The Expectation Formation Process along the Business Cycle: More Information or Better Information ?

Jérémy Boccanfuso*

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Abstract

Recent studies have shown that information rigidities decrease during recessions. Using a Sticky and Noisy Information model, this paper proposes novel explanations of how expectations are affected by economic shocks. The expectation formation process (EFP) is decomposed into an extensive margin – i.e. the choice to acquire new information – and an intensive margin – i.e. the information processing intensity (as in Sims [2003]). Using surveys on professional forecasters, the paper demonstrates that only the extensive margin is related to economic fluctuations. Then, a realistic EFP is introduced in an otherwise standard Permanent Income Model. The state-dependence of expectations makes the resolution particularly original and results in large deviations from mainstream models of rational inattention both in terms of consumption dynamics and savings behavior.

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Introduction

When people make decisions, their rationality is limited by the information, the cognitive capacities and the time they have. This viewpoint was central to Simon's theory of bounded rationality (1972) and led to the development of a large variety of theories that override the standard hypothesis of rational expectations. Four main methods have been developed by economists in the last decades to study the implications of bounded rationality on the behavior of individuals: adaptative and eductive learning (e.g. Evans and Honkapohja [2001] and Guesnerie [1992]), robustness (e.g. Hansen and Sargent [2008]), behavioral models using heuristic rules (e.g. Brock and Hommes [1997]) and rational inattention (e.g. Sims [2003] and Mankiw et al. [2003]). The latter theory focuses on the rigidities to acquiert and process information when people make a choice and is thus a natural candidate to

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introduce both limited information and limited cognitive capacities in economics. It was pioneered by Sims [2003, 1998] who used Information Theory to constrain the flow of information an individual can process in order to reproduce the limited capacities of the human brain, and was then extended by many authors who proposed different constraints on information flows or replaced these constraints by a cost to acquire information. One of these extensions is the well-known Sticky Information model of Reis [2006], which indicates that whenever there exists a fixed-cost to update expectations, people may rationally choose to remain inattentive for a possibly long time.

A decade after these theories were first developed, we now have large evidences that they are able to reproduce important stylized facts on the expectation formation process. Particularly, empiricists did find in the data that all the relevant information is not entirely used when people form their expectations[1] – as predicted by the noisy information theory of Sims [2003] – and that individuals update their expectations only sporadically[2] – as predicted by the sticky information theory of Mankiw et al. [2003] and Reis [2006]. However, taken appart, none of these theories can explain the coexistence of these two types of rigidities. Thus, this paper first develops a general process for the expectation formation process that is able to reproduce the facts that people do not systematically update their expectations and that whenever they update, they don't use all the relevant information. To do so, the expectation formation process is decomposed between an extensive and an intensive margin. On the one hand, the extensive margin reproduces individuals' choices to acquire information and to do complex economic tasks. On the other hand, the intensive margin stands for the limited capacities to collect and process all the relevant information for a given economic problem. Using surveys on professional forecasters from both the European Central Bank and the Federal Reserve Bank of Philadelphia, I show that both types of rigidities coexist simultaneously in the data, and that the rigidities at the intensive margin appears to be the most important.

Recent studies from Coibion and Gorodnichenko [2015], Dräger and Lamla [2013] and Loungani et al. [2013] have shown that information rigidities tend to decrease during recessions, thus giving a strong argument in favor of a state-dependent representation of expectations, as opposed to a time-dependent representation. Therefore, the second contribution of the paper – and probably the most important one – is to study the state-dependence of the expectation formation process in the data and to link it to each margin of the expectation formation process. The methodology used aims to asses whether people update more after an economic shock (i.e. rigidities at the extensive margin decrease) and whether the treatment of information is also better after a shock (i.e. rigidities at the intensive margin decrease). The results indicate that only the share of people updating at each period is affected by economic fluctuations, whereas the rigidities at the intensive margin are not. Thus, people are somehow always partially attentive to the evolution of their environment and their attention is a

[1] See for example Coibion and Gorodnichenko [2015, 2012].

[2] See for example Andrade and Le Bihan [2013] and Dräger and Lamla [2013].

scare ressource that they choose, consciously or not, to optimally allocate between their economic and non-economic daily tasks. From a psychological point of view, my representation of the expectation formation process may be reconciled with the Dual-Process theory (e.g. Evans and Frankish [2009]), which postulates that there exists two systems, or minds, in one brain. The choice to update is thus taken when people are the least attentive by the automatic, fast thinking, System 1.[3] Then, when people think it would beneficial for them to update their expectations and to resolve complex economic problems, the System 2 activates and they are able to collect and process more information.

Some papers have already studied a state-dependent expectation formation process. The first ones are those that used a setