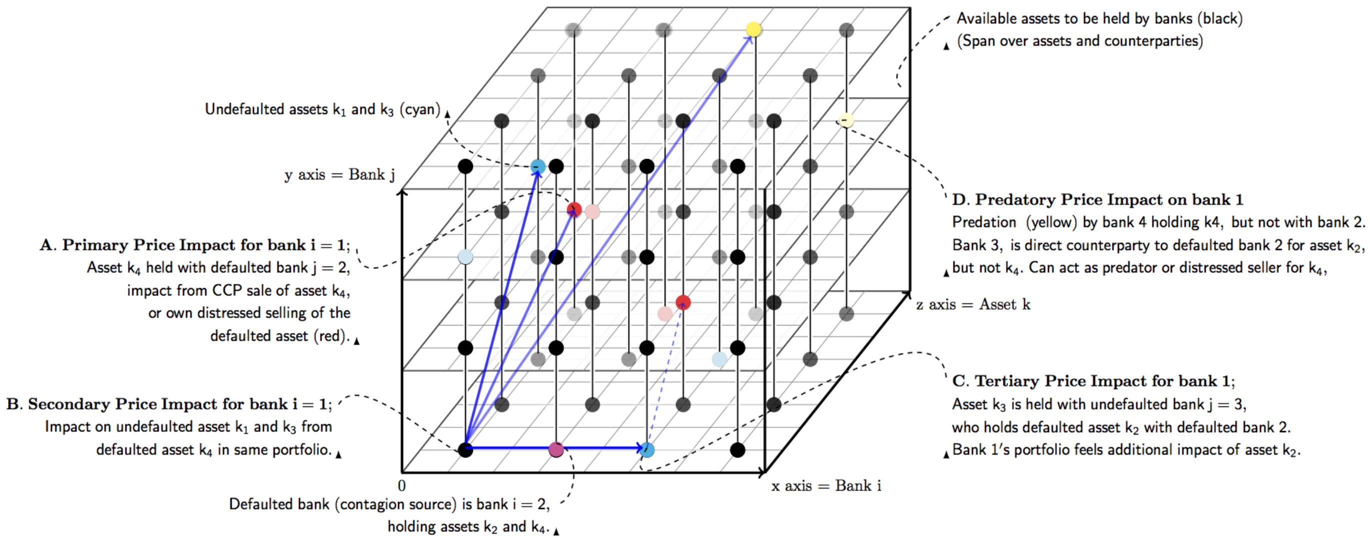

Figure 1: **Illustration of covariance relationships of banks in financial network in terms asset holdings (colour) and of spatial distance to defaulted assets.**

distressed selling, or selling a large position for other reasons through the liquidation rate a_i^k assigned to each bank. The reason for liquidation by a direct counterparty (to defaulted assets) is not important, only that it drives cds-spread change.

This primary effect is augmented by a further price impact, caused by predation. Predation is the liquidation of assets by bank i which have defaulted, but which i does not hold directly with a defaulted counterparty. To explain further, a bank which holds the asset k, but as a direct counterparty to a default, is considered *in distressed*; while the CCP liquidates the defaulted bank's positions, the distressed bank doesn't really have a choice whether to liquidate, especially if facing illiquidity. Furthermore, this bank will not be in a position to engage in a buyback strategy after liquidation. However, if a healthy bank holds asset k with a healthy counterparty, it is free to choose its strategy. It can predatorily engage in liquidation along with the CCP, under the assumption that it will re-purchase those positions at a lower price during the buy-back period, and make a profit.

A secondary and tertiary price impact also occurs[20]. The secondary impact is due to covariance between the defaulted CDS, k, and other CDS, k', held in bank i's portfolio. The tertiary impact is due to the covariance between defaulted assets i doesn't hold and different CDS, k', in its portfolio. The reasoning is that covariance can still occur between assets (k, k') which are not held mutually by two parties i and j. Instead, the asset k is alongside a defaulted asset (k"), in bank j's portfolio, where j though solvent is a direct counterparty to another defaulted bank z. Therefore, this quantifies a spillover caused by a distant, indirect link, further away from bank i.

The choice of exactly three price impacts is not arbitrary; the three price impacts define the most important covariance relationships between assets. The last two being longer distance relationships, take time to be fully transmitted through the market and incorporated in the cds-spread. As this is a temporary effect, it is reasonable (and empirically supported[21]) to stop at two time lags; past this point the content of information is lost to noise present in the market.

[20][Minca and Amini, 2012] show that price feedback effects from a distressed liquidation (fire-sale) uncovers hidden exposures between two banks, based on common holdings in their portfolios. These exposures can be much larger than any direct exposures they have to each other.

[21]In [Tang and Yan, 2013], the authors find that there is a significant part of CDS spread movement that is driven by excess demand/liquidity changes in the market, and is short-lived. This temporary component augments the fundamental-drive component. The excess supply-demand effect is transitory and dissipates after, at most, three trades.

Thus, these last two price impact functions have a time lag of $\triangle S^k(t_{\ell-2})$; since OTC markets are opaque, and contracts are negotiated bilaterally through search-and-quote methods, distant market information is incorporated into prices with a delay causing only an indirect price impact. The counterparties negotiating any CDS contract cannot see the whole market (market prices) at any one time, instead they must move through the market and gain market information slowly. This point is key to the model. Since predators are as any other member bank, they do not realise they are experience fluctuations in cds-spread due to there own predatory behaviour, and instead assume this is caused by some unexplained component of cds-spread fluctuations. It is this fact which can be exploited in order to dis-incentivise predation.

Thus, the cd-pricing structure takes a form similar to a taylor-expansion/power series of the pricing function,

$$V_i^k = \frac{1}{0!}\, X_i^k\, F(X_j^k) \,+\, \frac{1}{1!}\, X_i^k\, F'(X_j^k) \,+\, \frac{1}{1!}\, X_i^k\, \mathcal{F}'(X_j^k) \,+\, \frac{1}{2!}\, X_i^k\, F''(X_j^k) \,+\, \frac{1}{3!}\, X_i^k\, F''(X_j^k) \tag{5}$$

2.3.2 Covariance Structure of Assets

The previous section determined the structure of cd-pricing function, taking into account price-impact, predation and covariance. This section elaborates on the covariance structure, incorporating the dynamics of liquidation. The cd-pricing function can now be re-written as,

$$X_i^k \triangle S^k(t_\ell) = \underbrace{[\, X_i^k \triangle S^k(t_{\ell-1})\,]^+}_{P_0} + P_1 \underbrace{a_j^k \tau}_{\Gamma_j^k} + \mathcal{P}\, a_j^k \tau \,+\, P_2\, a_j^k \tau \,+\, P_3\, a_j^k \tau \tag{6}$$

where P_0 is the fundamental long term value of the cds-spread, $\mathcal{P}$ is the predation impact, $P_1,\ P_2,\ P_3$ are the primary, secondary and tertiary price impacts and a_j^k is the liquidation rate per period τ. Γ encapsulates the sign and strength of the liquidation effect while untangling it from the effect of price impact.

From this I get the exact form of the *cds-spread pricing functional*, $\triangle S$ (provided as Proof 2 in Appendix C), while the full value of bank i's portfolio, $V_i^k = X_i^k \triangle S^k$, is given by the proposition on the following page. Each term in the function – fully explained in Appendix B – should viewed in terms of the effect it exerts on *bank i's* portfolio. With liquidation, the party initiating the sale is given by the first subscript in a^k and the counterparty to the trade is given by the second subscript.[22] The first term is the fundamental value at time $t_{(\ell-1)\tau}$; it decreases the value of bank i's position in asset k if this asset was liquidated in the previous trading period.

The second and third terms are the primary price impact. The second term shows asset k, held in bank i's hold portfolio, which is being liquidated by the CCP for every defaulted bank j. The model differentiates between the liquidation of an asset held directly with the defaulted counterparty, and an undefaulted counterparty. In both cases, bank i is affected by holding an asset which the CCP is liquidating, thus, all CCP initiated liquidations are accounted for. With no liquidation this term is zero as $a_{ji'}^k = 0$.

The third term describes the effect of bank i's own liquidation of any defaulted asset it holds – its own distressed selling. This further ensures that all direct counterparty liquidations of asset k are accounted for; liquidations on both sides of trade where one side is a defaulter. The asset k which bank i holds with an undefaulted bank j experiences a smaller impact from liquidation, as not being direct counterparty to bank j, it avoids other negative externalities.[23] Furthermore, if it holds no defaulted

[22] This leads to a liquidation rate matrix for each bank, accounting for it's liquidation rate on each separate position.

[23] This can be a delay of receivables, lower receivable payment due to low recovery value, or complete loss of owed variation margin, due to variation margin haircuts as a recovery measure.

assets with direct counterparties, this term is just it's own predatory behaviour on asset k.

The fourth term quantifies the effect of predation on bank i by all other banks; this is seen in the first part of the term, which accounts for bank i's holding of asset k with any other bank. Notice that bank i can choose to predate if it holds CDS k, which is being liquidated with any undefaulted counterparty. Though this seems like double-counting – the price impact of direct liquidations of bank i's side of the portfolio are accounted for – here banks don't hold the asset directly with a defaulted counterparty. Also notice that the last part of the term accounts for selling initiated by j (not i or i'), and thus, is the counterparty's side of the distressed/predatory trade for bank i.

> *Proposition 1: The value of a bank's portfolio is determined by the size of its holding of an asset and the various degrees of covariance relationships that asset has with liquidated assets in the market, through the pricing functional.*

$$
\begin{aligned}
\sum_k X^k_{ij}(\ell\tau)\,\triangle S^k(\ell\tau) \;&=\; \sum_k \Big(X^k_{ij}((\ell-1)\tau) \;+\; a^k_{ji}\tau\Big)\,\triangle S^k(\ell\tau) \\
&= \sum_k \Bigg\{ \underbrace{[\, X^k_{ij}((\ell-1)\tau)\triangle S^k((\ell-1)\tau)\,]^+}_{fundamental\ cds-spread} \\
&\text{primary price impact}\begin{cases}
+\underbrace{\left(\sum_{j\in\mathcal{D}}\left|\frac{X^k_{ij}}{X^k_{ij}}\right|X^k_{ij}+\varepsilon\sum_{j'\notin\mathcal{D}}\left|\frac{X^k_{ij'}}{X^k_{ij'}}\right|X^k_{ij'}\right)\sum_{i'=1}^{m}|\triangle S^k((\ell-1)\tau)|\left(\frac{X^k_{ji'}}{D_k}\right)\left(\frac{a^k_{ji'}\tau}{X^k_{ji'}}\right)}_{CCP\ liquidation} \\
+\left(\underbrace{\sum_{j\in\mathcal{D}}\left|\frac{X^k_{ij}}{X^k_{ij}}\right|X^k_{ij}\sum_{j\in\mathcal{D}}}_{distressed\ selling}+\underbrace{\varepsilon\sum_{j'\in\mathcal{D}}\left|\frac{X^k_{ij'}}{X^k_{ij'}}\right|X^k_{ij'}\sum_{j\notin\mathcal{D}}}_{distress/predation}\right)|\triangle S^k((\ell-1)\tau)|\left(\frac{X^k_{ij}}{D_k}\right)\left(\frac{a^k_{ij}\tau}{X^k_{ij}}\right)
\end{cases} \\
&+\underbrace{\varepsilon\sum_{j=1}^{m}\left|\frac{X^k_{ij}}{X^k_{ij}}\right|X^k_{ij}\sum_{j'\notin\mathcal{D}}\sum_{i'=1}^{m}|\triangle S^k((\ell-1)\tau)|\left(\frac{X^k_{j'i'}}{D_k}\right)\left(\frac{a^k_{j'i'}\tau}{X^k_{j'i'}}\right)}_{predation} \\
&+\underbrace{\left(\frac{1}{2!}\right)\left(\left(\frac{3}{2!}\right)\sum_{j\in\mathcal{D}}\left|\frac{X^k_{ij}}{X^k_{ij}}\right|X^k_{ij}+\sum_{j'\notin\mathcal{D}}\left|\frac{X^k_{ij'}}{X^k_{ij'}}\right|X^k_{ij'}\right)\sum_{k'}\sum_{j=1}^{m}\left|\frac{X^{k'}_{ij}}{X^{k'}_{ij}}\right|\sum_{i'=1}^{m}|\triangle S^{k'}((\ell-2)\tau)|\left(\frac{X^{k'}_{ji'}}{D_{k'}}\right)\left(\frac{a^{k'}_{ji'}\tau}{X^{k'}_{ji'}}\right)}_{secondary\ price\ impact} \\
&+\underbrace{\left(\frac{1}{3!}\right)\left(\left(\frac{9}{3!}\right)\sum_{j\in\mathcal{D}}X^k_{ij}\sum_{k''=1}\left|1-\frac{X^{k''}_{ij}}{X^{k''}_{ij}}\right|+\sum_{j\notin\mathcal{D}}X^k_{ij}\sum_{k''=1}\left|1-\frac{X^{k''}_{ij}}{X^{k''}_{ij}}\right|\right)\sum_{i'=1}^{m}|\triangle S^{k''}((\ell-2)\tau)|\left(\frac{X^{k''}_{ji'}}{D_{k''}}\right)\left(\frac{a^{k''}_{ji'}\tau}{X^{k''}_{ji'}}\right)}_{tertiary\ price\ impact}\Bigg\}
\end{aligned}
\tag{7}
$$

The ϵ serves as a dampening of the effect. In the primary price impact, this acknowledges the smaller effect made by CCP liquidation of any defaulted assets held by bank i with undefaulted counterparties; this indirectly held asset is shielded, somewhat, from the full negative effects, but it still gives bank i some distress. The dampening on distressed selling and predation acknowledges that banks holding assets with undefaulted counterparties can choose to liquidate and are not as time constrained as the CCP, thus, they can mitigate their price impact.

The fifth term measures the secondary price impact on the asset k held by bank i, subject to the liquidation effect of the other types of assets k' in bank i's portfolio. The first part of the equation ensures that bank i is holding the asset with a counterparty j. The summation outside the brackets ensures that bank i is also holding the other asset k' with the same or any other j. The last part, running over i', ensures that all liquidations of the other assets, by j, are counted, even those with bank i itself. This identifies a cross-impact of liquidations of different asset types. Those assets which bank i holds directly with a defaulted bank feel a mildly reduced price impact at 75%, while those held indirectly only feel 50%.

The sixth term is the tertiary price impact. It accounts for the price impact of liquidation of assets which bank i doesn't hold directly. Bank i's assets may be affected by the assets it's counterparties are holding and liquidating. Thus, this accounts for ways bank i's portfolio may be affected through distant links or spillover from holding assets with counterparties, who themselves, hold assets with defaulted counterparties. The terms in the bracket account for the fact that bank i holds asset k with bank j, but also that it doesn't hold asset k" with the same bank j. Instead, the last term accounts for the liquidation of any assets by j with another bank i'.

The terminating term in each line, $\frac{a^{k''}_{ji'}\tau}{X^{k''}_{ji'}}$, is the Γ term or the liquidation rate of the CCP and of the other dealers. This form differentiates the effect of price impact from the augmentation effect caused by liquidation rate. This allows one to separate and evaluate the interaction of these two different factors.

2.3.3 Liquidation Rate & Predation

The last piece of the framework concerns the method the CCP and member banks use to determine their liquidation rate. This model builds on the framework of [Brunnermeier and Pederson, 2005] adapted and expanded for use in an OTC market.[24] This paper provides a novel extension to an opaque over-the-counter market with a noisy price process; an imperfectly visible price process allows all members to become victim to predation, even predators themselves.

The CCP is composed of dealer banks $i \in \{0, ..., I\}$ where the CCP is represented by i_0. The focus is on the large dealer banks mentioned previously. The CCP is a distressed dealer bank, in crisis and so it must liquidate, it belongs to the set I^d. The comparably unconstrained dealer banks can engage in predation. Should they choose to liquidate alongside the CCP they then belong to group I^p.

Each strategic dealer has a position in the risky asset,

$$X_i^k(\ell\tau) = X_i^k(0\tau) + \sum_{\ell=1}^{T} a_i^k \ell\tau \tag{8}$$

Also, all large dealer banks are subject to the same regulatory restrictions (concerning how much risk they can hold in their portfolio), are of similar size and have roughly the same balance sheet capacity. In this way, they are roughly homogenous[25]. However, the homogeneity does not extend to

[24] For setup, see the original article [Brunnermeier and Pederson, 2005], for the extension, see appendix B.

[25] This assumption is for the purpose of tractability. The model will easily permit its relaxation, accommodating heterogeneity in banks' maximum buyback size and access to information, giving further varying trading rates.

the composition of their portfolio or their trading rate, shown further on. Thus, each dealer bank i is restricted to hold,

$$X_i^k \in [-\bar{X}, \bar{X}] \tag{9}$$

This restriction on dealer holdings, ensures that no large dealers can hold enough of asset k to move the cds-spread to their expectation of its value $\mathrm{E}[\mu] = v$ through price manipulation. This fact that the combined holdings of largest dealers for CDS makeup 80% of the market supports this assumption. Therefore, $S > I\bar{X}$ is the supply of CDS in the market.

The CCP must liquidate its position at $t_\ell = 1$ when all banks' liabilities are realised. The CCP chooses its liquidation rate in order to balance the market risk and liquidity risk of holding defaulted banks' positions (possibly incurring variation margin penalties). In choosing to liquidate with minimal price impact, the CCP chooses to liquidate over all available $T\tau$ periods. The CCP liquidates at a rate equal to, or below, the average trading rate of the market – information it is privy to since it clears *all* market transactions. The total net trading of all dealer banks at the CCP is,

$$A^k = |\sum_i^I a_i^k| \tag{10}$$

The CCP then liquidates according to $a_i^{k,p,-}(\ell\tau) \leq -\frac{A^k}{I}$ (selling), $a_i^{k,p,+}(\ell\tau) \geq \frac{A^k}{I}$ (buying), or the trivial rate $a_i^{k,p,0}(\ell\tau) = 0$.

The CCP liquidates over the 5 day clearing period; each trading period τ is 1 day and $T = 5$. In keeping with the results of [Brunnermeier and Pederson, 2005], it liquidates at constant speed A/I for the duration of the liquidation period, regardless of the number of predators. This is a strong assumption, but considering that the CCP wants to minimise both market risk and price impact, a smooth liquidation strategy is reasonable. Since, it does this for $\tau\frac{X_{t_0}}{A/I}$ periods, one can determine the trading rate A.

The predators decide their rate by trying to minimise their own price impact given the optimal trading strategy of other dealers. Thus, predator chooses its strategy by solving its trading problem given the optimal trading strategy of other predators.[26] [Brunnermeier and Pederson, 2005]. They liquidate at any point of profit and so only healthy banks can choose to predate.

The trading problem is given by,

$$\min_{a_i \in \mathcal{A}_i} \quad \sum_0^T a_i^{k,p}(\ell\tau) \sum_{j\neq i} X^j \ell\tau \qquad s.t. \quad X_i^k(T\tau) \; = \; X_i^k(0\tau) \; + \; \sum_{\ell=1}^T a_i^k \, \ell\tau \; = \; \bar{X}$$

The homogeneity in predator trading rate occurs as all predators optimise their rate assuming they will be able to buyback their maximum allowable holding. Given that the market is opaque, at any given time predators can only see the CCP's trade size and that of one other counterparty. This makes the price process extremely noisy. Thus, considering the total supply of assets in the market and the combined demand of all parties, the market-clearing price process is given by the next proposition.

> *Proposition 2: In an opaque market with a noisy price process, the covariance structure of market trade relationships is directly determined by market supply and demand. This supply/demand can be related back to the market clearing price (see Appendix A for proof). That is, the market clearing price indirectly embeds, outside consumer demand as well as the effects of price impact and predation.*

[26] See appendix A. This format is used as I assume excess demand is driven by the dealer's buy-side clients – trades on customer accounts. Also, dealers operate in opaque market; the price used to make their trading decisions is noisy and not an accurate reflection of the true cds-spread.

$$\triangle S^k(t_{\ell\tau}) = \underbrace{v}_{P_0} - \underbrace{\frac{1}{D_k}(\mathcal{S}^k(t_{(\ell-1)\tau}) - \sum_i^I X_i^k(t_{(\ell-1)\tau}))}_{P_1,\, \mathcal{P},\, P_2,\, P_3} \tag{11}$$

This shows that all traders or banks in the market, see a noisy price process, where they can only pin down the fundamental value $v = P_0$ from common market information. This common market information also contributes to dealer homogeneity. An explicit explanation and derivation of the market clearing price is given in the appendix A. The noisiness of the price process is a novel contribution of this model, since this feature allows a large predator to eventually become prey[27]!

When there is only one predator – or multiple predators acting as one (collusion) – the predator liquidates at the same rate as the CCP. As the market is opaque, the predator bank cannot see the majority of the trades made by the other dealers. However, the CCP is readily visible to all banks and has signalled its distress by initiating liquidation; banks assume that all other banks are 'homogeneously' trading at this rate. In response, the predator trades at the same rate and in the same direction as the CCP until it has finished liquidating. Later it buys back its positions at the rate A for $\frac{\bar{X}}{A}$ periods.

A competition effect appears if there are multiple, independent predators, as in [Brunnermeier and Pederson, 2005]. The predatory bank must then stop liquidation earlier, before the CCP has finished, and start buying back its positions. This reduces its expected profits. With increasing competition, the predator must start buying increasingly early. Thus, with $I^p \geq 2$, if the size of the predatory bank's initial holding/position in the CDS is larger than the position limit,

$$X_{t_0} = \frac{I^p - 1}{I - 1}\bar{X} \tag{12}$$

and each predator will sell at same rate as CCP for,

$$\frac{X_{t_0} - \frac{I^p-1}{I-1}\bar{X}}{A/I} \tag{13}$$

periods. Each predatory dealer then buys back positions at rate $\frac{AI^d}{I(I^p-1)}$ until $\frac{X_{t_0}}{A/I}$, the end of the CCP's liquidation. Thus, the competitive predator terminates his profit-making strategy just as the CCP finishes liquidation, at the end of period $t_\ell = 1$. This contrasts the case with one predator, where buyback ends in period $t_\ell = 2$.

3 Preliminary Analysis

3.1 Fundamental Relations

A preliminary analysis of results give the first implications of the model. The full illustration of the model demands a simulation, which is provided in later section. Considering liabilities of the form,

$$\mathbf{L^k_{ij}}(\ell\tau) = \mathbf{X^k_{ij}}(\ell\tau)\triangle\mathbf{S^k}(\ell\tau)$$

Here, $L_{ij}^{k,S}$ is the liability bank i has to bank j on a sell position for reference entity k. In the same manner, $L_{ji}^{k,S}$ is the receivable that bank i is owed from bank j for the same position. Similarily, $L_{ji}^{k,B}$ would be a receivable for bank i on a buy position.

> *Definition 2: The liability of one bank on the buy (sell) side of a CDS position, is the receivable of the counterparty on the sell (buy) side of the position. With clarification, all else equal,*

[27] In [Duffie, 2010], at least one counterparty in a derivative trade is a dealer. In the CDS market, counterparties are most likely both dealers. With the concentrated nature of trades, it's reasonable to assume a default by a large dealer can bring down one or two other large dealers. This also provides an incentive for dealers to predate.

$$L^{k,S}_{ij}(\ell\tau) = -L^{k,S}_{ji}(\ell\tau) = L^{k,B}_{ji}(\ell\tau) \qquad and \qquad L^{k,S}_{ij}(\ell\tau) = -L^{k,B}_{ij}(\ell\tau) = L^{k,B}_{ji}(\ell\tau)$$

$$That\, is, \quad iff \quad |L^{k,S}_{ij}(\ell\tau)| = |L^{k,B}_{ij}(\ell\tau)|, \quad otherwise \quad L^{k,S}_{ij}(\ell\tau) \neq L^{k,B}_{ji}(\ell\tau).$$

The above relation shows that the liability on a sell[28] position, for bank i (from bank i's perspective), is the same as a bank j holding a receivable on the buy position; except that from bank j's perspective, it is bank i. Note that it is assumed there is one CDS, held between two counterparties, which hold the same amount of the asset.

With multiple assets and multiple counterparties, a liability on a sell position for asset k is not necessarily the opposite of a liability on a buy position for the same asset, as each buy/sell position is distinct and may very in magnitude. That is, each side of the position can be sold off and multiple holdings on one CDS are possible.

The following relations are derived from the model and act as a preliminary sensitivity analysis. For the case where bank i has a sell position, the effect on bank i's portfolio is:

$$\mathbf{1}.\ \frac{\partial L^{k,s}_{ij}(\ell\tau)}{\partial \triangle S^k\big((\ell-1)\tau\big)} < 0$$

$$\mathbf{2a}.\ \frac{\partial L^{k,s}_{ij}(\ell\tau)}{\partial a^{k,-}_{ji}\big((\ell-1)\tau\big)} > 0 \qquad \mathbf{2b}.\ \frac{\partial L^{k,s}_{ij}(\ell\tau)}{\partial a^{k,+}_{ji}\big((\ell-1)\tau\big)} < 0$$

$$\mathbf{3a}.\ \frac{\partial \mathcal{P}^{k,s}_{ij}(\ell\tau)\ \Gamma^k_j}{\partial (X^{k,s}_{ij} + a^{k,-}_{ij}\big((\ell-1)\tau\big)\tau}) < 0 \qquad \mathbf{3b}.\ \frac{\partial |\mathcal{P}^{k,s}_{ij}(\ell\tau)\ \Gamma^k_j|}{\partial a^{k,-}_{ij}\big((\ell-1)\tau\big)} > 0$$

$$\mathbf{4a}.\ \frac{\partial P^{k,s}_{ij}(\ell\tau)\ \Gamma^k_j}{\partial a^{k,-}_{ij}\big((\ell-1)\tau\big)} < 0 \qquad \mathbf{4b}.\ \frac{\partial |P^{k,s}_{ij}(\ell\tau)\ \Gamma^k_j|}{\partial a^{k,-}_{ji}\big((\ell-1)\tau\big)} > 0$$

From relation (1), a decrease in price will increase the liability of bank i holding a sell position. From relation (2a), an increasing sell position liability, with any liquidation/selling of positions by others. This is due to the price impact and the cds-spread decrease of asset k. From relation (2b), purchases of the position by others, decreases the cds-spread and lowers the liability of bank i. This is due to the market's increased demand for sell positions. It signals the market's willingness to take the risk of providing insurance on reference entity k, and subsequently the belief in the increased creditworthiness of that reference entity.

Relation (3a) illustrates that the impact of predation, $\mathcal{P}$, is decreased with liquidation of the asset k for bank i, provided that bank i is holding asset k and also predating by selling asset k. A decreased holding of asset k, means bank i is also decreasing its exposure to the impact caused by all other entities engaging in predation. From relation (3b), liquidation by others increases the impact of predation on bank i's own portfolio, all else equal. From relation (4a), liquidation of bank i's own holding in asset k, reduces bank i's exposure to the three sources of price impact, P. From relation (4b), liquidation of others, increases the effect of all the price impacts on bank i's portfolio. The same relations exist for bank i holding a 'Buy' position, with the opposite conclusions for the first three relations.

The CDS market has many subtleties. The market dynamics are generated through an interplay

[28] To avoid confusion in terminology, the buyer of protection will be referred to as one holding the *buy* position, and the seller of protection as being the one holding the *sell* position.

between the cds-spread and demand. Market demand for positions is measured by dealer quotes for buy or sell positions and resulting dynamics depend on the side of the position. An analysis of price movements (cds-spread) and trading patterns provides the dynamics for the sell and the buy positions.

On the sell side, a decrease in price suggests there is a decreased demand for sell positions, or insurance, on underlying reference entity k. Thus, if a bank holds a sell position, a decrease in price increases the banks liabilities in terms of variation margin. Also, a seller can closeout an initial CDS position by obtaining a buy position, or an existing sell position on a CDS can be sold between banks[29]. Thus, a bank with a sell position submitting a sell quote decreases demand for sell positions, but a bank seeking to buy a sell position with a buy quote increases demand for sell positions. A decrease in the demand for sell positions suggests that the market is downgrading the creditworthiness of the reference entity k, which is then reflected in a drop in the cds-spread for that entity.

A demand for buy positions works in the same way, except for a small difference. If a bank seeks to obtain a buy position with a buy quote, that shows increased demand for insurance on reference entity k and decreases the cds-spread for k (temporarily). However, the same bank can offload the buy position by selling it, decreasing the demand for insurance; this suggests increased confidence in reference entity k's creditworthiness which increases the cds-spread (also increasing the liability for any bank holding a buy position).[30] Thus, one can see that trading in both initial and existing CDS contracts can affect market demand and cds-spreads.

The following relations describe the the trading behaviour in terms of investor's trading strategies:

During the ***Liquidation Period***,

$$\mathbf{5}.\ \text{From } t_{\ell\tau} \in [0, T\tau] \ \text{ means } \begin{cases} P := X_{ij}^{k,B} \mapsto a^- \quad P := X_{ij}^{k,S} \mapsto a^-, a^+ \\ \\ \mathcal{P} := X_{ij}^{k,B} \mapsto a^- \quad \mathcal{P} := X_{ij}^{k,S} \mapsto a^-, a^+ \end{cases}$$

During the ***Buy-back Period***, with one predator (or many predators colluding,)

$$\mathbf{6}.\ \text{From } t_{\ell\tau} \in \left[T\tau, \frac{\bar{X}}{A}\right] \ \text{means } \begin{cases} P := X_{ij}^{k,B} \mapsto a^+ \\ \\ P := X_{ij}^{k,S} \mapsto a^+ \end{cases}$$

or with an extension to many 'homogeneous' predators,

$$\mathbf{7}.\ \text{From } t_{\ell\tau} \in \left[T\tau, \frac{X_{t_0} - \frac{I^p-1}{I-1}\bar{X}}{A/I}\right] \ \text{means } \begin{cases} P := X_{ij}^{k,B} \mapsto a^+ \\ \\ P := X_{ij}^{k,S} \mapsto a^+ \end{cases}$$

Outside of the Buy-back Period $a^-/a^+ = 0$.
There are cases of interest and thresholds, which effect exposures non-trivially. The following relations govern the behaviour of the pricing functional and the direction/magnitude of exposures:

8. If during a trading period, the change in the fundamental cds-spread is smaller than that from the combined effects of price impact and predation, the receivable of bank i becomes

[29] This is called novation, where one counterparty is replaced with another.

[30] In-depth review of the temporary nature of the demand effect on the cds-spreads in [Tang and Yan, 2013]

a liability.

$$\text{If} \quad P^S_{(0,\,(\ell-1)\tau)}\Gamma < \left(\{P_1, \mathcal{P}\}_{(\ell-1)\tau}, \{P_2, P_3\}_{(\ell-2)\tau}\right)^S$$

$$\text{Then} \quad L^{k,S}_{ji}(\ell\tau) \to L^{k,S}_{ij}(\ell\tau) \quad \text{as} \quad L^{k,S}_{ji} = 0, \quad L^{k,S}_{ij}(\ell\tau) > 0$$

9. All else equal, if bank i holds all assets with all defaulted counterparties then,

$$P^S_1\big((\ell-1)\tau\big) \; > \; \mathcal{P}^S\big((\ell-1)\tau\big).$$

The pricing functional gives more weight to the effect of primary price impact (liquidation by direct counterparties of a defaulted asset) than to the impact caused by predation (liquidation of the defaulted asset k to which bank i is not a direct counterparty).[31]

10. All else equal, if bank i holds all assets with all defaulted counterparties, and if bank i holds more assets from defaulted counterparties than undefaulted one, then

$$P^S_2\big((\ell-2)\tau\big) \; > \; P^S_3\big((\ell-2)\tau\big).$$

The pricing functional gives more weight to the effect of secondary price impact (effect of liquidations on all assets of direct holders of a defaulted asset) than to the impact caused by tertiary price impact (the effect of liquidations of the defaulted asset k on all assets of non holders of the defaulted asset, but who may hold asset in common with bank who is also holder of the defaulted asset[32].)

3.2 Effects of Liquidation & Predation On The Net-Worths of the CCP and Banks

With the fundamentals and dynamics of CDS trading established, the basic model can now be embedded in a larger framework. The following sections describe the flow of liabilities and receivables within the CCP, and determine the effects of liquidation and predation on the net worth of the CCP and banks.

In considering the structure of the CCP and its members, the initial framework of [Amini et al., 2015] for two CCP guarantee fund designs is extended. Model results will show that the CCP design is *integral* in determining, both, the predation incentives for members and the CCP's power to punish this behaviour. The analysis employs the current CCP guarantee fund structure, the *Pure Fund*, and a proposed *Hybrid Fund* structure. In the pure fund, a bank whose liabilities exceed receivables, must first use cash and proceeds – from liquidation of external assets – to meet any shortfall to the CCP; only then can its margin contribution be netted against the remaining shortfall. However, the hybrid fund structure allows the initial margin (guarantee fund contribution) to be netted directly against the shortfall. In this way, the probability that a bank will have to undergo a loss from forced liquidation is reduced. For this reason, [Amini et al., 2015] have shown the hybrid fund to be more incentive compatible for member banks. This model will further demonstrate that with liquidation, a threat of predation and the possibility of failure, the hybrid type is *also* better aligned with the CCP's incentives. In fact, I extend this seminal static, two-period CCP liability-exchange model with one asset, to a multi-period model, dynamic model which accommodates multiple assets, variation margin, price impact, and predation.

All CDS contracts are formed bilaterally between banks, but must be cleared through the CCP. The CCP charges a flat fee to member banks based on the volume cleared. Banks, in order to maintain

[31] Note that we are considering the pricing functional and not the full effect on bank i's portfolio from the positions of bank i, $X^{S,B}_{ij}$, or the liquidation rate $\Gamma^{+/-}_{ij}$

[32] Bank i may not hold defaulted asset k, but may have asset k-1 in common with bank j, who also holds asset k. Bank j's trading strategy for it's whole portfolio is, in part, dictated by holding asset k, potentially increasing the covariance between asset k and k-1. Thus, bank i is indirectly affected by asset k without directly holding it.

their membership, must contribute an initial margin g_i to a guarantee fund G_{tot} based on the size, direction and risk of their position. The rules governing the disbursement of this fund depend on the chosen CCP structure.

Member banks must also contribute to a CCP *default fund*, which is much smaller than the guarantee fund. The full default fund contribution, D_{tot} is accessible to resolve the shortfall of *any* bank, whose outstanding liabilities are larger than the guarantee fund contribution they can access. Finally, the member banks also have outside assets (non-CDS), Q_i, which they can access and liquidate in another market/CPP, in order to settle a shortfall and avoid potential default. The timing and magnitude of these liquidations is also governed by the structure of the CCP guarantee Fund.

3.2.1 Liabilities, net exposure, and shortfall

This section outlines model trading mechanics, building from simple one-sided liabilities to overall net worth. It focuses on the sell position as this side is subject to wrong-way risk, and can have asymmetrically large realised payments. Extending [Amini et al., 2015] from one to multiple assets, and to multiple trading-steps per period, allows for description of the full trading dynamics of all participants. There are four periods; the *Initial* (t=0), *Liquidation* (t=1), *Buyback* (t=2) and *Resolution* (t=3) period.

A bank i holds a portfolio of buy/sell CDS positions on an array of underlying reference entities, $k = 1...K$. It holds these assets with other banks $j \neq i$. During the *Initial* period at t=0, the banks form the positions that will lead to liabilities and receivables at $t_\ell = 1$.

The banks begin t=1 with a stylised balance sheet, common in the financial network theory literature, particularly that of [Amini et al., 2015, Cont et al., 2013b, Minca and Amini, 2012]. It is comprised of assets ($\mathcal{A}_i$) and liabilities ($\mathcal{L}_i$) given by,

$$\mathcal{A}_i\,(t_{1\tau}=1) = \sum_{k=1}^{K}\left(\underbrace{\gamma_i}_{cash} + \underbrace{\sum_{j=1}^{m} L_{ji}}_{receivables} + \underbrace{Q_i}_{external\ assets} \right)$$

$$\mathcal{L}_i\,(t_{1\tau}=1) = \sum_{k=1}^{K}\left(\underbrace{\sum_{j=1}^{m} L_{ij}}_{L_i} + \underbrace{\left(\gamma_i + \sum_{j=1}^{m} L_{ji} - L_i\right)}_{nominal\ net\ worth} \right)$$

Note, the nominal net worth is both an asset and a liability, depending on the realisation of bank i's receivables – the bank may have to use all sources of liquid funding to meet its liabilities.

The net exposure of bank i to only bank j is trivial if both banks fully repay each other. However, in the CCP bank i's *net exposure* is the sum of all its receivables minus the sum of it's liabilities, over each trading period.

$$\Lambda_i^S(\ell\tau) = \sum_{k=1}^{K} \Lambda_i^{k,S}(\ell\tau) = \sum_{k=1}^{K}\left(\sum_{j=1}^{m} L_{ji}^{k,S}(\ell\tau) - \sum_{j=1}^{m} L_{ij}^{k,S}(\ell\tau)\right) \tag{14}$$

If a bank's liabilities exceed it's receivables, it is *illiquid.* However, if a bank's liabilities are larger than it can repay through other means – guarantee fund or external assets – it is *insolvent* and defaults. A defaulted bank's positions are then taken on by the CCP, and the dealer bank can no longer function as a trading member.[33] As the CCP liquidates the defaulter's positions on k assets, only the liquid (illiquid) banks which can engage in predation (distressed selling).

[33] I assume when the bank *is* insolvent, it defaults, and the CCP tries to offload its positions before it taps into the full resources at its disposal.

Definition 3: If bank i changes its buy/sell position in any asset k, its portfolio holding changes according to,

$$X_{ij}^k(\ell\tau) = X_{ij}^k\big((\ell-1)\tau\big) + \mathbf{a_{ij}^k} \quad \text{where} \quad \begin{cases} a_{ij}^{k,+} = +a_{ij}^k & a_{ij}^{k,+} > 0 \\ a_{ij}^{k,-} = -a_{ij}^k & a_{ij}^{k,-} < 0 \end{cases} \tag{15}$$

Importantly, predatory liquidation by bank i, reduces it's position in the asset, thereby, also reducing it's exposure to the change in value. However, should bank i choose not to predate, while holding asset k , it feels the full impact of predation by others. In this way, there is a dangerous *predatory incentive*; if one bank chooses to predate, then *any* bank which can predate, is made better off by doing so.

The following analysis provides the conditions under which banks defaults and a CCP takeover of positions ensues. At period $t_\ell = 1$, bank i's liabilities and receivables are realised.[34]. The bank has a liquidity problem if its liabilities exceed receivables during any phase of the trading period, $t_{\ell\tau}$.

Thus, the bank's net exposure becomes, $\Lambda_i^-(\ell\tau)$, disregarding it's margin contribution, if liabilities outweighs receivables. Since the liquidation evolves over time, the net exposure cumulates over daily trading steps according to, $\sum_{\ell\tau=0}^{T\tau} \Lambda_i^S(\ell\tau) = \sum_{\ell\tau=0}^{T\tau} \sum_{k=1}^{K} \Lambda_i^{k,S}(\ell\tau)$.

If the bank has a positive net exposure (receivables are greater than liabilities), the CCP charges a flat fee on the amount cleared. Thus, when payments are cleared the CCP takes on a liability to bank i,

$$L_{0i} = (1-f)\Lambda_i^+ \quad \text{s.t.} \quad L_0 = (1-f)\sum_{i=1}^{m} \Lambda_i^+ \tag{16}$$

which is the return of bank i's profits net the clearing volume fee.

In the case that bank i cannot meet it's liability directly from it's receivables, it may have a debt with the CCP, which may demand liquidation of external assets. The timing of this liquidation will depend on the type of guarantee fund structure the CCP employs. In the *Hybrid Fund* structure, the initial margin contribution of bank i to the guarantee fund, can be netted directly against it's net exposure,

$$\hat{L}_{i0} = (\Lambda_i + g_i)^- \quad s.t. \quad g_i(\ell\tau) = \Big(\Lambda_i^-\big((\ell-1)\tau\big) - g_i\big((\ell-1)\tau\big)\Big) \tag{17}$$

Since this happens at each point in the trading period, the initial margin, $g_i(\ell\tau)$, of bank i decreases with shortfall in receivables, over time. Therefore, bank i will only have a liability to the CCP, if it's outstanding liabilities are larger than it's guarantee fund during any period, $\Lambda_i^-(\ell\tau) > g_i(\ell\tau)$. In this case, it must liquidate it's external asset Q_i, receiving a reduced liquidation value R_i. The amount that bank i needs to liquidate is,

$$\hat{Z}_i = \frac{(\gamma_i - g_i - \hat{L}_{i0})^-}{R_i} \wedge 1 \tag{18}$$

In each CCP fund structure, the nominal cash balance of the bank at t=1 is $\gamma_i - g_i + L_{i0} - L_{0i}$.

In a CCP with a *Pure Fund*, the initial margin contribution is only netted against the net exposure after the external asset has been liquidated. This can been written as,

$$\bar{L}_{i0} = \Lambda_i^- \quad \text{then} \quad \bar{L}_{i0}(\ell\tau) = \Lambda_i^-(\ell\tau) \tag{19}$$

which implies that,

$$\bar{L}_{i0}(\ell\tau) = \Lambda_i^-(\ell\tau) \geq \hat{L}_{i0}(\ell\tau) = \big(\Lambda_i(\ell\tau) + g_i(\ell\tau)\big)^-$$

[34]I use the formulation based on [Amini et al., 2015] which provides background for this framework

Clearly the bank cannot simultaneously have a receivable from and a liability to the CCP, so that $L_{0i} \times L_{i0}$. Thus, the bank must rectify any shortfall by first liquidating the external asset, and then may the bank use it's initial margin contribution. The amount that bank i needs to liquidate is,

$$\bar{Z} = \frac{(\gamma_i - g_i - \Lambda_i^-)^-}{R_i} \wedge 1 \tag{20}$$

[Amini et al., 2015] show that $\bar{Z} \geq \hat{Z}$ and that this is a fundamental reason the hybrid fund is more incentive compatible for member banks.[35]

The bank contribution to the guarantee fund is trivial in the pure fund case. Each bank's own contribution can only be netted against their account. Thus, the contribution to the guarantee fund is $G_i = g_i$ for all of $t_{\ell\tau} \in [1, 2]$ until the external asset is totally liquidated and deemed insufficient.

In the hybrid fund, the share of bank i in the guarantee fund depends on the realisation of Λ_i. As in [Amini et al., 2015] this is denoted as,

$$\hat{G}_i = (\Lambda_i + g_i)^+ - \Lambda_i^+ = \begin{cases} g_i & \text{if} \quad \Lambda_i > 0 \\ g_i + \Lambda_i & \text{if} \quad -g_i < \Lambda_i \leq 0 \\ 0 & \text{otherwise (limited liability)} \end{cases} \tag{21}$$

In the pure fund the share of bank i in the guarantee fund depends not only on the realisation of the net exposure for bank i, but also on the level of external assets the bank holds,

$$\bar{G}_i = \Big(\Lambda_i + \gamma_i + R_i\Big)^+ - \Big(\Lambda_i + \gamma_i + R_i - g_i\Big)^+ \tag{22}$$

For both of the above, we see that if the total guarantee fund held by the CCP is $G_{tot} = \sum_{i=1}^m G_i$ then $\bar{G}_{tot} \geq \hat{G}_{tot}$; that with any shortfall the guarantee fund is larger in the pure case rather than in the hybrid case. Though this may first appear to be more beneficial to the CCP, it will shown that this may not necessarily be the case in a scenario with an extremely large default and subsequent liquidation.

3.2.2 Clearing payments, and default condition

The clearing equilibrium depends on the realisation of *all* member banks' liabilities. Depending on each bank's liabilities, shortfall and liquidation value for outside assets, at $t_{\ell\tau} = 1$, the clearing payment of/to the bank may not necessarily be the full payment. In this section, the model identifies banks' default condition and the CCP's corresponding seizure rule.

The CCP begins t=1 with a nominal balance sheet like the one in [Amini et al., 2015], composed of assets, $\mathcal{A}_0$, and Liabilities, $\mathcal{L}_0$ with,

$$\mathcal{A}_0(t_{\ell\tau} = 1) = \underbrace{\gamma_0}_{cash} + \sum_{i=1}^m g_i + \sum_{k=1}^K \sum_{i=1}^m L_{i0}^k$$

$$\mathcal{L}_0(t_{\ell\tau} = 1) = \underbrace{\sum_{k=1}^K \sum_{i=1}^m L_{0i}^k}_{L_0} + \underbrace{\sum_{i=1}^m G_i}_{G_{tot}} + \underbrace{\Big(\gamma_0 + f \sum_{k=1}^K \sum_{i=1}^m \Lambda_i^{k+}\Big)}_{nominal\ net\ worth}$$

What is an receivable, at one time, can quickly become a liability for the CCP depending on the realisation of payments and the clearing equilibrium.[36]

[35]This is a fundamental result of the work by [Amini et al., 2015], which has the same terminal net worth in both cases.
[36]Note that in the above, $\sum_{k=1}^K \sum_{i=1}^m L_{i0}^k = f \sum_{k=1}^K \sum_{i=1}^m \Lambda_i^{k+}$.

In the case that bank i does not have enough cash assets to pay the liability $\gamma_i - g_i < L_{i0}$, at this time, the bank must liquidate a fraction Z_i of it's external assets. The bank's clearing payment to the CCP is,

$$L_i^\star = \begin{cases} \hat{L}_{i0} \wedge (\gamma_i - g_i + R_i) & \text{in Hybrid Case} \\ \bar{L}_{i0} \wedge (\gamma_i - g_i + R_i) + g_i & \text{in Pure Case} \end{cases} \tag{23}$$

When a quantity appears with neither a hat (hybrid fund) nor a bar (pure fund) it implies that the quantity, either, applies to both guarantee fund systems or that it has previously appeared with the corresponding bar or hat and should continue to be interpreted as such.

The CCP's assets now become,

$$\mathcal{A}_0(t_{\ell\tau} = 1) = \gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} L_i^\star(t_{\ell\tau} = 1) \tag{24}$$

The CCP now has has a total clearing liability payment that is determined by the equilibrium clearing receivables from banks,

$$L_0^\star(t_{\ell\tau} = 1) = \mathcal{A}_0(t_{\ell\tau} = 1) \wedge L_0(t_{\ell\tau} = 1) \tag{25}$$

The clearing payment that the CCP must make to bank i is made according to [Eisenberg and Noe, 2001, Amini et al., 2015] and follows the *Proportionality Rule*, $L_0^\star \times \Pi_{0i}$. The relative weights of each bank are given according to,

$$\Pi_{0i} = \begin{cases} \frac{L_{0i}}{L_0} = \frac{\Lambda_i^+}{\sum_{j=1}^{m} \Lambda_j^+} & \text{if} \quad L_0 \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

In the case where a bank's liability shortfall was larger than any of these personal assets could cover, $\gamma_i - g_i + R_i < L_{i0}$, the bank has liquidated its total external assets, $Z = 1$. At this point, if the bank still has a shortfall in repaying its liability, due to limited liability, this shortfall becomes the liability of the CCP. The liability of defaulted banks makes up part of the liability of the CCP according to,

$$L^{k,p}_{0\, i \in \mathbb{D}_{m-1}} = \sum_{i=1}^{m} (\Lambda_i^{k,p} + g_i + \gamma_i + R_i)^-$$

Therefore, until liquidation of defaulted positions occurs, the CCP's full liability to all surviving banks is augmented by the unpaid liability of each defaulted bank $i \in \mathbb{D}_{m-1}$[37]. This liability can increase (or decrease) in value over the liquidation period.

The liability of the CCP $\mathcal{L}_0$ now changes to accommodate this additional liability, as the positions of the defaulted banks remain on the balance sheet throughout the *Liquidation* period.

$$\begin{aligned} L_0\big(t_{\ell\tau} = 1\big) \quad \Longrightarrow \quad L_0\Big(\ell\tau \in [1\tau, T\tau]\Big) &= L_0^{1-\mathbb{D}} + L_0^{\mathbb{D}} \\ &= (1-f) \sum_{i=1}^{m} \Lambda_i^+ + \sum_{i=1}^{m} (\Lambda_i + g_i + \gamma_i + R_i)^- \end{aligned}$$

The above comes from the assumption that the CCP has a rule/threshold over which it finds it too risky to allow the defaulted party to continue trading on it's own account. This is a concrete and realistic, however, the timing of the CCP's seizure of positions is slightly arbitrary due to the limited institutional

[37] Note, i $\in \mathbb{D}_{m-1}$, since for $\mathbb{D}_m$ the maximum amount of banks would have failed. Thus, the CCP failed.

knowledge which is publicly available. Thus, the above illiquidity condition has been chosen.

For the hybrid fund, the CCP allows the distressed bank to deplete its initial margin contribution, before liquidating it's outside assets. If the bank only needs it's initial margin contribution, it's outside assets can be used to replenish that margin contribution, should the CCP make a margin call. If the bank needs, both, its initial margin contribution and the total liquidation of its outside assets, then the bank no longer has any funds to pay a fundamental part of its membership g_i[38], and it also doesn't have any remaining liquid funds, $\gamma_i + R_i$, to replenish its initial margin.

The next step would be for that bank to use it's own default fund contribution, d_i, however, this is a much smaller than the guarantee fund contribution. Since the CCP's equity tranche is the next line of defence in the CCP Waterfall, the CCP finds it too risky to allow the bank to keep clearing while using up its default fund contribution. Instead, the CCP seizes positions when the guarantee fund and assets are depleted, and applies the default fund contribution d_i itself, as needed. As many aspects of CCP functioning are not privy to the public, one is forced to apply this kind of assumption.

There is a subtle, but important, difference between the hybrid and the pure fund. The hybrid CCP, in allowing the distressed bank to net directly against it's exposure, allows the continual use of g_i up to the possible depletion of the guarantee contribution by the end of liquidation period. Since the CCP needs this contribution to offset market risk during the liquidation window, the depletion of g_i means that upon default, the CCP must instead use *profitable* members contributions. This fact will be the key that dis-incentivises member banks from predation.

In the pure fund, the bank must use it's liquid assets, and external liquidations, to meet its obligations and only then can the bank uses its initial margin contribution for the remaining shortfall. Thus, upon depleting it's initial margin, bank i already cannot replenish this fund in and the CPP cannot use a margin call on other members' contributions to save this bank. Thus, this resembles any situation where the CCP seizes positions when the remaining shortfall, netted against liquid assets, is greater than the guarantee fund, $\Lambda_i^- - \gamma_i - R_i \geq g_i$. Therefore, it waits to use the g_i until it has seized the positions and then initiates liquidation.

Definition 4a: Each bank i has the following systemic ***default condition****,*

$$\Lambda_i^{k,p,-} \geq (\gamma_i + R_i + g_i) = \begin{cases} \hat{L}_{i0} = (\hat{\Lambda}_i^{k,p} + g_i)^- & \geq (\gamma_i + R_i) \quad \text{(Hybrid)} \\ \bar{L}_{i0} = \quad \bar{\Lambda}_i^{k,p,-} & \geq (\gamma_i + R_i + g_i) \quad \text{(Pure)} \end{cases} \tag{27}$$

This says that the bank defaults if it cannot meet its full liability through all its available means: liquid assets and guarantee fund. In default, the bank faces the same situation in both structures.[39] Otherwise, when shortfall is of the size of the initial margin contribution, the bank i is better off in the hybrid structure, as it can avoid liquidation.

The CCP must decide at which point it needs to take over positions. The CCP will seize assets when the shortfall, netted against liquid assets, is still larger than the guarantee fund in either structure.

Definition 4b: The CCP ***Seizure Rule*** *is,*

$$\Lambda_i^- - \gamma_i - R_i \geq g_i$$

This is a more explicit way of re-writing the banks' default condition from the view of the CCP; in terms of seizure, the CCP is indifferent to a hybrid or pure fund as it just requires sufficient funds to cover liabilities.

[38] The membership clearing fee, f is not an issue for a distressed bank in this situation.

[39] Note that a bank in the hybrid fun will use it's g_i first, and one in the pure fund will use its g_i last.

Note that if in the final stage a bank i has an insufficient margin account, the CCP will require it's replenishment by making a *margin call* on bank i, $\mathcal{G}_i = G_i - gi$. This is especially important in the hybrid fund, when the CCP can make margin calls on surviving member banks (predators included) to replenish the reduced guarantee fund.

3.2.3 Liquidation phase

This section outlines the dynamics of the liquidation process and presents it's cumulative effects. At t=1, liabilities are realised. There is a fundamental default $\mathbb{D}_0 \neq \{0\}$; it is assumed that some bank experiences an exogenous default, not due to the liquidation, predation or a default in any of the k positions it is holding. Thus, the network forms at time $t_0 = 0\tau = 0$, when contracts are entered into. At this time, contracts are set to have zero value[40], such that, $\Lambda_i^{k,S}(0\tau) = 0$.

Due to $\mathbb{D}_0$, at time $t_{1\tau} = 1$ the CCP must start liquidating for the period $\ell\tau \in [1\tau, T\tau]$. T is the underlying liquidation time assumption used to establish initial margin contributions, g_i, and τ is the time-step/increment of the trading period. The length for the trading period is the standard 5 days ($T = 5$) and the trading increment is 1 day ($\tau = 1$).

At $t_{1\tau} = 1$, the CCP begins liquidating the holdings of the defaulted bank. As the liquidation progresses that holding experiences the value fluctuations (outlined in previous sections) and the CCP incrementally feels the various price impacts. At each period, as the CCP liquidates, it reduces its holding of by, $X_i^k(\ell\tau) = X_i^k\big((\ell-1)\tau\big) + \mathbf{a}_{\mathbf{ij}}^{\mathbf{k},\pm}$.

> *Proposition 3: The price impacts and predation effects are cumulative, and transmitted through the pricing functional for the cds-spread change. For all banks' portfolios, this amplifies unfavourable cds-spread movements and dampens positive cds-spread movements associated with a buy or sell position (Proof in appendix C).*

$$\mathbf{P}_1\ \big(3\tau, \mathbf{X}_i^{k,S}\big(3\tau, a_{ji}^{k,\pm}(2\ell)\big), \triangle\mathbf{S}^{k,S}\big(3\tau,\ X_i^{k,S}(2\tau),\ \triangle S^{k,S}(2\tau),\ P_1(2\tau),\ \mathcal{P}(2\tau),\ P_2(1\tau),\ P_3(1\tau),\ a_{ji}^{k,\pm}(2\ell)\big)\big)$$

$$\mathcal{P}\ \big(3\tau, \mathbf{X}_i^{k,S}\big(3\tau, a_{ji}^{k,\pm}(2\ell)\big), \triangle\mathbf{S}^{k,S}\big(3\tau,\ X_i^{k,S}(2\tau),\ \triangle S^{k,S}(2\tau),\ P_1(2\tau),\ \mathcal{P}(2\tau),\ P_2(1\tau),\ P_3(1\tau),\ a_{ji}^{k,\pm}(2\ell)\big)\big)$$

This is the key reason that, under any circumstances, predators decrease their own predation profits. The above mechanism illustrates how a simple liquidation sale of one banks single asset, coupled with predation and price impact as amplifiers, can create an avalanche of multiple defaults, further liquidations, resulting in a full-scale fire-sale of multiple assets. Thus, increased predation and price impacts, pose an increased chance of default for any bank at each time period. This means that, at each time-step in the trading period, there can be a cascade of defaults. Thus, any bank i can default at each time-step and join the set,

$$\mathbb{D}(t=1) = \{\forall\, i \ni \mathbb{D}_{m-1} \longrightarrow i \in \mathbb{D}_m \mid \Lambda_i^{p,-} \geq (\gamma_i + R_i + g_i)\} \tag{28}$$

This illustrates two important considerations; there can be at most m defaults, and that the CCP fails if all member banks $i = 1...m$ default.

The shortfall of bank i can then be written as,

$$i\ \in \mathbb{D}(t_{\ell\tau}) \quad \text{if} \quad C_i^-(\ell\tau) = L_{i0}(\ell\tau) - L_i^\star(\ell\tau)$$

Recall from the previous section that the CCP, in the clearing payment equilibrium, receives in any time-step of the trading period $L_i^\star(\ell\tau)$ and has a liability $L_0^\star(\ell\tau) = A_0(\ell\tau) \wedge L_0(\ell\tau)$. If any banks default

[40]This is the nature of a standard CDS contract; the up-front payment makes the contract value zero at start.

in the time-step $(\ell\tau)$, and the bank fails to pay to the CCP the amount $C^{-}_{i\in\mathbb{D}}(\ell\tau) = L_{i0}(\ell\tau) - L^{\star}_{i}(\ell\tau)$, then the CCP has an increase in liabilities in this time-step of the trading period,

$$C^{-}_{0}(\ell\tau) = L_0(\ell\tau) - L^{\star}_{0}(\ell\tau) = \big(A_0(\ell\tau) - L_0(\ell\tau)\big)^{-} \tag{29}$$

The CCP carries any shortfall, from defaults, into the next time-step of the trading period, where it will attempt to liquidate the position. It starts the next time-step with a hang-over in the previous time-steps liabilities.

Lemma 1: The shortfall to the CCP caused by defaulted banks' positions has a cumulative effect on the CCP, which can increase or decrease the liability of the CCP, depending on the outcome of liquidation at each time-step.

$$L_0\big((\ell+1)\tau\big) = L^{1-\mathbb{D}}_{0}\big((\ell+1)\tau\big) + L^{\mathbb{D}}_{0}\big(\ell\tau\big) \tag{30}$$

The strength of the effect depends on how fast the CCP liquidates the positions, and on the profits that it makes from liquidations; profits may be reduced by cumulative price impact effects and predation. The longer the CCP holds the positions on its account, the longer it is exposed to these effects, but the faster it liquidates, the more it increases the short-term strength of these effects. Finally, the CCP's exposure may become more precarious, as other member banks predate and lower the revenue on the liquidation of these defaulted positions.

Then in order to see how the CCP's assets change at time-step during each period we have,

$$\mathcal{A}_{\mathbf{0}}\ (t_{\ell\tau}) = \gamma_0(\ell\tau) \ + \sum_{i=1}^{m} g_i(\ell\tau) \ + \sum_{k=1}^{K}\sum_{i=1}^{m} L^{k,\star}_{i0}(\ell\tau) \ + \sum_{k=1}^{K}\sum_{j\in\,\mathbb{D}}^{m} \Bigg(\underbrace{X^{k}_{ij}\big((\ell-1)\tau\big) + \mathbf{a^{k,\pm}_{ij}}(\ell-1)\tau}_{X^{k}_{ij}(\ell\tau)}\Bigg) \triangle S^{k}(\ell\tau)$$

$$\mathcal{L}_0\ (t_{\ell\tau}) = \underbrace{\sum_{k=1}^{K}\sum_{i=1}^{m} L^{k,\star}_{0i}(\ell\tau) + C^{-,\star}_{0}((\ell-1)\tau)}_{L^{\star}_{0}(\ell\tau)} \ + \underbrace{\sum_{i=1}^{m} G^{\star}_{i}(\ell\tau)}_{G^{\star}_{tot}} \ + \underbrace{\Bigg(\gamma_0 + f\sum_{k=1}^{K}\sum_{i=1}^{m}\Lambda^{k+}_{i}\Bigg)}_{\textit{nominal net worth}}$$

As the time period progresses through the time-steps, the clearing payment equilibrium evolves as bank variables interact. On the asset side, clearing payments $(L^{k,\star}_{i0})$ may not be fully repaid to the CCP. As well, there is an inflow of incoming profits from the liquidation of defaulted assets. The previous periods positions change by the amount liquidated during the time-step $(\mathbf{a^{k,\pm}_{ij}}(\ell-1)\tau)$.

On the liability side, it now reflects any shortfall from the previous period, which the CCP is trying to cover through the liquidation of positions. Liquidation may fail to fully cover the last periods liability, and any shortfall will persist until the next time-period. As this occurs, banks may be tapping into their own initial margins. At this point the CCP, in the hybrid fund, could be using the combined members initial margins, or the total guarantee fund, in order to meet its shortfall.

3.2.4 Buyback period and state of guarantee fund

At t=2, the CCP has finished liquidation, the predators now enter the *Buyback* period, and the CCP determines the state of the guarantee fund. Based on the clearing payment equilibrium from the liquidation period, it is evident which banks have used their guarantee fund contributions and which banks have defaulted. This information established the terminal net worth of banks at the end of period $t_{\ell\tau} = 1$ and beginning of the period $t_{\ell\tau} = 2$

In this period, there is still a netting difference between the two fund structures. However, at this stage banks are allowed to use their own initial margin contribution g_i, and if that does not suffice, their own default fund contribution, d_i. In later periods, the CCP can access further funds toward banks' shortfalls.

Over the liquidation and buyback periods, the CCP guarantee fund changes according to,

$$G^{\star}_{tot} = \sum_{i=0}^{m} G^{\star}_i = G_{tot} \wedge \left(A_0 - L^{\star}_0 - \gamma_0 - f \sum_{k=1}^{K} \sum_{i=1}^{m} \Lambda^{k+}_i \right)^{+} \tag{31}$$

The above illustrates that each bank cannot take more than it's own contribution, at this point in time. Here, $G^{\star}_i = \left(G_i - \Lambda^{-}_i\right)^{+}$ explicitly shows that the bank cannot exceed its own contribution at $t_{1\mathcal{T}} = 2$. Also, the guarantee fund may not be the same for the two structures, as in the pure the bank needs to liquidate earlier, and for small shortfalls, may never tap into its initial margin, g_i, so that,

$$\hat{G}^{\star}_i \,(t_{\ell\mathcal{T}} = 2) \;\geq\; \bar{G}^{\star}_i \,(t_{\ell\mathcal{T}} = 2) \quad \text{and} \quad \hat{G}^{\star}_{total} \,(t_{\ell\mathcal{T}} = 2) \;\geq\; \bar{G}^{\star}_{total} \,(t_{\ell\mathcal{T}} = 2)$$

The portion of the guarantee fund owned by bank i is attributed using the proportionality rule. If this is applied to the guarantee fund the CCP must eventually return (at t=3) to each bank i the amount,

$$G^{\star}_i = \frac{G_i}{G_{tot}} \times G^{\star}_{tot} \qquad (= 0 \quad \text{if} \quad G_{tot} = 0) \tag{32}$$

The terminal net worth of the CCP is,

$$\mathbf{C_0} \,(t_{\ell\mathcal{T}} = 2) = \mathcal{A}_0 - L_0 - G^{\star}_{tot} \tag{33}$$

The shortfall during this period is again,

$$\mathbf{C^{-}_0} \,(t_{\ell\mathcal{T}} = 2) = \; L_{0\ (t_{\ell\mathcal{T}}=2)} - L^{\star}_{0\ (t_{\ell\mathcal{T}}=2)} \;=\; \Big(\mathcal{A}_{0\ (t_{\ell\mathcal{T}}=2)} - L_{0\ (t_{\ell\mathcal{T}}=2)}\Big)^{-} \tag{34}$$

The nominal assets of bank i become,

$$\mathcal{A}_i \,(t_{\ell\mathcal{T}} = 2) = \; \gamma_i \;+\; Z_i R_i \;+\; (1 - Z_i) Q_i \;+\; L^{\star}_0 \times \Pi_{0i} \;+\; G^{\star}_i - g_i \tag{35}$$

The first term is, again, the cash that bank i has on hand, the second term is the liquidated portion of the external asset, while the third term is what remains unliquidated. The fourth term is the portion of cleared receivables that bank i receives from the CCP, which depends on whether the CCP has experienced any shortfall. The final term is the portion of the guarantee fund that bank i is entitled to, net its own contribution. Since the guarantee fund portion owed to bank i obeys the proportionality rule, any loss which the fund incurs, is absorbed proportionally by all banks. In the same way, the receivables are dispersed proportionally, so any losses incurred by the CCP in liquidation, are passed on to each member bank.

The net worth of bank i can be written, $\quad \mathbf{C_i} \,(t_{\ell\mathcal{T}} = 2) = \mathcal{A}_i - L_{i0}$.

The shortfall of bank i is then, $\quad \mathbf{C_i}^{-} \,(t_{\ell\mathcal{T}} = 2) = L_{i0} - L^{\star}_i$.

Thus, when the bank is in shortfall this can be rewritten,

$$\mathbf{C}^{-}_i \,(t_{\ell\mathcal{T}} = 2) = \hat{C}^{-}_i \,(t_{\ell\mathcal{T}} = 2) = \bar{C}^{-}_i \,(t_{\ell\mathcal{T}} = 2) = \Big(\Lambda_i + \gamma_i + R_i\Big)^{-}$$

The surplus that bank i makes will be important in future sections. This is especially relevant for any profits that bank i makes from predation, by buying back positions into period t=2. The surplus of bank i is,

$$\mathbf{C_i}^{+}\ (t_{\ell\tau}=2) = \ \gamma_i \ + \ Z_i R_i \ + \ (1-Z_i)Q_i \tag{36}$$

This quantity is more relevant in the aggregate – for all member banks – when considering the failure resolution measure for the CCP with a massive shortfall in funds,

$$\mathbf{C_{tot}}^{+}\ (t_{\ell\tau}=2) = \ \sum_{i=1}^{m}\gamma_i \ + \ \sum_{i=1}^{m} Q_i \ - \ \sum_{i=1}^{m} Z_i(Q_i - R_i)$$

The difference between the two guarantee structures is seen in rewriting $G^{\star}_{tot} = \Big(G_{tot} - \sum_{i=1}^{m} C_i^{-}\Big)^{+}$.

The CCP net worth at the beginning of the period is now rewritten,

$$\mathbf{C}_0\ (t_{\ell\tau}=2) = \gamma_0 + \sum_{i=1}^{m} f\Lambda_i^{+} - \Big(G_{tot} - \sum_{i=1}^{m} C_i^{-}\Big)^{-}$$

such that $\bar{C}_0\ {}_{(t_{\ell\tau}=2)} \geq \hat{C}_0\ {}_{(t_{\ell\tau}=2)}$ is explicitly visible .

For the moment, it is best to provide the net worth of the banks in both guarantee fund scenarios though in future pure (bar) and hybrid (hat) quantities will be implicitly implied where respective quantities are almost identical. Bank i's terminal net worth at the beginning of this period is,

$$\begin{gathered}\hat{C}_i\ {}_{(t_{1\tau}=2)} = \gamma_i + Q_i + \Lambda_i - \Pi_{0i}\hat{C}_0^{-} - \hat{Z}_i(Q_i - R_i) - f\Lambda_i^{+} - \frac{\hat{G}_i}{\hat{G}_{tot}}(\hat{G}_{tot} - \hat{G}^{\star}_{tot}) \\ \text{or} \\ \bar{C}_i\ {}_{(t_{1\tau}=2)} = \gamma_i + Q_i + \Lambda_i - \Pi_{0i}\bar{C}_0^{-} - \bar{Z}_i(Q_i - R_i) - f\Lambda_i^{+} - \frac{\bar{G}_i}{\bar{G}_{tot}}(\bar{G}_{tot} - \bar{G}^{\star}_{tot})\end{gathered} \tag{37}$$

The net worth of the bank may appear identical to [Amini et al., 2015], but that is not the case; the underlying driver of net worth is different from their analysis, as there is the price impact of liquidations and predation incorporated here. The first three terms describe the cash, external assets and net exposure of the bank. Since the net worth will progress for the undefaulted banks which choose to trade within period t=2, one should think of the net exposure in terms of time $\Lambda_i(t_{\ell\tau}=2) = \sum_{1\tau}^{T\tau}\Lambda_i(\ell\tau)$.

The second term is the loss that bank i incurs if the CCP defaults on its clearing receivable. The third term is the loss due to the liquidation of external assets; this may be bigger for bank i in the pure fund scenario barring default. The last term is the loss bank i incurs on its share of the guarantee fund from defaulting banks. In the hybrid fund, this loss can occur due to another banks shortfall, as well as its own.

Concerning surviving banks, predators continue their strategy, provided predation by others hasn't put them in distress. Also, some predatory banks may have defaulted during the liquidation stage. Thus, surviving predators buy back positions during this period, trying to make a profit. This profit is not subject to the liquidation price pressure from the CCP, but from predation behaviour and fundamental values of the cds-spread. From $t_{\ell\tau}=2 \in [t_{1\tau}, t_{T\tau}]$ the modified liabilities appear as,

$$\mathbf{L^{k,S}_{ij}}(\ell\tau) = \big(\mathbf{X^{k,S}_{ij}}_{\ (\ell-1)\tau} \triangle \mathbf{S^{k,S}}_{(\ell-1)\tau}\big)^{+} + \mathcal{P}^{\mathbf{S}}_{(\boldsymbol{\ell}-\mathbf{1})\tau}\ a_{ji}^{k,+},\ +\mathbf{P}^{\mathbf{S}}_{\mathbf{2},\ (\boldsymbol{\ell}-\mathbf{2})\tau}\ a_{ji}^{k,+},\ +\mathbf{P}^{\mathbf{S}}_{\mathbf{3},\ (\boldsymbol{\ell}-\mathbf{2})\tau}\ a_{ji}^{k,+}\tau$$

and we have the net exposure $\Lambda_i(t_{\ell\tau})$ associated with the predation profits. It may appear that predation will increase profits and benefit those who practice it. It will be shown in later that this is not always the case, and also depends on the state of the CCP guarantee fund. Furthermore, since predation profits are decreased with multiple predators because combined price and predation impacts on the defaulted assets mean that banks that can predate should, there are competing effects which determine how much profit the predators can obtain. This is addressed more explicitly in future sections.

3.2.5 Default contribution and large losses

This section introduces a novel addition to the previous CCP framework; the default fund. It contains identities for fund contributions by bank i, the total contribution of all banks, and the first tranche of CCP equity. Furthermore, the model characterises the use of these contributions is utilised.

Previous examples of fire-sales and CCP liquidations have shown that losses caused by defaulted members can be very large. Respected dealer members can hold extremely large, well-balanced positions that demand only small margins by comparison. Economic intuition and this work, both, demonstrate that liabilities owed to the CCP can grow like an avalanche as the cds-spread incorporates the temporary price impacts of liquidation and predation. These fire-sale drivers can push a small fundamental price/cds-spread change toward extreme volatility. This mechanism can turn a previously healthy predatory bank into the next defaulted bank – a victim of its own strategy.

There is the possibility that a defaulting bank has a shortfall which is greater than its assets, its initial margin, and its default fund contribution, d_i. The default fund contribution is an important part of the CCP waterfall. Thus, the CCP demands both the guarantee fund G_i and the default fund D_i contributions to use against any shortfall of a bank. For the pure fund, bank i's own default contribution d_i which it deposits with the CCP as D_i is,

$$\bar{D}_i = \Big(\Lambda_i + \gamma_i + R_i - g_i\Big)^+ - \Big(\Lambda_i + \gamma_i + R_i - g_i - d_i\Big)^+ \tag{38}$$

For the hybrid fund, the share of bank i in the default fund depends on the guarantee fund g_i,

$$\hat{D}_i = (\Lambda_i + g_i + d_i)^+ - (\Lambda_i + g_i)^+ = \begin{cases} d_i & \text{if} \quad -g_i < \Lambda_i \leq -d_i \\ d_i + g_i + \Lambda_i & \text{if} \quad -d_i < \Lambda_i + g_i \leq 0 \\ 0 & \text{otherwise (limited liability)} \end{cases} \tag{39}$$

The total default fund is the cumulative contribution of all members banks, $D_{tot} = \sum_{i=1}^{m} D_i$. After the clearing payment equilibrium is established this is,

$$D^\star_{tot} = \sum_{i=1}^{m} D^\star_i = \begin{cases} \bar{D}_{tot} \wedge \sum_{i=1}^{m} \Big(\bar{D}_i - \big(\bar{G}_i - \bar{C}_i^-\big)^-\Big)^+ & \text{(Pure)} \\ \hat{D}_{tot} \wedge \sum_{i=1}^{m} \Big(\hat{D}_i - \underbrace{\sum_{i=1}^{m} \big(\hat{G}_i - \hat{C}_i^-\big)^+}_{G^\star_{tot}}\Big)^+ & \text{(Hybrid)} \end{cases} \tag{40}$$

For the pure fund formulation, only the guarantee fund contribution is accessed right after the bank's own initial margin is depleted. The hybrid fund permits the use of the full guarantee fee against the bank's shortfall, ahead of accessing the total default fund.

Once the CCP has netted the defaulted bank's own guarantee fund and default fund contributions against its shortfall, it may still need funds. Next, it must use a tranche of its own equity before the remaining guarantee and default funds. The CCP equity tranche is notoriously small – very little *skin-in the-game.* This is denoted as a fraction of nominal worth,

$$(1-\epsilon)\Big(\gamma_0 + f \sum_{i=1}^{m} \Lambda_i^+\Big) \tag{41}$$

This allows the full quantification of the net worth of the bank and CCP, at the end of the buyback period ($t_{T_\mathcal{T}} = 2$). The terminal net worth of the CCP is,

$$C_0(t_{T_\mathcal{T}} = 2) = (1-\epsilon)\Big(\gamma_0 + f \sum_{i=1}^{m} \Lambda_i^+\Big) + \Big\{\epsilon\left(\gamma_0 + f \ \textstyle\sum_{i=1}^m \ \Lambda_i^+\right) - \sum_{i=1}^m \left(G_i^\star + D_i^\star + C_i^-\right)^- \tag{42}$$

In effort to reduce expansive mathematical content due to double notation, the bracket after the first term implies that there are two separate statements for the respective fund-specific quantities; *bars* for the pure fund and *hats* for the hybrid fund (see the fully notated expressions in appendix C).
The CCP equity tranche is depleted when $\Big(\epsilon\left(\gamma_0 + f \ \sum_{i=1}^m \ \Lambda_i^+\right) - \sum_{i=1}^m \left(G_i^\star + D_i^\star + C_i^-\right)^-\Big)^+ \geq 0.$

The terminal net worth of bank i is,

$$C_i(t_{T_\mathcal{T}} = 2) = (\gamma_i + Q_i + \Lambda_i) - \Bigg\{\Big(\Pi_{0i}C_0^- + Z_i(Q_i - R_i) + f\Lambda_i^+\Big) + \left[\tfrac{G_i}{G_{tot}}(G_{tot} - G_{tot}^\star + \tfrac{D_i}{D_{tot}}(D_{tot} - D_{tot}^\star)\right] \tag{43}$$

Only at the end of the period does the default fund become available to the CCP, since only at this time is all trading is complete and the equilibrium fully realised. This is a reasonable assumption, since a decision to use the default fund, signals a potentially precarious situation for the CCP to both members and regulators.

3.2.6 Resolution period and guarantee fund replenishment

Finally, in this section, the *predation disincentive tool* is outlined and quantified for both CCP structures. Furthermore, in the face of a large liquidation, the model identifies the *incentive compatible guarantee fund structure* for the CCP.

The final period is the *Resolution*, or recovery period, at $t_{\ell_\mathcal{T}} = 3$. During this period, all trading has ceased and the CCP has determined the state of all accounts. If necessary, the CCP utilises the guarantee and default fund contributions. In a pure fund, the CCP's only financial resource is the remaining default fund contribution, $D_{tot}^\star$. In the hybrid fund, it can access the rest of both the guarantee and default funds, $G_{tot}^\star + D_{tot}^\star$, to buffer it's potential failure.

At the end of this period, the CCP must return the guarantee fund contributions, $G_i^\star$, to the proper accounts. In the pure fund this is trivial, the member simply gets back its own contribution. However, in order to retain membership in the CCP, it must replenish its own initial margin account,

$$\bar{G}_i^{\mathfrak{R}}(t_{T_\mathcal{T}} = 3) = (g_i - \bar{G}_i^\star)$$

In the case that all cash and external assets in the margin account are depleted, and the member has defaulted, even if some of the initial margin remains. The CCP simply returns the remaining contribution to the bank and cancels its participation. Thus, for the pure fund $\bar{G}_i^{\mathfrak{R}}$ is a guarantee *refund*, which may be used for filling one's own margin account.

For the hybrid fund, the situation for member banks is more financially punitive. The banks which have defaulted may impose a burden on the CCP, which demands the use of the full guarantee fund. The likelihood of this event increases with the amount of predation and distress; predation can increase liabilities such that further distress and defaults occur. Therefore, in period t=3, surviving and profitable members are called upon by the CCP to replenish their own margin contribution, if it has been depleted by another member! In this case, the CCP, as a recovery measure, makes a margin call for full guarantee fund *replenishment*.

Proposition 4: In a hybrid guarantee fund structure, the CCP has a natural disincentive tool for predation in the initial margin. The CCP can make margin call on each bank (including predators) to replenish their initial margin contribution after a shortfall to in order to maintain the bank's membership in the CCP and clearing privileges.

$$\hat{G}_i^{\mathfrak{R}}(t_{T\mathcal{T}} = 3) = (g_i - \hat{G}_i^{\star}) \tag{44}$$

The member bank must meet the initial margin requirement in order to maintain membership in the CCP, thus, it must use its profits (possibly from predation) and the CCP has a natural, predetermined, punitive mechanism to dis-incentivise predation.

With the expectation by the CCP, that it's fire-sale liquidation could possibly cause a shortfall greater than the magnitude of it's equity tranche plus the available total guarantee feed,

$$D_{tot}^{\star} + (1-\epsilon)\Big(\gamma_0 + f\sum_{i=1}^{m}\Lambda_i^{+}\Big) \;<\; \epsilon\Big(\gamma_0 + f\sum_{i=1}^{m}\Lambda_i^{+}\Big) + G_{tot}^{\star} \;\leq\; \mathbb{E}\left[C_0^{-}(t_{\ell\mathcal{T}} = 3)\right]$$

then all members are in a better (aggregate) position, in terms of their default fund contributions, having instituted the hybrid fund and preventing the whole CCP's default. That is, if the hybrid fund is instituted, all guarantee fund contributions can be used to meet remaining liabilities. This increases the chance that single banks avoid early liquidations, that default fund contributions are preserved, and that predatory banks must pay for increasing the liabilities of other banks.

Proposition 5: In the case that the is shortfall is the size of the guarantee fund (+ CCP tranche) or smaller, the CCP is expected to be better off using the hybrid approach and protecting its own equity.

$$\mathbb{E}\left[\hat{C}_0(t_{\ell\mathcal{T}} = 3)\right] \;\geq\; \mathbb{E}\left[\bar{C}_0(t_{\ell\mathcal{T}} = 3)\right]$$

Thus, it is in the CCP's best interest, in terms of dis-incentivising predation and protecting it's own equity, to institute a hybrid guarantee fund!

The net worth of the CCP, in period $t_{\ell\mathcal{T}} = 3$, reflects the access to the full guarantee and default fund for the CCP in either guarantee fund structure,

$$C_0(t_{T\mathcal{T}} = 3) = \Big(\gamma_0 + f\sum_{i=1}^{m}\Lambda_i^{+}\Big) - \Big\{\Big(G_{tot}^{\star} + D_{tot}^{\star} + \textstyle\sum_{i=1}^{m} C_i^{-}\Big)^{-} \tag{45}$$

The net worth of bank i is,

$$C_i(t_{T\mathcal{T}} = 3) \tag{46}$$

$$= (\gamma_i + Q_i + \Lambda_i) - \left\{\Big(\Pi_{0i}C_0^{-} + Z_i(Q_i - R_i) + f\Lambda_i^{+}\Big) - \left[\tfrac{G_i}{G_{tot}}(G_{tot} - G_{tot}^{\star}) + \tfrac{D_i}{D_{tot}}(D_{tot} - D_{tot}^{\star})\right] + G_i^{\mathfrak{R}}\right. \tag{47}$$

Note again, that if a healthy bank loses a part of its guarantee contribution, it must replenish that contribution to remain a member of the CCP for future clearing services.

4 Simulation Results

4.1 Simulation specifications and calibration data

The following section provides a a simulation, calibrated to financial market data, in order to gauge the magnitude and impact of the theoretical model. For realistically meaningful results, multiple default scenarios – varying market liquidity, number of distressed banks and number of predatory banks – must be taken into account.

The simulation considers 14 core dealer banks, holding 100 different CDS each. Due to the opacity of the OTC market, multiple reputable data-sources must be used. For current financial market liquidity and CDS market features, data from Bloomberg (2017) is used. For accurate past market depth, spread movements, turnover, as well as gross and net CDS holdings, I cross-reference among three vetted sources [Oehmke and Zawadowski, 2017, Duffie et al., 2015, Amini et al., 2015]. For brevity, the various variable magnitudes and an in-depth explanation of simulation design are given in appendix D. As well, further results (ie. different calibrations and multiple runs) are also available in the appendix and online.

[Duffie et al., 2015] analyse the daily data[41] of 184 single-name CDS reference entities, composed of 22 sovereigns and 153 global financial entities. Furthermore, they include any counterparty/CCP member as long as they hold a CDS on 1/184 reference entities. [Oehmke and Zawadowski, 2017] specifies the analysis of about 1000 single-name CDS, with at least three dealers holding 1/1000 reference entities; they also analyse a sub-sample of 97 reference entities. The number of CDS analysed is not provided for [Amini et al., 2015]. All of the above data sources analyse CDS positions held at multiple CCP's.

In the scenario of interest in this paper, one is looking at CDS positions held at one CCP, by the top 14 dealers, liquidated over the period of a week, and bought back over a period from one day to a month.[42] Since, it is unlikely that any dealer will hold the full universe of possible CDS, in one CCP, during a short period of time, I use a maximum of 100 contracts for each dealer. The positions are assigned autonomously between counterparties, by random assignment. This means that each dealer randomly holds one side of each (buy/sell) position with another dealer, this is done position by position, thus, allowing all positions to act as separate entities. In this simulation, the number of non-matched pairs is both, chosen and assigned, randomly. This is to ensure that the matrix of bilateral contracts is not completely saturated, and that I get a realistic distribution of pairings in the market.

In order to create the matrix of position sizes I use three vetted sources, [Oehmke and Zawadowski, 2017, Duffie et al., 2015, Amini et al., 2015]. I start with a similar approach to [Amini et al., 2015] in setting positions X_i^k using a probability distribution obtained from data. I use their approach taking the gross (rather than net) notional, which encompasses both buy and sell positions. However, I do not use their value of $19e12$ taken from 2010 BIS data since this is yearly data and approximates the whole CDS market. Instead, I reference $4.91e12$ from [Duffie et al., 2015] which has daily data for single-name global CDS over a period which includes the financial crisis and $4e11$ from [Oehmke and Zawadowski, 2017] over a similar period.[43] The former sample takes into account 31.5% of the global single-name CDS market, and 18.9% of the total CDS market. It also encompasses all the core dealers. The latter, [Oehmke and Zawadowski, 2017], gives monthly data. They use the Trade Information Warehouse of the Depository Trust & Clearing Corporation (DTCC) which captures 95% of the total globally traded single-name CDS market. They use data obtained on a weekly basis, for positions from October 31, 2008, and for trading from July 16, 2010. They use 1,000 reference entities and 5-year CDS. Finally, to round out the calibration, I obtain Bloomberg data for February 21, 2017, where 1,241 reference entities are available. This last set of data, is used to approximate normal CDS market size, and depth, as well as the size of dealer bank holdings.

Thus, [Oehmke and Zawadowski, 2017, Duffie et al., 2015, Amini et al., 2015] have gross notionals of $13.41e12$, $4.91e12$ and $19.0e12$, respectively. I reference the [Oehmke and Zawadowski, 2017] for out-

[41] Their data period takes into account css-spreads from January 2008 until December 2011.

[42] Other possible scenarios, constructed with the above data, are given in the robustness section. These may produce extended time periods of buyback, into months and years.

[43] Data for the former encompasses a period of January 2008 to December 2011, and for the latter a period of October 2008 to December 2012 for notional data, but August 2010 to December 2012 for trading data.

standing positions in each asset, but with caution; the mean is $13.3e9$ and the standard deviation is $12.19e9$. I inflate it by a factor of 10 in order to more closely match the other data sets. This adjustment comes from sparse, but available data on CCPs; ICE (the biggest Credit CCP) with ICE Clear Credit and ICE Europe, it's much smaller rival CME, barely competitive LCH CDS Clear, and JSCC. According to [IOSC, 2012], as of March 23, 2015, each have a single-name CDS market share of approx. 77.1%, 18.8%, 3.69%, 0.369% and 0.0369%, respectively. This gives a market share of 20% for the average CCP. Furthermore, of the whole cleared CDS market, the actual proportion of single-name CDS cleared is small at 14.63%. ICE itself, cleared 29% of the global CDS market of $24e12$ in 2010 according to [Terhune, 2010], though it is not state whether this is through both ICE Credit Clear and ICE Europe. Only ICE has been designated as a systemically important institution by the FOSC. However, this is no reason to disregard the important of CCP's of much smaller size as they tend to have a membership composition of very different creditworthiness and risk profiles. According to [Council, 2012] ICE has 27 clearing members, 14 of which are financial. In 2011, they cleared contracts on $1,145$ single-name reference entities.[44] They clear 200 of the most liquid reference entities daily. However, ICE cleared only approx. 15% of the contracts on any reference entity in the market.

Using the above data and the gross notional from [Duffie et al., 2015], I start with the value of $4.9e12$ taking 20% of that as the market share of the CCP, and taking a further 20% as the number of reference entities cleared, this gives $16e10$, which is fairly close to the [Oehmke and Zawadowski, 2017] value (inflated by a factor of ten) of $13.3e10$. Thus, I use inflated values for the mean and standard deviation originally given by Oehmke. Since I am using these values as the mean gross notional for the market in one average CCP, rather than the mean outstanding positions in each asset, I must divide it by the number of banks, number of counterparties and the number of assets available. I do this in order to ensure that the value is properly distributed among each contract. Thus, dividing it among each bank, each counterparty and each asset, gives a mean=$13.3e10/m(m-1)k$ and standard deviation=$12.19e10/m(m-1)k$. I use a normal random distribution to assign both sides of the positions.

I check that the final total sum over all positions over all banks gives a realistic value, which for the analysis presented is $1.50e11$. This is close to the data for the daily market size of $2.21e11$ from Bloomberg (2017), and the sub-data set of 97 reference entities of $4e11$ from [Oehmke and Zawadowski, 2017], but further from their value for the top 1000 reference entities, $12.6e11$, and for $4.91e12$ from [Duffie et al., 2015]. Taking 31.5% of the full market (1000 reference entities) in [Oehmke and Zawadowski, 2017] gives $1.11e11$, which approximates the size of the of the market sample used in [Duffie et al., 2015]. This gives a rough range of $1.11e11 - 12.6e11$ for a market with approximately 100 CDS and adds to the credibility of using both data sets in order to get a total picture of the market. A further investigation for robustness and sensitivities in these values is provided in appendix D.

I use the data from [Amini et al., 2015] to set the level for the initial margin at g=$11.2e9$ (maximum incentive compatible), though this gives 0.09% margin on each banks holdings. In [Duffie et al., 2015] the system wide collateral demands on gross positions is 0.78% and dealer-to-dealer collateral demand is 1.37% of gross notional. Thus, over all 14 dealers, the dealer-to-dealer collateral demand of the gross notional is 1.26% using margin level of [Amini et al., 2015]. The default contribution of banks is set at 10% of the initial margin as is often mentioned in the literature. The cash and external asset holdings of the CCP and bank are set according to [Amini et al., 2015] at $\gamma_0 = 5e9$, $\gamma_i = 10e10 - g_i - d_i$ (endowment - initial and default fund contributions), $Q_i = 1.1e10$, respectively. This is in close agreement with [Oehmke and Zawadowski, 2017] which gives average banks asset holdings as $1.06e10$.

The fundamental price, S_0^k, is also drawn from a normal distribution, with mean and standard deviation provided by data on cds-spread movements from [Oehmke and Zawadowski, 2017]. The normal distribution is a rational assumption as this is only used for the fundamental part of the cds-spread, which is determines by the integration of public information into prices and accounts for 40% of the explainable movement in the cds-spread. I set the mean equal to 249.2 bps (basis points) and stan-

[44] As well as, 821 index and 397 sovereign, and have a daily trading volume of approx. $300e9$ for all contracts. The peak daily cleared is $14,708$ contracts.

dard deviation at 269 bps. I then devise a function which calculates, both, the full spread and the full price (with price impacts); the full spread determines the fundamental cds-spread for the next time-step, while the full price gives the net exposure (Λ_i) of each bank. The liquidation matrix is also determined randomly form a normal distribution based on [Oehmke and Zawadowski, 2017] on CDS turnover; mean=$0.516e9$ and standard deviation=$0.646e9$ both adjusted for the number of banks, m, and the number of available CDS reference entities, k.

I illustrate the model's default scenario under three realisations of market liquidity: normal, decreasing, and crisis. In a normal market, liquidity is at healthy/stable levels, producing the least severe instability effects. Then, decreasing market liquidity illustrates the downturn which can lead to large-scale instability; market liquidity decreases proportional to the amount of liquidation and increases during the buyback period. Finally, financial crisis liquidity replicates a market dry-up, much like that which occurred during the recent financial crisis. A final assumption, unless stated otherwise, assumes that that the CCP can continue offloading[45] its positions throughout the model and that distressed traders can no longer trade in the buyback period.

For market liquidity, I use market volume from both Bloomberg (2017) and [Oehmke and Zawadowski, 2017] in setting the market depth, D_k, during profitable market functioning at $221e9$, and at $12e9$ for financial crisis liquidity. In order to obtain changing market liquidity, I also run a scenario where I decrease normal market liquidity, per period, until it reaches crisis liquidity after liquidation and permit liquidity increase during buyback, in an amount equal to the periods of trading that ensue; increments are set at $38e9$ per period. I have set the model so that one can determine the round in which the CCP defaults, in the liquidation and/or the buyback rounds. In the buyback round, the model will stop once predators have reached their maximum allowable threshold holding. The simulation accommodates multiple CCP trading strategies, both, when the CCP stops liquidating the original defaulted position at the end T=5 day/period window (for the original defaulted asset), and when it continues to liquidate into buyback round. Moreover, it accommodates both distressed banks selling and not selling in the buyback round, along with the predators. The simulation further allows for the specification of the maximum number of allowable predators and distressed banks. These values are then used to determine the level at which parameters for the liquidation and buyback values are set.

For each period, I calculate a final worth/value and then track both, the losses and gains from period to period, for the CCP and for banks. The evolution of default is tracked for the banks, thus, one can track which banks fail in the liquidation and which fail in the buyback period. The simulation starts with one exogenously determined defaulted bank and this bank's parameters are set to default levels. The gains and losses for this bank (and all subsequent defaulting banks) go on the account of the CCP, at the end of each period. Also, at each stage, the assets of each subsequently defaulted bank are added to the pool of defaulted assets, $k \in \mathbb{D}$. They are then accounted for in the spread and pricing functions, through covariance and price impact. Thus, the effects of liquidation and predation are transmitted throughout the periods. Furthermore, the liquidation function accommodates, both, positive and negative movements in each asset, assuring that cds-spreads on the positions move in both directions, according to marking demand.

The Matlab® code and example output data tables will be available in an online appendix. In this simulation, I have tried to devise a large-scale and novel computing approach for the modelling of dynamic contagion in a financial network. However, I acknowledge there are some drawbacks. Without doubt, there are issues concerning the accuracy of the calibration of the OTC data for CDS due to the opaqueness and availability of market data. However, the simulation admits ongoing updates of the calibration values easily. As well, any future clarification of CCP guidelines and procedures can also be input easily into the simulation. However, currently, the model does not admit the formation of new CDS relationships, though this could be added at some stage. All bilateral trading relationships are formed at period 0 and then change according to bank defaults. Also, currently, the model doesn't

[45] However, the CCP has finished liquidating the holdings of the original defaulter within the liquidation window.

estimate the number of counterparty relationships in the network or for each bank, though this could be added. Furthermore, the simulation doesn't stop when the CCP first reaches a shortfall, leading to the least possible default values. Though this could be done, it is instructive to see how large a shortfall the CCP could incur over a short period of time. Finally, in a simulation of such magnitude, there may be variations in the result or computing issues which arise when varying calibration inputs to a large degree. However, I try to account for these possibilities with robustness checks available in the following section and in Appendix D.

4.2 Simulation results: implications for policy and regulation

The first result (Fig.2) occurs in a healthy market, under normal/stable market liquidity, and looks at how the distribution of defaulted banks changes with the number of increasing predatory banks and decreasing distressed banks. I define distressed banks as any non-predatory bank, holding the asset(s) being liquidated. Strikingly, the major effect on the number of defaults is the amount of distressed banks. The number of bank defaults jumps with large numbers of distressed banks; from the one external default to six defaults when 12/14 banks are distressed. However, there is a clear effect caused by predator competition, which serves to dampen defaults to a small extent (Fig.3). Maximal defaults occur when there is a monopolistic predator. However, with the increased competitive pressure of two predators and up to 11 distressed banks, bank defaults are suppressed to only the external default. This is because competitive pressure decreases the length of time over which predators can engage in liquidation, mitigating some of the instability effect provided the market is liquid. As well, for normal liquidity, there is no failure of the CCP during the liquidation round. However, the liquidation loss is large and pushes the CCP to default during the predatory buyback phase. This occurs with varying magnitude according to the distribution of distressed and predatory banks. The most dire situations (yellow) occur with a low number of predators and a high, increasing number of distressed banks (Fig.4). This suggests that simply suppressing the number of predators is insufficient to mitigate loss. In general, predatory profits are quite volatile (Fig.5). However, interestingly one can tease out the interplay (aqua) of competitive pressure (green) and distress (yellow) on predation profits; the combined effect is synergistic, but not necessarily beneficial, suggesting a complicated inter-reaction.

The decreasing liquidity scenario is the most realistic scenario if a large CCP initiates a fire sale. Turning to preliminary results, it turns out, there is no difference in defaults based on predator collusion. The competitive effect has been diminished due to reduced market depth. The major effect on the number of defaults is still the number of distressed banks (Fig.6 and Fig.7). However, maximal defaults (13) occur and with fewer distressed banks required (8). This case may be instructive for a lender-of-last-resort (LOLR) trying to stave off a crisis scenario at the beginning of a financial downturn; containing distressed bank insolvency (to less than half the core dealers), either through provision of liquidity to the banks (ex-ante) or to the CCP (ex-post), can prevent default contagion among major dealer banks (Fig.8). Strikingly, the CCP loss is immensely minimised ($\approx -0.5 \times 10^{10}$ vs. -6.5×10^{13}) if the overall number of both, distressed and predatory banks, is kept low (aqua-blue scale) (Fig.9). More resolution on this market liquidity scenario is provided later in this section.

If a CCP launches a liquidation in a state of financial crisis, with low and constant market liquidity, the results are dire; the CCP fails during the second or third day (out of five days) of liquidation in all cases. Although one bank survives the liquidation round (Fig.10), there is maximal default (14 banks) during the buyback round. This maximal default combined with the previous insights shows that predators, have effectively, predated themselves into default. The CCP loss is again the lowest with low levels of distressed banks (green), in general. The highest losses occur with intermediate levels of distressed banks (7-10) for any level of predators (Fig.11). This suggests market illiquidity augments the loss of liquidation in a way the cannot be recompensed by profits (predatory or otherwise) accrued in buyback. This also implies that a LOLR may mitigate losses by intervening at the beginning of a crisis, keeping a low levels of distressed banks. With an high level of distressed banks (10), the LOLR might want to wait until the end of the crisis to bail out the CCP, when there is an increased chance that further distress and buyback profits will mitigate the intervention required. However, intervention in the middle of a crisis, when distressed dealer survival is as probable as failure, may be the least efficient course of action, unless large amounts of liquidity can be provided. Remarkably, with very low market depth and the impending possibility of extremely large losses for the CCP, the CCP will almost always incur less of a loss than with a hybrid fund structure (Fig.12) than with a pure fund, as predicted.

Turning again towards the decreasing market liquidity scenario, with more fine-grained detail. In regard to losses for a hybrid vs. a pure fund for decreasing market liquidity, the smaller hybrid loss effect is only partially visible; it disappears for very low levels of distressed banks. This is due to the large losses that predators suffer when there are few positions on which to make predation profits. The predators themselves become distressed and themselves become a burden on the hybrid guarantee fund. Furthermore, the simulation yields interesting results concerning the hybrid fund as a disciplinary mechanism where predator profits are garnished to bailout distressed banks and margin accounts must be replenished in a margin call. One can see that systemic loss in the whole market (Fig.14) is driven by the number of distressed banks; particularly, when an intermediate number of distressed banks and competition mean that there is only a little predation profit which can be realised, the price impact of liquidation takes its toll on predatory banks. In the case of a low, stable number of distressed banks, predators lose a third of their wealth to the recovery margin call (Fig.13, Panel 2). Increasing the number of distressed banks gives the predators a chance at maximum predation profits, though they experience a huge loss at the point when just less than half the market (6/14) is distressed (Fig.13, Panel 1, 3). In fact, when the market makes a profit (under increasing distressed and decreasing predatory banks), the predators yield only a small loss of their income (Fig.13, Panel 1). This suggests that the hybrid mechanism can be used as extremely powerful disciplinary mechanism (Fig.15). In particular, the major loss for intermediate distressed banks, seen in the pure fund financial crisis scenario for the LOLR, is partially born by the predators themselves in a hybrid fund.

One can also look at the profit/loss predators make on buying back shares (Fig.16); for low, stable levels of distressed banks (yellow) predators losses are driven by competition and quite volatile. This implies that the threat of a LOLR, ready to keep bank defaults low, will make engaging in predation a very risky business. In the case of low, stable predation (purple), there is a marked increase in profits for an increase in distressed banks. However, these profits are made during the buyback round, and may not survive the margin call. In the recovery stage, the predators pay the lowest margin refill when there is low predation, and a high number of distressed banks (Fig.17); this is in line with what is seen for CCP losses. In the case of no collusion (orange), encroaching competition and an increase in distressed banks (>7) can lead to a volatile profit and debilitating losses. In this case, predation can become costly, and the threat of margin refill may dissuade this behaviour.

Concerning the the CCP profit and loss from liquidation, one sees that the actual liquidation loss is greater (Fig.18) in the pure fund. This is apparent in most cases, but particularly, in the no-collusion (yellow)/stable predator (red) cases. During the buyback round, the loss incurred is similarly the same or greater in the pure fund. Concerning the profit/loss of all banks in the market, they also tend to profit more in the hybrid structure (especially in low, stable predator bank scenario) (Fig.19). As well, in the hybrid fund the banks experience a higher aggregate surplus in both the liquidation and buyback rounds (Fig.20). Finally, the CCP experiences its largest loss if it can't trade past the liquidation window, that is, laying off positions of the most recent defaults. As well, the distressed banks lose the least when they continue to sell into the buyback round, provided that the CCP also continues selling (Fig.21). The CCP's loss is magnified, at the point of intermediate bank distress, if it can't sell past its liquidation window and the distressed banks can, yielding maximum price impact. Now, the distressed banks' selling has, itself, a predatory effect on the CCP. This last threat further suggest, that liquidation is an extremely inefficient strategy to close out positions due to the many routes through which losses can be augmented.

As a last note, though the default fund is completely exhausted in each fund scenario, the final the hybrid CCP guarantee fund is never depleted. This fact combined with the smaller CCP loss in the hybrid fund suggests that the amount in the default fund is insufficient, in either fund scenario, to cover the banks' default deficit which is left over after their initial margin contributions in the guarantee fund have been depleted (see online table). This also implies strongly that, in most cases, the hybrid CCP can cover as many risk scenarios as can the pure CCP, with added benefits. Further analysis is available in appendix D.

4.3 Simulation robustness and sensitivities

I have used the specification above after determining that, with the information available, the values chosen are most illustrative of the realistic OTC financial market in the situation the model attempts to address. However, I have acknowledged that arguments can be made for other calibration values, and in any case, investigation of alternative specification can give good insights on the possibilities/limits of the simulation as well as sensitivities in values. I will address these supplementary cases and any marked or novel features/problems which appear, below.[46] For the first case I will outline all three scenarios and then focus on the market depth scenario for the other cases. All additional tables and figures can be seen online.

Case 1 - More Assets: 150 CDS, Mean=$\frac{13.3e10}{k{*}m(m-1)}$, Sd=$\frac{12.19e10}{k{*}m{*}(m-1)}$

Total size of the gross notional in the market provided by the model is $1.51e11$, which is roughly the same as the official case. For a normal, stable market depth, with more assets, there is an increase in the number of defaults (fig.2), for a high number of distressed banks, and effect which flattens out quickly. CCP loss is now more prominent with the amount of banks which are distressed, and greatly minimised in the case of a low, stable level of distressed banks (fig.4). Predation profits are volatile, but profits are maximised in the no collusion case and minimised in the case of a low, stable distressed level of banks (fig.5).

Under crisis market depth, the CCP liquidation and final loss is further maximised when half of the banks are in distress for reasons I have outlined in the official case (fig.10). The pure and hybrid funds yield increasingly similar income outcomes in most cases except when number of distressed banks is at its lowest (fig.12).

Under decreasing market depth, with more assets, the CCP final loss is minimised overall and the burden of increasing distressed banks is minimised (fig,9). Predator loss due to the margin call are increased in most cases, but disastrous in cases of low level of predators and half of banks being in distress(fig13). Profits are equally as volatile (fig15). Predation profits on original positions are now more clearly increased by more predators in buyback case, as more buying increases price (fig.16). Overall, average margin refill required of each predatory bank is diminished as losses can be recovered by the CCP as price rises in buyback round, as long as it is still selling (fig.17). For the CCP there is an increasing advantage in having a hybrid fund over a pure fund when it comes to its liquidation loss (fig.18). This is true for the banks overall and for the predators alone (fig.19). Preventing the CCP from selling in the buyback round and allowing this same selling for distressed banks has less dire consequences with increasing assets.

Case 2 - Larger Market : 100 CDS, Mean=$\frac{13.3e9}{m(m-1)}$, Sd=$\frac{12.19e9}{m{*}(m-1)}$

Total size of the gross notional in the market provided by the model is $1.50e12$, which is a factor of 10 larger than the official case, approaching the [Oehmke and Zawadowski, 2017] scenario which expands the number of available CDS from100 towards 1000 reference entities. Note, that in the data, the matrix of holdings between dealer banks may be incredibly sparse (ie. 3 dealers holding 1/1000 CDS with each other).

Under decreasing market depth, the dealer default rate is maximal in all scenarios, as larger positions in the same amount of asset lead to lower diversification(fig.6). Losses are increased in all scenarios for

[46]Note we call the case used in the paper, the *Official Case: 100 CDS and Mean net notional=13.3e10/m(m-1).*

the CCP (fig.9). Predator profits are tiny in all cases and the banks as a whole experience large losses (fig.14) Predator profits and losses become larger and increasingly more volatile, but percent margin lost to the margin is much lower, as predators can make larger profits (fig.15 & 16). The margin refill required by the CCP becomes more volatile as well (fig.17). The CCP loses increasingly more in the pure fund over the hybrid fund when there is a low, stable level of distressed banks. Interestingly, an increase in the market, has led to a reversal in the configurations (of the financial network) for which the CCP has an advantage using the hybrid fund rather than the pure fund (fig.18). The liquidation surplus for the banks is much higher in the hybrid fund, but there is no difference during buyback (fig.20). For the effects of the CCP and distressed selling strategies, the effects have shifted. There is no longer the possibility of an extreme loss for the economy (CCP and banks) due to one configuration of the financial network, but losses have have increases overall. Furthermore banks no longer see any final profit (fig.21).

Finally, under crisis market liquidity, There is now a clear benefit to having a hybrid fund over a pure fund (fig.12).

Case 3 - More Assets, Larger Market: 184 CDS, Mean=$\frac{13.3e9}{m(m-1)}$, Sd=$\frac{12.19e9}{m*(m-1)}$

Total size of the gross notional in the market provided by the model is $2.75e12$ which is approaching the [Duffie et al., 2015] scenario which includes a slightly larger market with multiple CCPs and admits CDS on sovereigns. This scenario produces the same affects as the previous scenario, with increased magnitude. Thus, increasing assets has the same augmenting effect in for both the smaller and larger market cases.

5 Extensions

5.1 CDS Credit Events: CDS Defaults and CDS written on member dealers

In the analysis, we have thus far neglected one nuance of CDS; the case where a CDS which a dealer has sold experiences a *credit event* and comes due, at which point he must pay the buyer. When there is a credit event, such as a default, which triggers this part of the contract, there is a CDS auction[47] held to determine the residual value that must be paid out to the buyer. This process is not normally expedient, and thus, it is possible to disregard this nuance in the model and get reasonable results. However, in order to build more accuracy into the model, one can include the residual restitution payments for a CDS contract that one dealer must pay another; that is, once the value of the CDS has been ascertained and comes due in the current period. This is important because it changes the financial situation of both the selling and buying dealer, perhaps sending it into distress or bringing it back to health. Also, it is another trigger for default contagion.

Another important nuance of a CDS CCP, concerning its dealer members, is that there may be a CDS written on any of the participating members (ie. Lehman Brothers). Hence, any member dealer j=1...m, can be the underlying entity for a CDS $k = j'$. Thus, it is possible that the default of a member dealer can trigger a credit event for a CDS written on that dealer. This one complication can have ripple effects that may increase the liquidation burden of the CCP and increase losses. Therefore, this scenario should be strongly taken into consideration, especially considering that there are many situations, falling short of a full default by a dealer, that may be contractually deemed as credit events. In order to accommodate this scenario, the model requires only that the above expressions for the liabilities (L) and net exposure (Λ) be adapted.

In the case that any dealer $j' = k$ may default and trigger a credit event for a CDS written on it as the underlying reference entity by some other dealer $j \neq j'$, we augment the liability structure, written concisely for the receivable (L_{ji}) to dealer i from dealer j as,

$$\mathbf{L_{ji}^{k,p}} = \mathbb{1}_{k \in \mathbb{D}} \left(R^{k,p} \; X_{ji}^{k,p} \right) + \triangle S^{k,p} \; X_{ji}^{k,p}$$

[47] For details on the CDS auction process please see [Du and Zhu, 2012]

where $\mathbb{1}$ is the indicator that the CDS in question is in default due to reference entity $k = j'$, R^k is the agreed upon residual compensation value determined for the CDS written on reference entity $k = j'$. The full compensation to dealer i from CDS seller dealer j is $R^k X_{ji}^{k,B}$, but only if dealer i is the buyer of the CDS. Conversely, the amount dealer i must pay to dealer j is $R^k X_{ij}^{k,S}$, but only if dealer i is the CDS seller. I denote the full compensation as $r_{ji}^k = R^k X_{ji}^{k,p}$. When embedding this augmented term for liability into the full expression for net exposure, one needs to account for possible default of either party, on each side of the CDS contract.

This gives the following structure,

$$
\begin{aligned}
\Lambda_i^{k,S}(\ell\tau) &= \sum_{j=1}^{m} L_{ji}^{k,p}(\ell\tau) - \sum_{j=1}^{m} L_{ij}^{k,p}(\ell\tau) \\
&= \sum_{j=1}^{m} \left(l_{ji}^{k,S}(\ell\tau) + \mathbb{1}_{k\in\mathbb{D}}\, r_{ji}^k \right) - \sum_{j=1}^{m} \left(l_{ij}^{k,S}(\ell\tau) + \mathbb{1}_{k\in\mathbb{D}}\, r_{ij}^k \right) \\
&= \sum_{j=1}^{m} \left(\triangle S^{k,S}(\ell\tau) + \mathbb{1}_{k\in\mathbb{D}}\, \mathcal{R}^k(\ell\tau) \right) X_{ji}^{k,S} - \sum_{j=1}^{m} \left(\triangle S^{k,S}(\ell\tau) + \mathbb{1}_{k\in\mathbb{D}}\, \mathcal{R}^k(\ell\tau) \right) X_{ij}^{k,S}
\end{aligned}
$$

where $L_{ij}^{k,S}$ is the full liability of i when accounting for the default risk of the CDS; this is written on dealer $j' \neq j$ who acts as the reference entity k. This dealer belongs to the set $k = \{j'_1, j'_2, ...j'_{m-1}, j'_m\}$. However, the expression also accounts for the default risk associated to a CDS written on all other reference entities, $k = \{j'_1, j'_2, ...j'_{m-1}, j'_m, k_{m+1}..., .k_{n-1}, k_n\}$.

Furthermore, $l_{ji}^k = \triangle S^k \; X_{ji}^k$ is the total variation margin liability described in previous sections, r_{ji}^k is the total agreed upon default compensation value that the buyer (i) will receive from the seller (j), on the appropriate portfolio holdings (X_{ji}^k), should the contracted credit event (on k) occur.

Finally, embedding this expression into the full model will illuminate the implications of this new layer of possible liabilities. It is interesting to note that the default of a member triggering a credit event introduces one more nuance into the model: It is unlikely that j is writing/buying a CDS on itself. Furthermore, due to the presence of the CCP, in the situation where dealer $k = j'$ fails, the CDS seller, dealer i, must still repay it's full variation margin liability to the CCP, that owing to both j and to $k = j' \neq j$. In turn, the CCP must still repay to i, the receivable owed by j', even though it has failed. This is in contrast to the situation with no CCP, where the failure of j' means that dealer i will probably receive nothing from the failed counterparty due to counterparty risk not secured by rigorous CCP margin requirements. Finally, dealer i is also owed any receivable accrued to it by dealer j, and eventually, the residual value from buying the CDS on $k = j'$, provided that dealer j is the seller of this CDS.

6 Conclusions

Recent regulation mandating standardisation and the central clearing of OTC derivatives, such as CDS, has turned CCP's into systemically important institutions. In light of this, the possible failure of a large CCP poses a serious threat to the stability of the global financial system. This work investigates whether the failure of a large dealer bank could, possibly, initiate the failure of a CCP through an effort to liquidate/unwind a defaulted bank's positions. The theoretical model looks at the variation margin exchanged between dealers and the price impact of liquidation and, subsequent, predatory selling. I provide a mathematical illustration of the distance between assets in banks' portfolios, in terms of their covariance. Furthermore, I show how price impact affects assets to varying degrees, based on their distance to the defaulted assets.

This yields four key theoretical results. First, the CCP will always lower its profits, if, it engages in liquidation in order to offload a defaulter's positions; devising another method of unwinding is imperative. Second, the price impact of predation decreases the profits of all members and pushes banks to default; member's should be educated on acting in their own interest. Third, the CCP, in instituting a hybrid CCP structure, has a natural disciplinary mechanism for predation in its ability to make a margin call on predator profits in recovery. Depletion of predators' initial margins, to pay the debts of defaulting banks in the hybrid structure, means the CCP is legally entitled to make this call, and requires no extra regulatory intervention. Fourth, in expectation of a default cascade, one large enough to deplete the guarantee fund, the hybrid fund structure, rather than the current (pure) structure, is more incentive compatible for the CCP, as it is more likely that it can protect it's own equity.

I, then, use a dynamic, multi-period, adaptive simulation, calibrated to market data, to further investigate parameter sensitivities and regulatory implications . I find that losses are most sensitive to the number of distressed banks, rather than number of predators. Furthermore, predators' losses are maximised when there are a few competing predators along with few distressed banks; predators cannot make enough profit to overcome the effects of, both, price impact and inter-predator competition. There are also implications for interventions by a Lender-of-Last-Resort (LLR). In times of increasing financial turmoil, it is most advantageous to attempt to keep the level of distressed banks low through an injection of liquidity. However, in the case of a sudden, severe event which causes prolonged market illiquidity (dry-up/crisis), the LLR is most effective in injecting liquidity, either, at the very start or the very end of the crisis.

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7 Figures

List of Figures

Unless otherwise indicated, the number of distressed banks decreases as the predator number increases. This a constraint due to a finite number of banks in the market. As well, numerics of the order of 10^9 can be interpreted as in billions and 10^{12} as trillions. They are shown in scientific notation to underline the magnitude of losses which can be incurred.

7.1 Under normal market liquidity

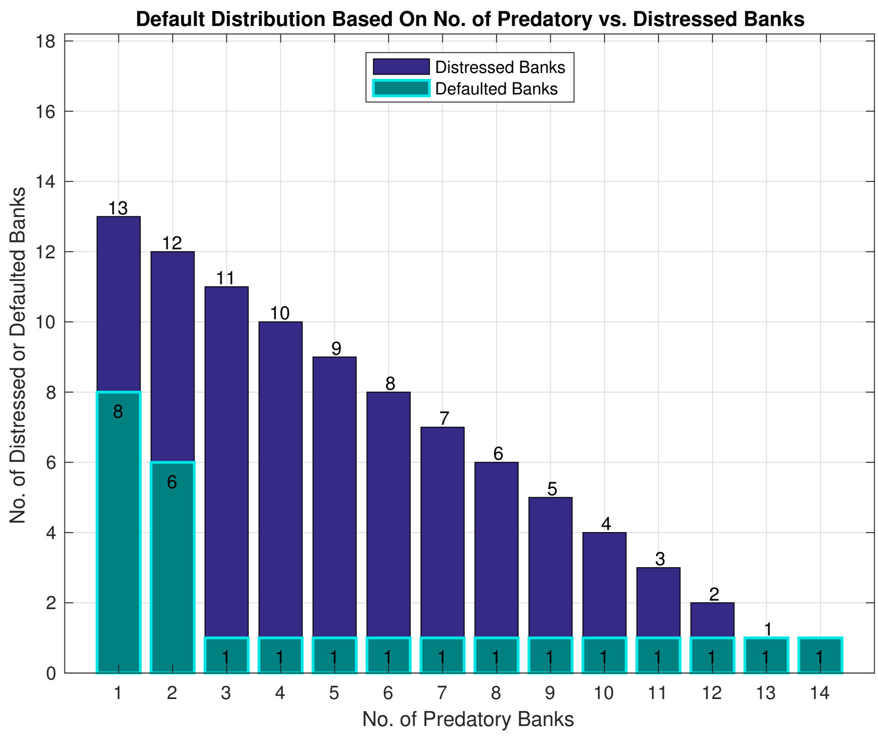

Figure 2: Number of distressed banks as drivers of defaults (no collusion, increasing predators).

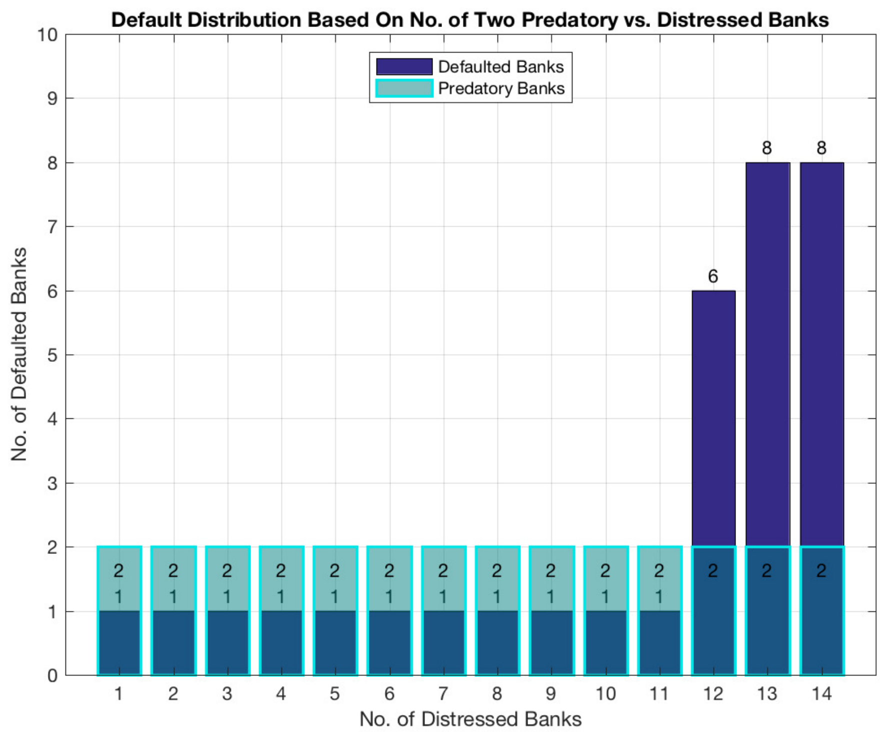

Figure 3: Higher defaults driven by increasing distressed banks vs. predator competition.

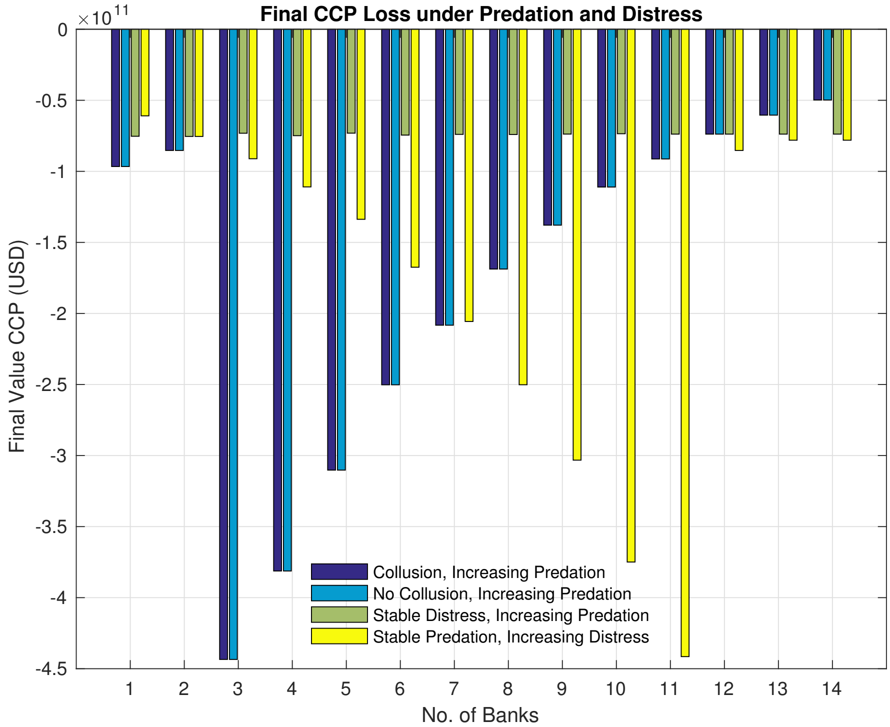

Figure 4: High number of distressed banks and a low number of predators as drivers of CCP losses.

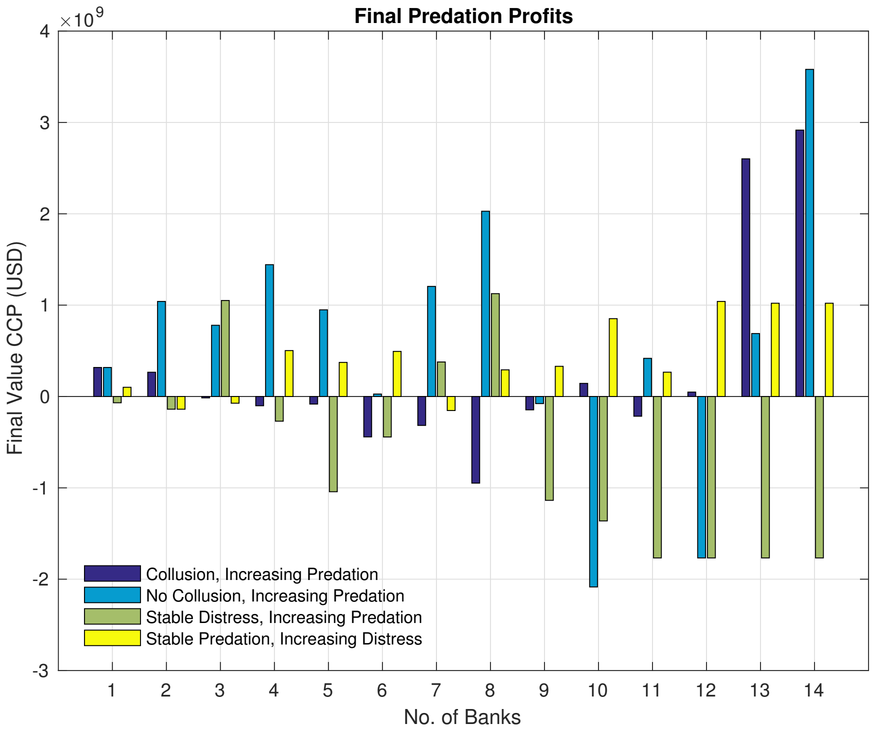

Figure 5: Illustration of the volatility of predators' profits.

7.2 Under decreasing market liquidity

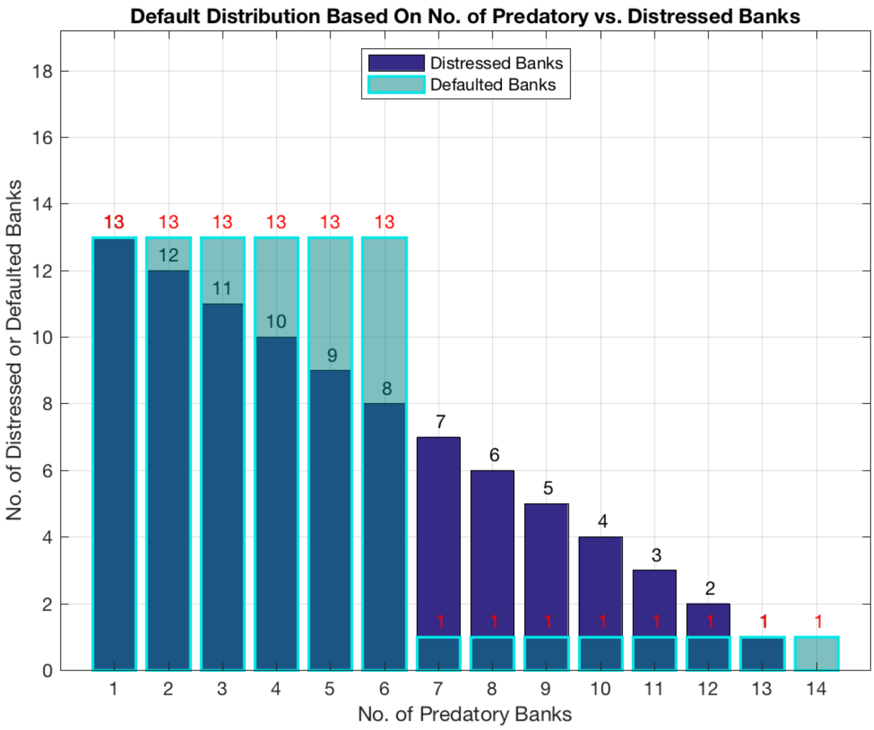

Figure 6: Effect of the increasing number of distressed banks on the increasing number of defaults.

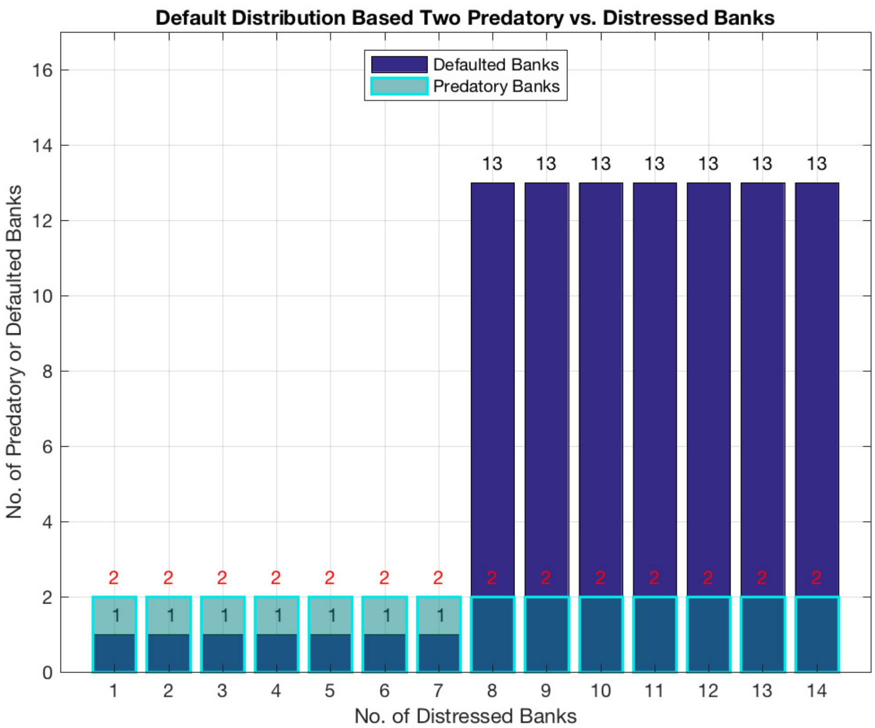

Figure 7: Illustration of lack of driving effect from the number of predatory bank on the number of bank defaults.

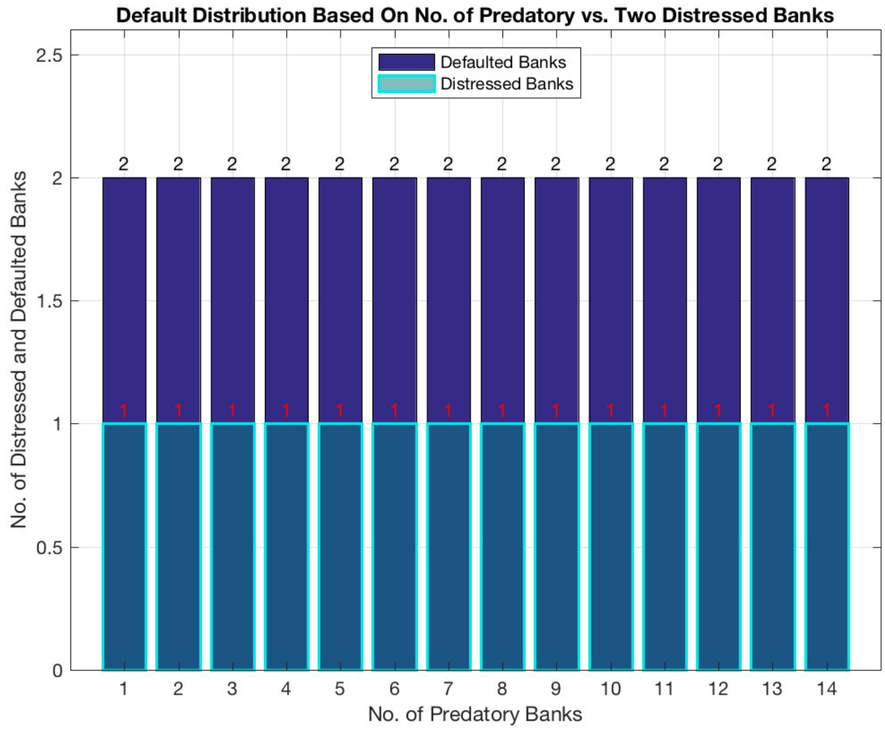

Figure 8: Illustration of effect of low number of distressed banks on the minimised number of defaults

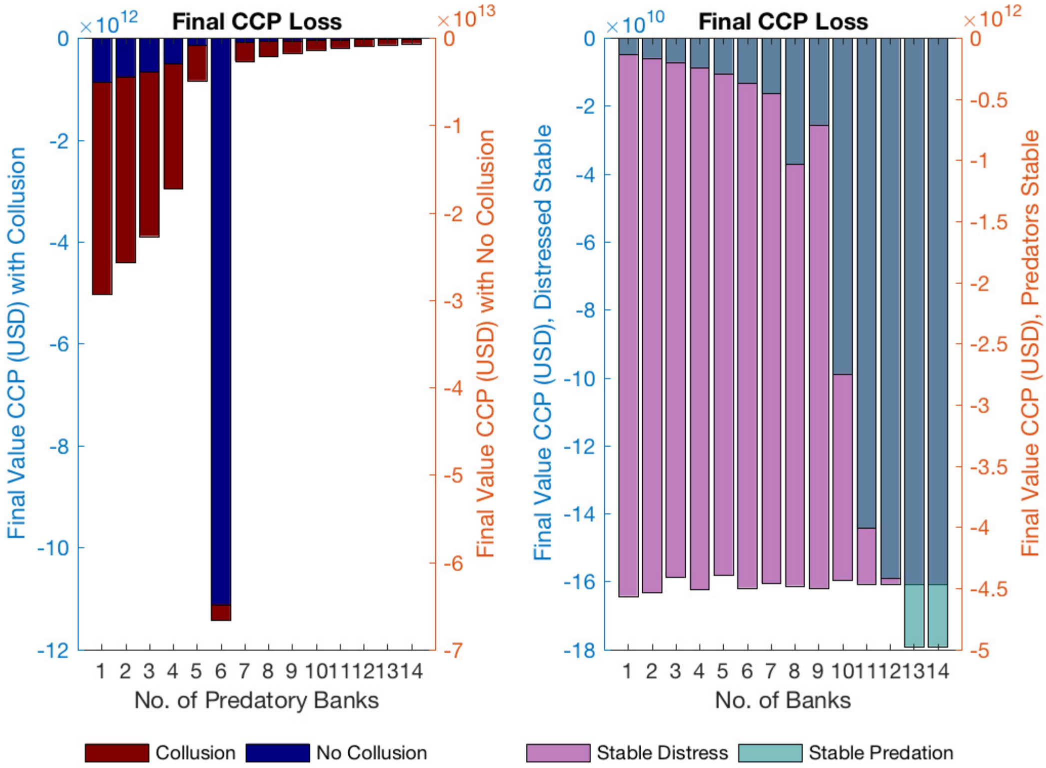

Figure 9: Minimisation effect of low levels of distressed banks on CCP losses.

7.3 Under financial crisis market liquidity

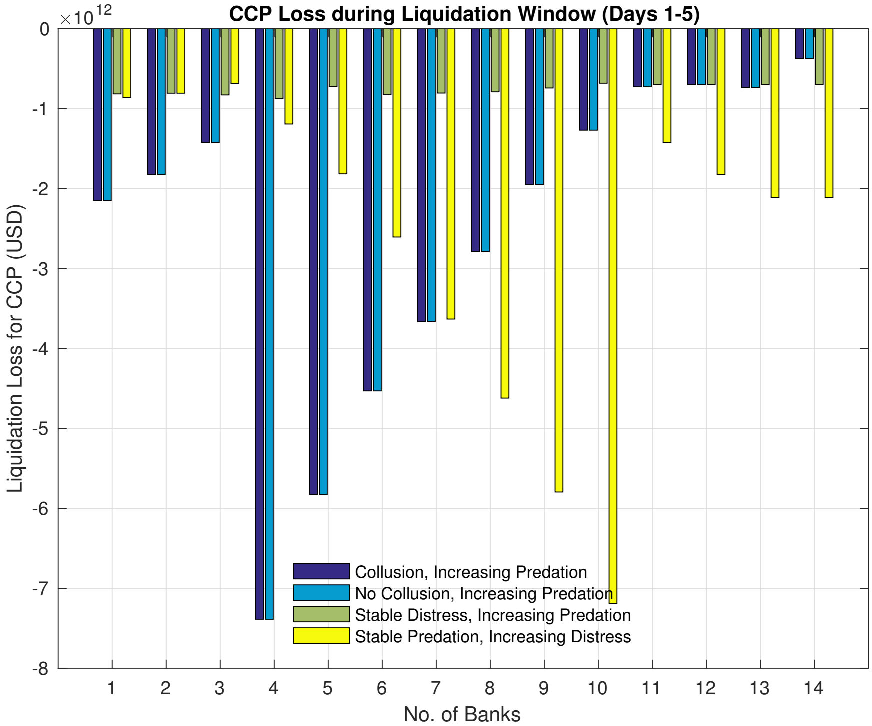

Figure 10: Levels of CCP loss at the end of liquidation window with failure of 13 of 14 banks.

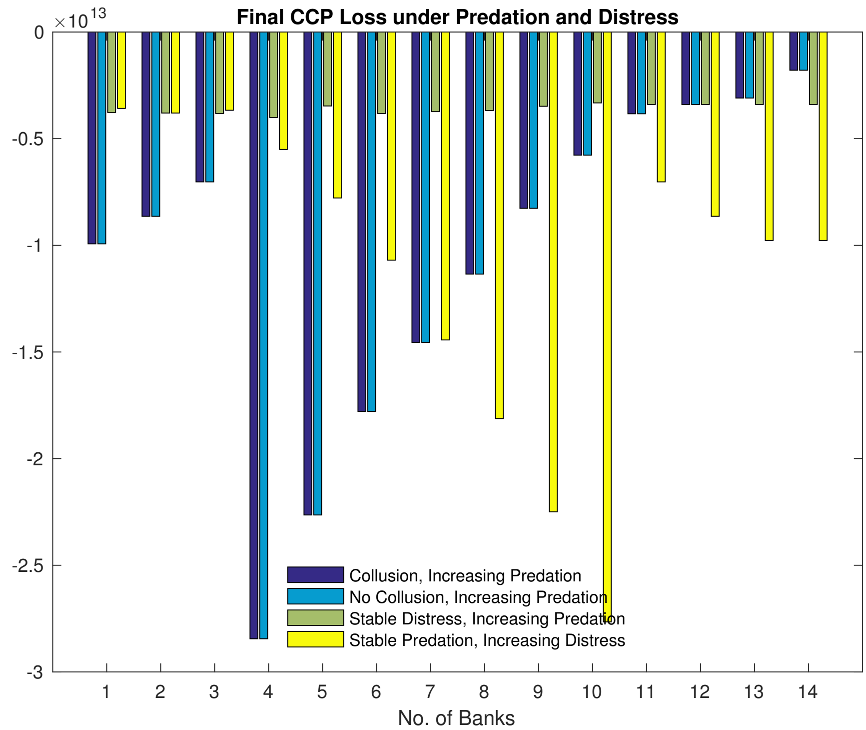

Figure 11: Uneven distribution of maximal final CCP Loss under low predation and high distressed banks.

*Note: no. of distressed banks is decreasing except in stable case.

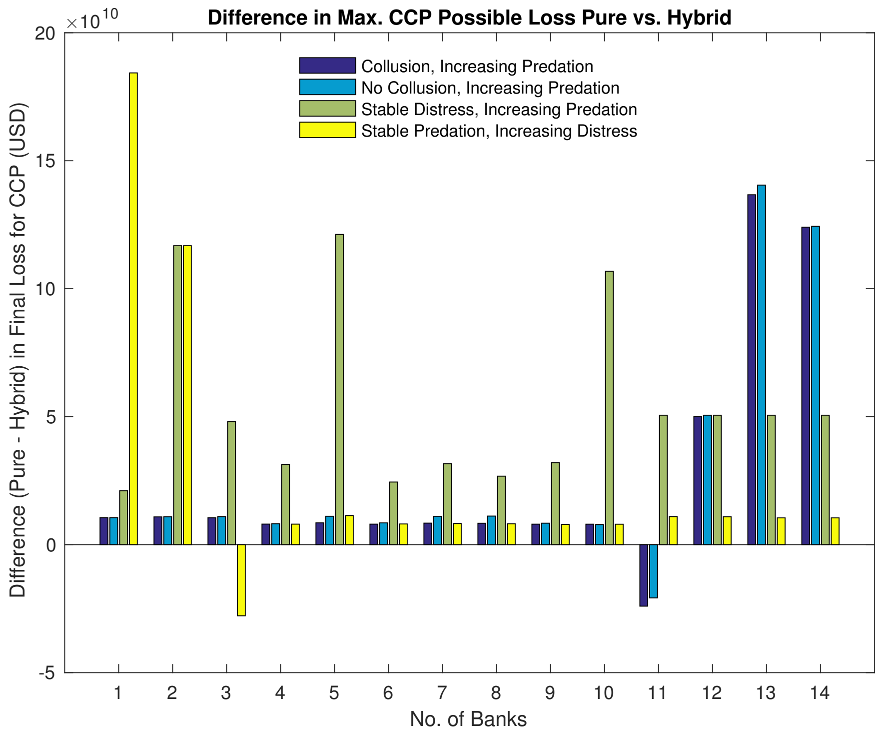

Figure 12: Illustration of the larger loss for CCP with the pure fund vs. hybrid fund.

7.4 Extended Investigation under decreasing market liquidity

Note that the no collusion case is reflective of all cases not plotted.

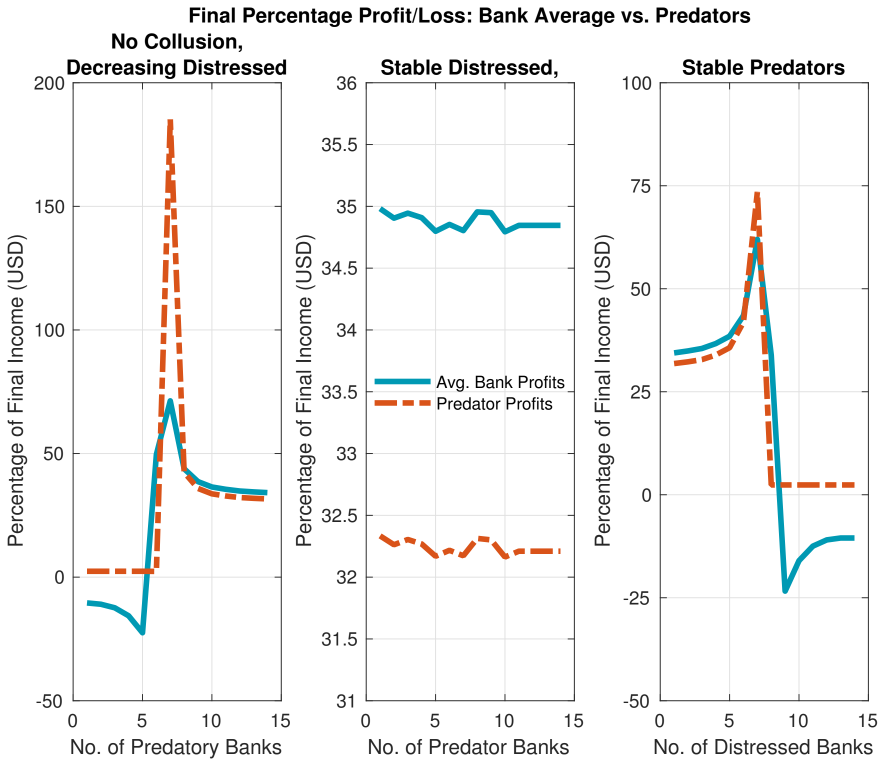

Figure 13: Potential avg. profit/loss on final income after recovery margin call by CCP for the total economy (defaulted, distressed, and predators) vs. predators alone.

*Note 1: Banks are composed of failed, distressed and predators (both positive and negative income)

*Note 2: Positive values are associated to percentage of income lost to initial margin.

*Note 3: Negative values are associated with negative income and illustrate the amount that predator margin refill cannot compensate for bank failure due to predatory behaviour.

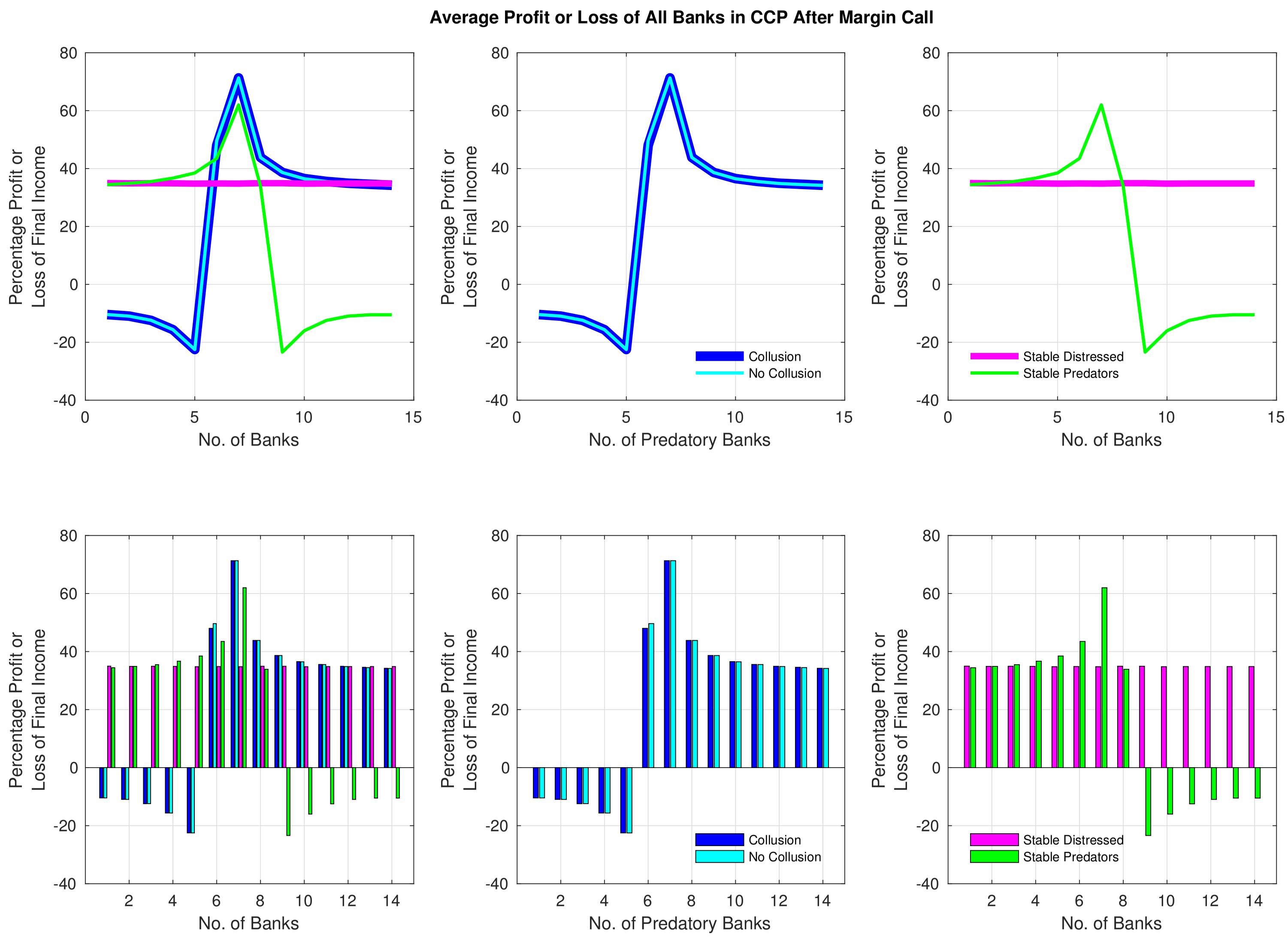

Figure 14: Illustration of the profit/loss on predators' final income after CCP's recovery margin call.

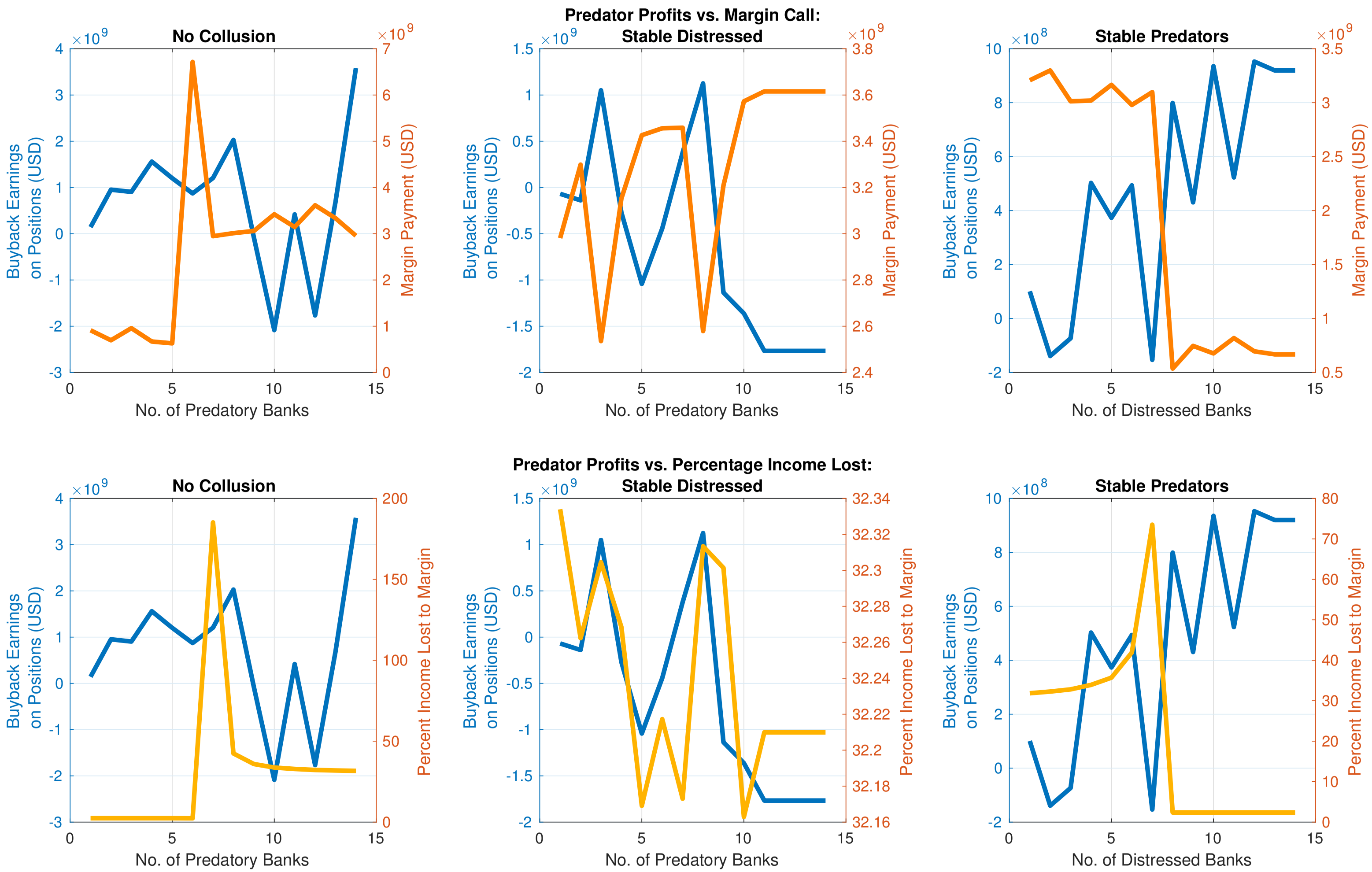

Figure 15: The effect of volatility from predator competition vs. price impact (increasing margin refill with decreasing predation).

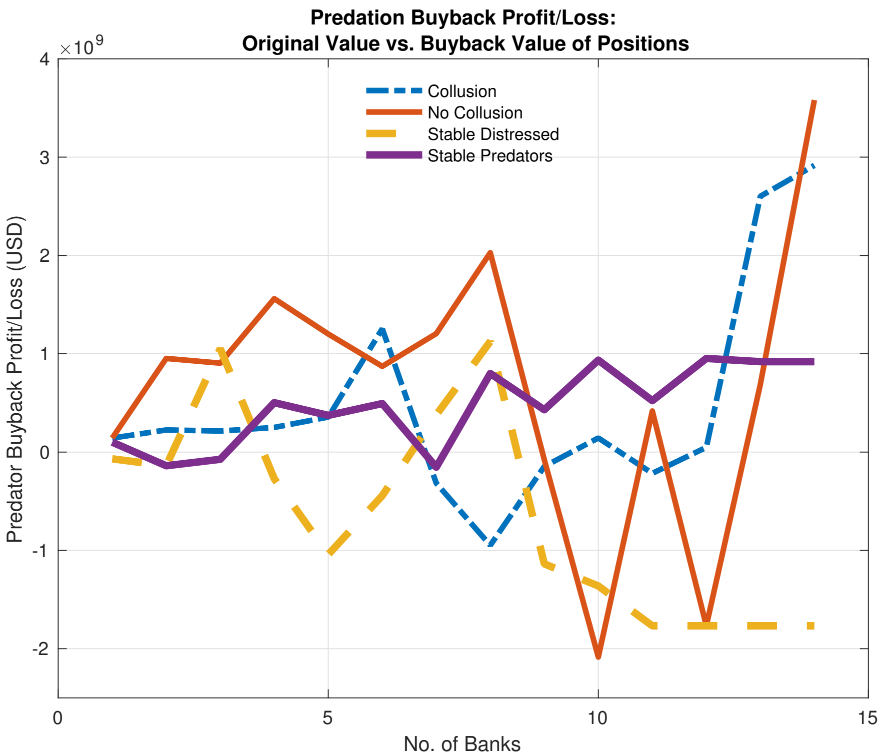

Figure 16: Effect of distressed bank number and competition on predators' profit/loss on buyback of original positions.

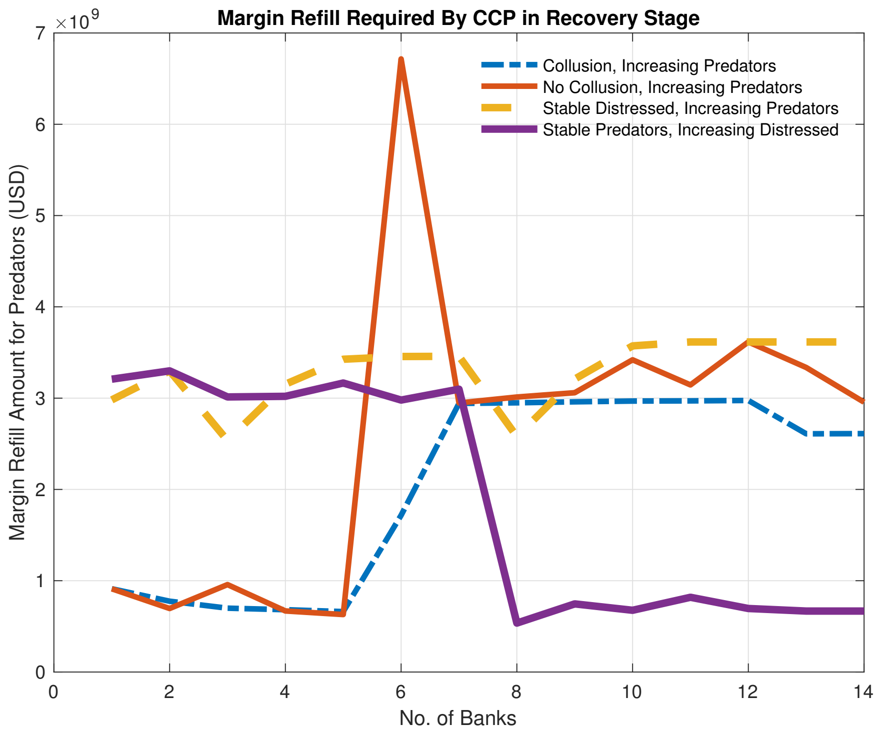

Figure 17: Illustration of increasing margin demand for predators with decreasing distressed banks

*Note: Margin can only be obtained from predators who survive, thus, low margin may mean few banks to pay it.

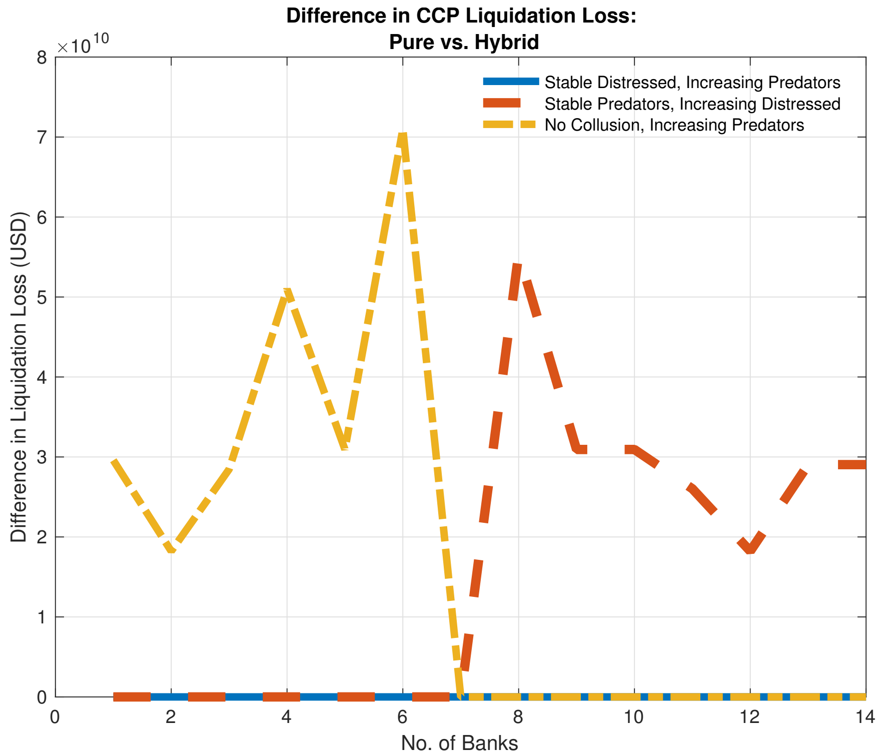

Figure 18: Illustration of lower CCP gain and higher loss in pure fund vs. hybrid fund

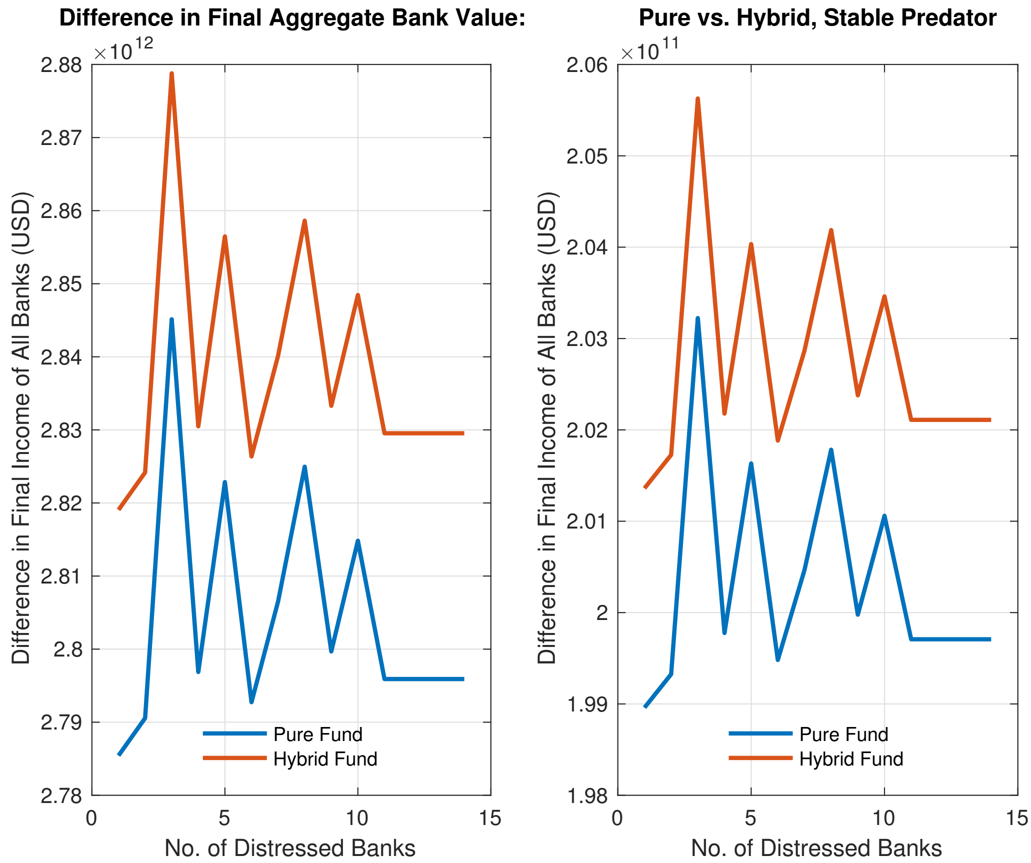

Figure 19: Illustration of the overall tendency toward higher final bank profit in hybrid fund

*Note: Group of banks contains both defaulting and profiting banks.

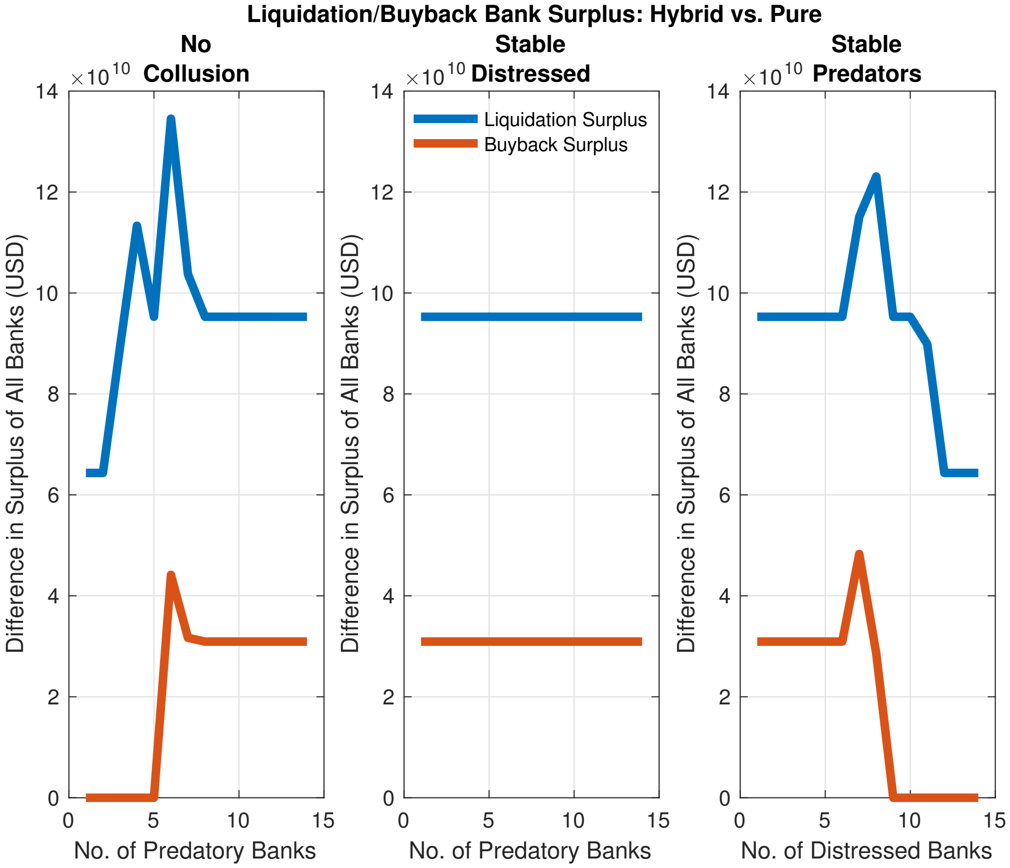

Figure 20: Illustration of, both, higher liquidation and buyback surplus for banks in hybrid fund

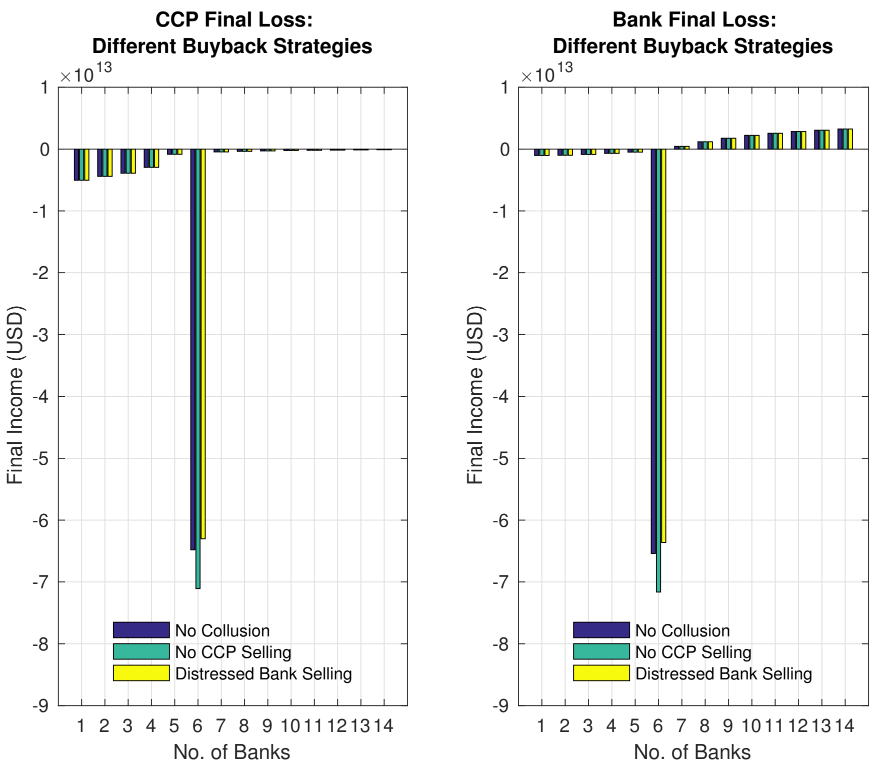

Figure 21: The effect of CCP's and distressed banks buyback selling on the CCP's loss.

*Note: The CCP case is reflective of the case with combine CCP and distressed bank selling effects.

8 Appendix

8.1 Appendix A: Background and assumptions in determining liquidation rate

In utilising and expanding [Brunnermeier and Pederson, 2005], the model has been adapted to this scenario, making some additional assumptions which yield novel and surprising implications. The assumptions will be outlined in this section.

The market is composed of I large strategic dealer banks, smaller banks and buy-side consumers which trade through the dealer banks. We take the transactions made by dealer banks on customers accounts, as non-strategic activities based on consumer demand. For trades on their own accounts, the dealer banks compose two groups; some of which may be distressed banks I^d, and others which may be predatory banks I^p. In the scenario, which has been have outlined in the paper, the CCP acts as a distressed bank, when a large dealer member defaults. When it begins to liquidate those defaulted assets, the member banks can be direct counterparties to those assets or not. A direct counterparty to a defaulted bank can also become distressed, and engage in forced selling, without the ability to buyback, entering the set I^d. If this distressed counterparty, itself, defaults it enters the set $\mathbb{D}$ and can no longer participate in trading. The CCP is then forced to additionally liquidate this party's assets. On the other hand, a large, healthy bank which is not a direct counterparty, but holds the defaulted asset, can predate and belongs to the set I^p. In trading strategically the predatory dealer bank seeks to profit by exploiting the trading behaviour of other banks since it is not powerful enough to engage in price manipulation.

This combined selling between the CCP, distressed traders and predatory banks determines the total market trading rate. The CCP attempts to minimise its price impact by liquidating at the average market rate. The rate of the CCP is determined by the size of positions sold and the number of sellers, according to,

$$a^{-}_{CCP} = \frac{A}{I} = \frac{\sum_{i=1}^{I} a^{\pm}_{ij}}{\sum_{i\in\mathbb{D}} I_d + \sum_{i\notin\mathbb{D}} I_p} \tag{48}$$

The largest market making dealers (the top 8-10 dealer banks) own close to 80% of the market for CDS, the other 20% is held by smaller banks. It is assumed that the smaller banks compose consumer demand, and they must take prices as given. They may or may not sell in the same direction, but are not big enough to make a large impact. Though as a group there impact is notable, they are especially at risk of becoming the initial group of distressed banks. It is assumed that total supply S is composed of these two groups. Thus, these largest strategic banks have a common holding according to,

$$\bar{X} = \frac{\mathcal{S} \times 0.80}{I}$$

Thus, the relation $S > \bar{X}I$ clearly shows that the largest strategic banks, even in colluding, cannot manipulate the price to reach its true expected value through their own trading (ie. strategic banks don't have infinite resources.)

It is assumed that CCP will liquidate its full holding of the defaulted assets during the liquidation period. In order to manage it's risk it liquidates at the average market rate, thus for a time,

$$\ell_T = \frac{X_{CCP}(t_0)}{A/I} = \frac{\sum_{j\in\mathbb{D}} X_{ji}(t_0)}{A/I}$$

where $X(t_0) = X_{ji}(t_0) \in [0, \bar{X}]$ is the initial holding of each strategic banks at the start of the liquidation period. Though, it is assumed that all dealer banks are homogenous in their ability to take on a *maximum* buy and sell position in an asset – due to CCP restrictions and general regulations on positions sizes and risk– this does not mean that that all banks must have homogeneous holdings in the asset, or similar endowments of wealth. This assumption is required for tractability at this point of the analysis, and can be relaxed at a later time. In fact, [Brunnermeier and Pederson, 2005] provides the result that the major commonality that arises due to this homogeneity is that the predatory[48] banks, in determining their trading rate, *attempt* to buy-up up to the maximal allowable holding $X_i(T\tau) = \bar{X}(T\tau)$. Though a bank may choose its trading rate in order to buy up the maximal allowable amount, it may not actually be successful in doing so. This is due to the opaqueness of market trading rates and the actually supply of and demand for the asset. Thus, at any time, the dealer bank is choosing his trading rate based on any observable trades he can see.

The predator achieves this holding by maximising his trading according to,

$$\min_{a_i()\mathcal{A}_i} \int_0^{T\tau} a_i(\ell\tau) \sum_{j\neq i} X^j(\ell\tau)$$
$$s.t. \quad X_i(T\tau) = X_i(0) + \int_0^{T\tau} a_i(\ell\tau)\, d\tau = \bar{X} \quad \text{if } \in I^p,$$

subject to upper and lower trading limits beyond which dealer bank will make a price impact. Thus, each trader/predator chooses his rate based on the liquidation time, his holding, other (visible) traders' trading and his goal of profit maximisation. An important note here is that, though no trader has an informational advantage, each trader has a different set of information based on who he interacts with and there particular trading behaviour.

Based on the expected liquidation time that the CCP sets out for its initial margin demand – for example, the common 5 days – the CCP will liquidate for,

$$T\tau = 5\tau = \frac{X_{CCP}(t_0)}{A/I} \quad \Rightarrow \quad A = \frac{X_{CCP}(t_0)}{T\tau/I} = \frac{X_{CCP}(t_0)}{5/I} \quad \text{with } \tau = 1 \tag{49}$$

Thus, one can determine the average trading rate of the market only knowing the number of strategic banks and the liquidation time. The CCP, by clearing all payments, has a snapshot of the full market at any time, and so can accurately determine the average trading rate of the market.

Given that the market is opaque, a search-for-quote market, each strategic bank can at most see the actions of the CCP and it's direct counterparty (1 other bank), at any moment in time[49]. Thus, each bank $i$nfers that the trading rate of the CCP is the trading rate of the market; the predatory banks know that the CCP knows the trading rate of all participants and is seeking to minimise risk. This assumption is still valid even if the bank doesn't believe the liquidation rate of the CCP accurately reflects the market, it has no choice but to use its only example of market trading rate as the upper limit for its own trading rate. Thus, with one trader the trader chooses to liquidate at the same time as the CCP for the full liquidation period, at the fastest rate that he can, front-running the CCP. With multiple predators, they all choose to liquidate at the same constant rate as the CCP (or other distressed traders), given the *assumed* trading rate of others and their *assumed* initial position $X(t_0)$, for the time period given by,

[48]The assignment of predator implies that the bank is healthy enough to take on the maximum position.

[49]We assume that the bank can see the CCP at most times, because the behaviour of the CCP is probably subject to more disclosure, regulation and media attention. Thus, its trading cannot be kept truly secret at all times. It is this fact that makes it so sensitive to predation.

$$\sum_{i=1}^{I-1} a_i \tau = \sum_{i=1}^{I-1} X^i(t_0) - \sum_{i=1}^{I^p-1} \bar{X}^i$$

$$(I-1)\underbrace{\frac{A}{I}}_{a^{CCP}}\tau = (I-1)X(t_0) - (I^p-1)\bar{X}$$

$$\tau = \frac{X(t_0) - \frac{(I^p-1)}{I-1}\bar{X}}{A/I}$$

However, as outlined above, they buy-back with a different trading intensity, $\frac{AI^d}{I(I^p-1)}$, until the liquidation period is finished.

It is very important to understand that the price/fundamental CDS value is generated only by fundamental information. This is the only information visible to the dealer banks. Thus, there is no valuable, extra information generated by the price process because the true price (fundamental plus excess demand) is not visible to market participants in an opaque market. Thus, the predators cannot explain the part of the price that comes from their own predation, making the price process very noisy. Furthermore, traders can't see the true trading rate or trade size of all other participants, thus, they assess the market largely based on their own actions and that of the CCP. With the limited information, the noisy, and not full observable price process is noisy, in this model[50], even the predator may eventually become prey!

An important distinction of this model from that of [Brunnermeier and Pederson, 2005] is the fact that in their model, optimisation of the trader's objective function leading to an optimal trading rate transparently feeds into a price process fully visible by all participants. This is because the price process is driven only by outside demand and the change in asset supply in the market. In this model, the price process is not fully observable and so the trader doesn't see why the price is changing. Thus, in our model we have internalised the all demand into the price, the outside demand and the excess demand. The aggregate demand of non-strategic investors, price-takers is[51],

$$Y(\triangle S^k(t_{(\ell+1)\tau})) = D_k(v - \triangle S^k(t_{\ell\tau})) \quad \text{with expected value of asset} \quad \mathbb{E}[\mu] = v$$

Then if supply is determined by price-takers and large dealer banks,

$$Y + \sum_i^I X_i = \mathcal{S}$$

,
we have that price, obeying market clearing, is seen by the traders to be.

$$\triangle S^k(t_{\ell\tau}) = \underbrace{v}_{P_0} - \underbrace{\frac{1}{D_k}(\mathcal{S}^k(t_{(\ell-1)\tau}) - \sum_i^I X_i^k(t_{(\ell-1)\tau}))}_{P_1,\, \mathcal{P},\, P_2,\, P_3}$$

,
which qualifies the use of the pricing functional we have given in the paper.

[50] In [Brunnermeier and Pederson, 2005] they address the possibility that predators can become prey in an extension of the model where the price process is noisy.

[51] The parameter D_k can be altered in the model to look at different aspects of the market. Looking at D_k one is looking at price impact with an increase in this parameter showing rising illiquidity in the market. However, with $1/D_k$ one is looking at market depth, where an increase in this parameter signals more liquidity. Choosing constant market liquidity allows either formulation.

8.2 Appendix B: Illustrations of covariance relationship for pricing function in section 2.3.2

If one looks at formula 12 each part of the formula can be explained explicitly. It is important to remember that one is looking for the effect of price impact on the portfolio of bank i. The fundamental cds-spread is composed of the position size multiplied by the current cds-spread change. Since the position and the change may take on signs in either direction, for liabilities to properly occur if the multiplied amount if positive or zero.

$$\sum_k X_{ij}^k(\ell\tau)\,\triangle S^k(\ell\tau) \;=\; \sum_k \Big(X_{ij}^k((\ell-1)\tau) \;+\; a_{ji}^k\tau\Big)\,\triangle S^k(\ell\tau)$$

$$= \sum_k \underbrace{\Big\{ [\, X_{ij}^k((\ell-1)\tau)\triangle S^k((\ell-1)\tau)\,]^+}_{fundamental\ cds-spread}$$

The second line entails CCP liquidation. The first part of the formula addresses the effect on bank i's holdings directly held with a defaulted counterparty. In addition there is the effect on bank i's holdings on defaulted assets held with non defaulted counterparties, these are both affected by the CCP liquidation of defaulted asset k with any defaulted counterparty, held or not held by bank i. Should the defaulted the CCP not liquidate this asset, the liquidation term a^k goes is zero and the whole line goes to zero.

$$\underbrace{\left(\sum_{j\in\mathcal{D}}\left|\frac{X_{ij}^k}{X_{ij}^k}\right| X_{ij}^k + \varepsilon\sum_{j'\notin\mathcal{D}}\left|\frac{X_{ij'}^k}{X_{ij'}^k}\right| X_{ij'}^k\right)\sum_{i'=1}^m |\triangle S^k((\ell-1)\tau)|\left(\frac{X_{ji'}^k}{D_k}\right)\left(\frac{a_{ji'}^k\tau}{X_{ji'}^k}\right)}_{CCP\ liquidation}$$

In the third line we address the effect of bank i's own selling of the defaulted asset on its portfolio. It's distressed selling of the defaulted asset which it holds with any bank j, captured by $\sum_{j=1}^m a_{ij}^k$ means that it will affect both the distressed assets it hold directly with a defaulted counterparty and the distressed assets it holds with safe banks. The epsilon captures the reduced potency of the effect, as negative externalities associated with holding the defaulted asset directly with the defaulted counteparty, are avoided. Attention must be paid to the reinitialisation of the sum for j in the middle of the equation; this is done in order to keep notation tractable. Note that should bank i not liquidate a holding, the whole term will go to zero as $a_{ij}^k = 0$. Note also, that if bank i is holding the asset with an undefaulted counterparty, then this is just bank i's own predation behaviour affecting the cds-spread and the value of it's portfolio. Furthermore, if this is the liquidation of an *undefaulted* asset with an undefaulted counterparty which is being sold in large amounts by bank i for any reason, then this simply accounts any cds-spread changes arising from the changing liquidity of an asset. Thus, the model can account for any reason that i's portfolio value is affected; distressed selling, predatory selling, and a large block trade.

$$\Bigg(\underbrace{\sum_{j\in\mathcal{D}}\left|\frac{X_{ij}^k}{X_{ij}^k}\right| X_{ij}^k}_{direct\ distressed\ selling} + \varepsilon\underbrace{\sum_{j'\notin\mathcal{D}}\left|\frac{X_{ij'}^k}{X_{ij'}^k}\right| X_{ij'}^k}_{distress/predation}\Bigg)\sum_{j=1}^m |\triangle S^k((\ell-1)\tau)|\left(\frac{X_{ij}^k}{D_k}\right)\left(\frac{a_{ij}^k\tau}{X_{ij}^k}\right)$$

The third term is the predatory term, which addresses the effect on bank i's portfolio, of predatory liquidations by other banks who hold the defaulted asset, but with an undefaulted counterparty. In the first part of the term, the model addresses all holdings of the asset k, which bank i has with any bank j'. Then the last term encapsulates all liquidations of the defaulted asset k by any bank j' (that is not i) who holds the asset with a undefaulted counterparty. Note that this could include bank i. This effect is dampened by ϵ from the primary price impact, since the liquidating party is, themselves, undefaulted, and shields bank i from the negative externalities mentioned above.

$$\underbrace{\varepsilon \sum_{j'=1}^{m} \left| \frac{X_{ij'}^{k}}{X_{ij'}^{k}} \right| X_{ij'}^{k} \sum_{j' \notin \mathcal{D}} \sum_{i'=1}^{m} |\triangle S^{k}((\ell-1)\tau)| \left(\frac{X_{j'i'}^{k}}{D_k} \right) \left(\frac{a_{j'i'}^{k}\tau}{X_{j'i'}^{k}} \right)}_{predation} \tag{50}$$

The fourth term is the secondary price impact on bank i's portfolio. This is the impact that bank i's other portfolio assets feel from the defaulted asset, or rather, the liquidation of the defaulted asset by any of it's counterparties, defaulted or undefaulted. The first, bracketed part of the equation ensures that i is hold the asset k with j. The subsequent summation ensures that bank i is also holding asset k' with the same bank or another bank j. Thus the last part captures all liquidations of all assets in i's portfolio that are for assets other than k. Note, that because the second term, outside of the brackets, runs over all j, it encapsulates all the counterparties in bank i's portfolio. As well, because the terms run over all counterparties i, this accounts for liquidations of assets with bank i as well. One can see here, that the effect of bank i's own liquidations, of it's side of the other assets k', on its own asset k, is taken into account here. This is because bank i is represented in the sum over m banks in both j and i. Note that terms with j=i' will be as a bank cannot hold assets with itself.

$$\underbrace{\begin{aligned} &\left(\frac{1}{2!}\right)\left(\left(\frac{3}{2!}\right)\sum_{j\in\mathcal{D}} \left|\frac{X_{ij}^{k}}{X_{ij}^{k}}\right| X_{ij}^{k} + \sum_{j'\notin\mathcal{D}} \left|\frac{X_{ij'}^{k}}{X_{ij'}^{k}}\right| X_{ij'}^{k}\right) \\ &\qquad \sum_{k'}\sum_{j=1}^{m} \left|\frac{X_{ij}^{k'}}{X_{ij}^{k'}}\right| \sum_{i'=1}^{m} |\triangle S^{k'}((\ell-2)\tau)| \left(\frac{X_{ji'}^{k'}}{D_{k'}}\right)\left(\frac{a_{ji'}^{k'}\tau}{X_{ji'}^{k'}}\right) \end{aligned}}_{secondary\ price\ impact} \tag{51}$$

The fifth term is the tertiary price impact on bank i's portfolio. This is the impact on the assets in bank i's portfolio's from assets which it doesn't directly hold, but which it's counterparties hold and have liquidated. The first bracketed part ensures that bank i is holding an asset with each counterparty; essentially listing it's counterparties for asset k. The second part in the absolute brackets, ensures that bank i doesn't also hold the other asset , which is being liquidated by it's counterparty j, with that counterparty. The summation over i' ensures that all liquidations by j with all it's other counterparties are taken into account. Naturally since I doesn't hold asset k", the liquidation rate for bank j with i would be zero.

$$\underbrace{\left(\frac{1}{3!}\right)\left(\left(\frac{9}{3!}\right)\sum_{j\in\mathcal{D}} X_{ij}^{k} \sum_{k''=1} \left|1-\frac{X_{ij}^{k''}}{X_{ij}^{k''}}\right| + \sum_{j\notin\mathcal{D}} X_{ij}^{k} \sum_{k''=1} \left|1-\frac{X_{ij}^{k''}}{X_{ij}^{k''}}\right|\right) \sum_{i'=1}^{m} |\triangle S^{k''}((\ell-2)\tau)| \left(\frac{X_{ji'}^{k''}}{D_{k''}}\right)\left(\frac{a_{ji'}^{k''}\tau}{X_{ji'}^{k''}}\right)}_{\textit{tertiary price impact}} \Bigg\} \tag{52}$$

Finally to clarify the terminating term in each line, $\frac{a_{ji}^{k}\tau}{X_{ji}^{k}}$, is the Γ term which we call the liquidation rate. This form is chosen in order to differentiate the effect of price impact $\frac{X_{ji}^{k}}{D_{k}}$ from the augmentation of the effect caused by liquidating at a chosen rate a_{ji}^{k}. This allows one to tease apart and evaluate the interaction of the two different factors. Note that the addition of τ to the rate a^{k} and dividing by $\left|X_{ji}^{k}/X_{ji}^{k}\right|$ allows one to obtain real number, representing the amount liquidated.

8.3 Appendix C: Proofs and Technical Details

Proof 1: Nominal Position Side From bank i's point of view:

$$X_{ij}^{B} = +X_{ij} \quad \text{and} \quad X_{ij}^{B} = +X_{ij} = X_{ji}^{S} \quad \text{with} \quad X^{B} > 0.$$

However, from bank j's point of view the same interaction appears as,

$$X_{ij}^{S} = -X_{ij} \quad \text{and} \quad X_{ij}^{S} = -X_{ij} = -(-X_{ji}) = X_{ji}^{B} \quad \text{with} \quad X^{S} < 0.$$

Considering that i and j hold different positions of the contract, the position between bank i and bank j for CDS k at any time is given by,

$$\Lambda_{i}^{k,B}(\ell\tau) = L_{ji}^{k,S}(\ell\tau) - L_{ij}^{k,B}(\ell\tau) \tag{53}$$

The liabilities under consideration are of the form:

$$\begin{aligned}
\mathbf{L_{ij}^{k}}(\ell\tau) &= \mathbf{X_{ij}^{k}}(\ell\tau)\triangle\mathbf{S^{k}}(\ell\tau) \\
&= \Big(\mathbf{X_{ij}^{k}}((\ell-1)\tau)\,\triangle\mathbf{S^{k}}((\ell-1)\tau)\Big)^{+} \\
&\quad + \ (\underbrace{\mathbf{X_{ij}^{k}}((\ell-1)\tau)}_{B/S}/D_{k})\,|\triangle\mathbf{S^{k}}((\ell-1)\tau)|\,(\underbrace{\mathbf{a_{ij}^{k}}}_{+/-}\tau\,/\mathbf{X_{ji}^{k}}) + ...
\end{aligned}$$

The bank i has liabilities with other banks j of the form[52],

$$\mathbf{L_{ij}^{k,S}}(\ell\tau) = \left(\mathbf{X_{ij}^{k,S}}_{(\ell-1)\tau}\triangle\mathbf{S^{k,S}}_{(\ell-1)\tau}\right)^{+} + \mathbf{P^{S}_{1,(\ell-1)\tau}}\ a_{ji}^{k}\tau + \mathcal{P}^{\mathbf{S}}_{(\boldsymbol{\ell}-\mathbf{1})\tau}\ a_{ji}^{k}\tau + \mathbf{P^{S}_{2,(\ell-2)\tau}}\ a_{ji}^{k}\tau + \mathbf{P^{S}_{3,(\ell-2)\tau}}\ a_{ji}^{k}\tau$$

Concentrating on only two banks, for simplicity, we can illustrate the receivable to bank i from bank

[52]Note, there is no term for bank i's own liquidation; the bank does not consider it's own price impact when it liquidates.

j. If we are looking from the point of view of the bank with a sell position, it's receivable is,

$$\mathbf{L_{ji}^{k,S}}(\ell\tau) = \big(\mathbf{X_{ji}^{k,S}}_{(\ell-1)\tau} \triangle\mathbf{S^{k,S}}_{(\ell-1)\tau}\big)^{+} + \mathbf{P^{S}_{1,(\ell-1)\tau}}\, a_{ij}^{k}\tau + \mathcal{P}^{\mathbf{S}}_{(\ell-\mathbf{1})\tau}\, a_{ij}^{k}\tau + \mathbf{P^{S}_{2,(\ell-2)\tau}}\, a_{ij}^{k}\tau + \mathbf{P^{S}_{3,(\ell-2)\tau}}\, a_{ij}^{k}\tau$$

From the point of view of the other bank, this will be a liability on a buy position, in asset k, with first bank (where it becomes bank i)[53].

Proof 2: CDS Pricing Functional and Proposition 1

$$\sum_k \triangle S^k(\ell\tau) = \sum_k \Bigg\{ [\,\triangle S^k((\ell-1)\tau)\,]^{+} + \sum_{j\in\mathcal{D}} \sum_{i'=1}^{m} |\triangle S^k((\ell-1)\tau)| \left(\frac{a_{ji'}^{k}\tau}{D_k}\right)$$
$$+ \sum_{j\in\mathcal{D}} \sum_{i'=1}^{m} |\triangle S^k((\ell-1)\tau)| \left(\frac{a_{i'j}^{k}\tau}{D_k}\right) + \varepsilon \sum_{j'\notin\mathcal{D}} \sum_{i'=1}^{m} |\triangle S^k((\ell-1)\tau)| \left(\frac{a_{j'i'}^{k}\tau}{D_k}\right)$$
$$+ \left(\frac{1}{2!}\right) \sum_{k'} \sum_{j=1}^{m} \sum_{i'=1}^{m} |\triangle S^{k'}((\ell-2)\tau)| \left(\frac{a_{ji'}^{k'}\tau}{D_{k'}}\right) + \left(\frac{1}{3!}\right) \sum_{j\in\mathcal{D}} \sum_{k''} \sum_{i'=1}^{m} |\triangle S^{k''}((\ell-2)\tau)| \left(\frac{a_{ji'}^{k''}\tau}{D_{k''}}\right) \Bigg\} \tag{54}$$

Proof 3: Cumulative Effect of Price and Proposition 3 At the first stage, the holdings are only exposed to changes in the holding, and a cds-spread shift. The net exposure can be written in terms of a function which is affected by the variables of interest,

$$\Lambda_i^{k,S}(1\tau) = \sum_{j=1}^{m} \mathcal{F}\Big(\mathbf{1}\tau, \mathbf{X}_i^{k,S}(1\tau), \triangle\mathbf{S}^{k,S}(1\tau,\, X_i^{k,S}(0\tau),\, \triangle S^{k,S}(0\tau))\Big)$$

The above shows the cumulative effects of variables propagating through the time-steps.

There is a progression throughout the period, the next time-step introduces the primary price impact of the liquidation, which occurred in the previous period, as the market absorbs this information. There is also the impact of the previous period's predation, as member banks choose to sell in the same direction. The predation can only happen a time-step after the first attempt at liquidation, since this is an announcement/signal of distress by the CCP.

$$\Lambda_i^{k,S}(2\tau) = \sum_{j=1}^{m} \mathcal{F}\Big(\mathbf{2}\tau, \mathbf{X}_i^{k,S}(2\tau, a_{ji}^{k,\pm}(1\ell)),\ \triangle\mathbf{S}^{k,S}(2\tau,\, X_i^{k,S}(1\tau),\, \triangle S^{k,S}(1\tau),\, P_1(1\tau),\, \mathcal{P}(1\tau), a_{ji}^{k,\pm}(1\ell))\Big)$$

It is clear that the current periods value now depends on both the impacts of the previous periods actions.

[53]This will appear as,

$$\mathbf{L_{ij}^{k,B}}(\ell\tau) = \big(\mathbf{X_{ij}^{k,B}}_{(\ell-1)\tau} \triangle\mathbf{S^{k,B}}_{(\ell-1)\tau}\big)^{+} + \mathbf{P^{B}_{1,(\ell-1)\tau}}\, a_{ji}^{k}\tau + \mathcal{P}^{\mathbf{B}}_{(\ell-\mathbf{1})\tau}\, a_{ji}^{k}\tau + \mathbf{P^{B}_{2,(\ell-2)\tau}}\, a_{ji}^{k}\tau + \mathbf{P^{B}_{3,(\ell-2)\tau}}\, a_{ji}^{k}\tau$$

At $t_{3\tau} = 1$, along with the primary price and predation impacts from the previous period, there is the incorporation of the secondary and tertiary price impacts, which have a two period lag in terms of price.

$$\Lambda_i^{k,S}(3\tau) = \sum_{j=1}^{m} \mathcal{F}\Bigg(\mathbf{3}\tau, \mathbf{X}_i^{k,S}\big(3\tau, a_{ji}^{k,\pm}(2\ell)\big),\ \triangle\mathbf{S}^{k,S}\big(3\tau,\ X_i^{k,S}(2\tau),\ \triangle S^{k,S}(2\tau),\ P_1(2\tau),\ \mathcal{P}(2\tau),\ P_2(1\tau),\ P_3(1\tau),\ a_{ji}^{k,\pm}(2\ell)\big)\Bigg)$$

These are incorporated late, as the market is opaque, and these impacts arise due to weaker ties, and longer distance relationships in the financial network. Note that in the next period, the primary price impact and the predation of this current time-step, are also affected by the previous time-steps effects, through the change in price (technically change in cds-spread). So that, the change in price and the predation impacts, are cumulative effects of the previous periods impacts. This makes economic sense as banks make their own liquidation/predation decisions based on the behaviour they see in the market and the outcomes of those previous actions.

Proof 4: CCP shortfall from defaults and Lemma 1 The CCP carries any shortfall, from defaults, into the next time-step of the trading period, where it will attempt to liquidate the position. It starts the next time-step with a hang-over in the previous time-steps liabilities,

$$\begin{aligned} L_0\big((\ell+1)\tau\big) &= L_0^{1-\mathbb{D}}\big((\ell+1)\tau\big) + L_0^{\mathbb{D}}\big(\ell\tau\big) \\ &= (1-f)\sum_{i=1}^{m} \Lambda_i^{+}\big((\ell+1)\tau\big) + \underbrace{\big(A_0(\ell\tau) - L_0(\ell\tau)\big)^{-}}_{C_0(\ell\tau)^{-}} \end{aligned} \tag{55}$$

Since at each time-step, there may be more and more defaults this effect is cumulative,

$$\begin{aligned} L_0\big((\ell+2)\tau\big) &= L_0^{1-\mathbb{D}}\big((\ell+2)\tau\big) + L_0^{\mathbb{D}}\big((\ell+1)\tau, L_0^{\mathbb{D}}\ell\tau)\big) \\ &= (1-f)\sum_{i=1}^{m} \Lambda_i^{+}\big((\ell+2)\tau\big) + C_0^{-}\left((\ell+1)\tau), C_0^{-}(\ell\tau)\right) \end{aligned}$$

Proof 5: Identities from section 3.2.5 with for both pure and hybrid fund

The terminal net worth of the CCP is,

$$C_0(t_{T\tau}=2) = (1-\epsilon)\Big(\gamma_0 + f\ \sum_{i=1}^{m}\ \Lambda_i^{+}\Big) + \begin{cases} \epsilon\ \left(\gamma_0 + f\ \sum_{i=1}^{m}\ \Lambda_i^{+}\right) - \sum_{i=1}^{m}\left(\bar{G}_i^{\star} + \bar{D}_i^{\star} + \bar{C}_i^{-}\right)^{-} & \text{(Pure)} \\ \\ \epsilon\ \left(\gamma_0 + f\ \sum_{i=1}^{m}\ \Lambda_i^{+}\right) - \sum_{i=1}^{m}\left(\hat{G}_i^{\star} + \hat{D}_i^{\star} + \hat{C}_i^{-}\right)^{-} & \text{(Hybrid)} \end{cases} \tag{56}$$

The terminal net worth of bank i is,

$$C_i(t_{T\tau}=2) = (\gamma_i + Q_i + \Lambda_i) - \begin{cases} \left(\Pi_{0i}\bar{C}_0^- + \bar{Z}_i(Q_i - R_i) + f\Lambda_i^+\right) + \left[\frac{\bar{G}_i}{\bar{G}_{tot}}(\bar{G}_{tot} - \bar{G}^{\star}_{tot}) + \frac{\bar{D}_i}{\bar{D}_{tot}}(\bar{D}_{tot} - \bar{D}^{\star}_{tot})\right] \\ \\ \left(\Pi_{0i}\hat{C}_0^- + \hat{Z}_i(Q_i - R_i) + f\Lambda_i^+\right) + \left[\frac{\hat{G}_i}{\hat{G}_{tot}}(\hat{G}_{tot} - \hat{G}^{\star}_{tot} + \frac{\hat{D}_i}{\hat{D}_{tot}}(\hat{D}_{tot} - \hat{D}^{\star}_{tot})\right] \end{cases} \tag{57}$$

The net worth of the CCP in period $t_{\ell\tau} = 3$,

$$C_0(t_{T\tau}=3) = \left(\gamma_0 + f\sum_{i=1}^{m}\Lambda_i^+\right) - \begin{cases} \left(\bar{G}^{\star}_{tot} + \bar{D}^{\star}_{tot} + \sum_{i=1}^{m}\bar{C}_i^-\right)^- & \text{(Pure)} \\ \\ \left(\hat{G}^{\star}_{tot} + \hat{D}^{\star}_{tot} + \sum_{i=1}^{m}\hat{C}_i^-\right)^- & \text{(Hybrid)} \end{cases} \tag{58}$$

The net worth of bank i is,

$$C_i(t_{T\tau}=3) \tag{59}$$

$$= (\gamma_i + Q_i + \Lambda_i) - \begin{cases} \left(\Pi_{0i}\bar{C}_0^- + \bar{Z}_i(Q_i - R_i) + f\Lambda_i^+\right) - \left[\frac{\bar{G}_i}{\bar{G}_{tot}}(\bar{G}_{tot} - \bar{G}^{\star}_{tot}) + \frac{\bar{D}_i}{\bar{D}_{tot}}(\bar{D}_{tot} - \bar{D}^{\star}_{tot})\right] + \bar{G}_i^{\mathfrak{R}} \\ \\ \left(\Pi_{0i}\hat{C}_0^- + \hat{Z}_i(Q_i - R_i) + f\Lambda_i^+\right) - \left[\frac{\hat{G}_i}{\hat{G}_{tot}}(\hat{G}_{tot} - \hat{G}^{\star}_{tot}) + \frac{\hat{D}_i}{\hat{D}_{tot}}(\hat{D}_{tot} - \hat{D}^{\star}_{tot})\right] + \hat{G}_i^{\mathfrak{R}} \end{cases} \tag{60}$$

8.4 Appendix D: Model simulation

8.4.1 Theoretical analysis behind model simulation

The following analysis will illustrate the most important results of the paper. The comparison of three scenarios for the evolution of bank and CCP portfolios illustrates the effects and flows of liabilities in the model:

- Scenario **A**: No liquidation or predation (assuming another method of offloading positions)
- Scenario **B**: Liquidation without predation
- Scenario **C**: Both liquidation and predation

Consider a simple case for a financial network with four homogenous banks (i, j, n, q) and 1 CCP. There are three assets k, k', K and K'. Only assets k, k', K' are held only by a distressed bank (n). I assume there are no CDS written on member banks.
At **t=0**: The CCP establishes the network; member banks establish margin accounts (g_i) and a default fund (d_i) contribution.

At **t=1**: Liabilities are realised. As in [Cont et al., 2013a], bank n exogenously defaults, $\mathbb{D} = \{n\}$,

Table 1: default

bank	**i**	**j**	**n**	**q**
i	0	$+X_{ij}^{\mathbf{k}}, -X_{ij}^{\mathbf{k}'}, -X_{ij}^{K}$	0	$-X_{iq}^{\mathbf{k}'}$
j	$-X_{ji}^{k}, +X_{ji}^{k'}, +X_{ji}^{K}$	0	$+X_{jn}^{k}, +X_{jn}^{k'}, +X_{jn}^{K'}$	0
n	0	$-X_{nj}^{k}, -X_{nj}^{k'}, , -X_{nj}^{K'}$	0	0
q	$+X_{qi}^{\mathbf{k}'}$	0	0	0

not due to the default of the underlying entity K', k' or k. Banks i and j are counterparties with each other for asset k; i has a *buy* position, and j has a *sell.* Also, they are both counterparties with bank n for k.

Scenario **A** $\rightarrow$

$$\Lambda_i^{k,S}(1\tau) \;=\; \sum_{j=1}^{m} L_{ji}^{k,S}(\ell\tau) - \sum_{j=1}^{m} L_{ij}^{k,S}(\ell\tau) \;=\; \sum_{j=1}^{m} \mathcal{F}\left(\ell\tau, X_i^{k,S}((\ell-1)\tau), \triangle S^{k,S}((\ell-1)\tau)\right)$$

In the case of no liquidation, the portfolio value is only determined by the position size, and the fundamental/underlying determinants of cds-spread on the reference entity k.

Example: The net exposure between banks j and n for CDS k is $\Lambda_j = L_{nj}^{k,B} - L_{jn}^{k,B}$ =20 bps.
If the cds-spread increases from 100bps to 120bps, $\triangle S^k > 0$, since j holds a 'buy' position, then n has a liability to j, $\frac{\delta L_{nj}^{k,S}}{\delta \triangle S^k} > 0$.
If n defaults, this liability is taken on by the CCP, $L_{nj}^{k,S} \rightarrow L_{0j}^{k,S}$.

Scenario **B** $\rightarrow$

$$= \sum_{j=1}^{m} \mathcal{F}\Bigg(\ell\tau, X_i^{k,S}((\ell-1)\tau),\; \triangle S^{k,S}((\ell-1)\tau),\; P_1((\ell-1)\tau),\; P_2((\ell-2)\tau),\; P_3((\ell-2)\tau),\; a_{ji}^{k,\pm}((\ell-1)\tau)\Bigg)$$

Now, a CCP liquidation adds a temporary price impact. This drags down the value of the position regardless of the the direction of fundamental cds-spread change $f(\triangle S^k)$. If the cds-spread increases (decreases), the price impacts dampen (strengthens) the increase (decrease). Furthermore, there is a phase change; a large enough price impact effect can drag the cds-spread down enough to turn a bank's receivable into a liability. One can see that CCP will always lower its own profits by using liquidation as a method of offloading the defaulted position in k of bank n.

Example: The CCP now holds bank n's position, $-X_{nj}^k$. With a decrease in the cds-spread from 100bps to 80bps, it should see an increased receivable of 20bps. However, with liquidation, the price impact drags down this receivable for the CCP. This decrease in profits occurs since with a negative liquidation rate, $\delta a_{k,-} < 0$, and positive price impacts, $\{P_1,\, P_2,\, P_3\} > 0$, gives a negative total impact term, $\frac{\delta |P^{k,S}\Gamma^k|}{\delta a_-} < 0$, decreasing receivables.

In turn, bank i has a buy position with j, and so it's liabilities are also increasing. However, the increase is smaller than that for j to n/CCP, as it is not a direct counterparty to bank n. Thus, it does not feel the primary price impact, but only the secondary an tertiary price impacts. We see that $\Lambda_{\mathbf{A}}^{-,k,S} < \Lambda_{\mathbf{B}}^{-,k,S}$, all else equal.

Result 1a: Firstly, the CCP will always lower its profits if it engages in a liquidation to offload a defaulters positions, hence, $\Lambda^{k,S,\mathbf{A}} < \Lambda^{k,S,\mathbf{B}}$. It is important that another method of relinquishing these positions be found, one which will allow the CCP to obtain maximum possible value for these positions.

Scenario $\mathbf{C} \rightarrow$

$$= \sum_{j=1}^{m} \mathcal{F}\Big(\ell\tau,\ X_i^{k,S}((\ell-1)\tau),\ \triangle S^{k,S}((\ell-1)\tau),\ P_1((\ell-1)\tau),\ \mathcal{P}((\ell-1)\tau),\ P_2((\ell-2)\tau),\ P_3((\ell-2)\tau),\ a_{ji}^{k,\pm}((\ell-1)\tau)\Big)$$

Once predation is added to liquidation, the secondary and tertiary price impact are intensified by the predation effect. Now it becomes apparent that bank i, in choosing to predate will decrease its own profits.

Example: Consider again that the cds-spread has decreased, and that bank i holds a sell position in asset k', just as the defaulted bank n. However, i holds this asset with j rather than n, thus, only bank j is a direct counterparty to n.

As the CCP begins to liquidate the assets of bank n, j can engage in distressed selling and depress the price. Bank i (thinking itself unaffected by bank n), begins to predatorily liquidate its holdings of asset k', in the same direction, further decreasing the price through predatory price impact. Furthermore, after one period, the cds-spread on bank i's other asset K, is being affected by holding k' in its portfolio. It feels the secondary price impact and through the connection to asset k' held by bank j, as j starts distressed selling of k and k', rebalancing it's portfolio an altering K. As well, asset K feels the added tertiary price impact for K through j for the same reason, increased pressure to rebalance j's portfolio due to liquidations of K'.

Result 1b: Predation will always lower the profits of the CCP and of the participating banks, hence, $\Lambda^{k,S,\mathbf{B}} < \Lambda^{k,S,\mathbf{C}}$. It is important that the CCP educate its member banks about the detrimental effect that predation has on their own profits, and survivorship.

Should only one of the banks, bank i choose not to liquidate their holding in asset k while others engage in predation or distressed selling, bank i will feel the full force of the price impacts on the holding at $X(\ell\tau)$. This is because his nominal position remains the same $X(\ell\tau) = X((\ell-1)\tau)$ while $\triangle S(\ell\tau) \leq \triangle S((\ell-1)\tau)$.

Result 2: If one bank predates then all other banks are better off predating. This creates the conditions for a fire-sale.

If bank q predates, it may cause the CCP to be unable to meet variation margin payments on bank n's holdings. Also, if this may cause j to fail, imposing further strain on the CCP and causing a shortfall. The CCP, may then, need to use bank q's hybrid guarantee fund contribution. Thus, in the recovery period, q, will have to use its predatory profits to refill its own initial margin contribution, $G_i^{\mathfrak{M}}$, in order to maintain membership. In the pure fund, the CCP can only demand replenishment of the default fund. However, in practice, this is often a complicated and drawn out process.[54]

Example: At first, predation drags down the cds-spread $\frac{\delta|\mathcal{P}^{k,S}\Gamma^-|}{\delta a^-} < 0$, and then it loads more weight onto the price impacts through previous cds-spread levels $\frac{\delta \triangle S^k(\ell\tau)}{\delta \mathcal{P}^{k,S}((\ell-1)\tau)} < 0$. Finally, we have that during all stages $\Lambda^{k,S,\mathbf{A}} < \Lambda^{k,S,\mathbf{B}} < \Lambda^{k,S,\mathbf{C}}$, all else equal.

Result 3: Third, the CCP, can use the contracted initial margin mechanism punitively to discourage predation, provided that it chooses a hybrid guarantee fund structure. It punishes predator's behaviour endogenously, according to previously established rules.

Result 4: The hybrid guarantee fund structure is more incentive compatible for the CCP over the pure

[54]The default fund is not subject to the strict legalities that surround initial margin contributions.

fund structure, taking into account the situation where large defaults can lead to a large shortfall for the CCP. A hybrid guarantee fund structure will, possibly, save the CCP's own equity, but also, with the enforceable margin call on the guarantee fund, make it increasingly more likely that the CCP will not fail. This last point is important for all concerned, banks and the CCP, since a smaller likelihood of failure means that, in the end, there is not a sudden, unexpected loss of every parties assets. It is with these points in mind, that it is tentatively suggested, that the network of global CCP would increase financial stability by instituting the hybrid guarantee fund structure.

8.4.2 Monte-Carlo-type Simulation

In this type of simulation is it most definitely certain that the random endowment or distribution of assets, and the random assignment of the liquidation rate among banks determines, to some degree, the results. However, a robust simulation, should show patterns and similarities in the results over many iterations with a randomised seed. In this section, average results from 50 separate randomised trials are provided. Over all trials, the majority of results remain similar, but present with more granularity. In the runs under variable market liquidity, there are a few interesting and surprising features.

Under normal market liquidity, the number of distressed banks remain largely the same, at 7.44, 6.16 and 1.32 defaulted banks for 1, 2, 3 predatory banks and 13,12, 11 distressed banks, respectively. The final CCP loss is increased by -2e11 with a new pronounced loss of -6e11 for the non-collusive case for 2 predatory banks. The final predation profits remain volatile, and keep largely the same pattern, except that profits are now largely positive for the collusive case; this is owing to the lack of competition and predators acting as a monopoly.

Under decreasing market liquidity. There is much more fine grained detail. The default distribution decreases more gradually with defaults jumping from 1 to 4.08 banks around 7 predatory banks and 7 distressed banks. When predatory banks are stable at 2 banks, at 7 distressed banks the default number jumps to 3.48. This gives an idea of the augmentation effect of competition on defaults, which is about 0.6 banks at this turning point. For 2 distressed banks, the default distribution remains unchanged. For the final CCP loss, in the case of collusion and non-collusion, the pattern remains the same except for the predatory case which takes on the pattern of the former two cases. This reinforces the idea of this threshold point where predation profit cannot overtake price impact. The average profit and loss for banks is extremely stable, as do predator profits and losses, though buyback values are depressed. Under financial crisis liquidity, all results are stable.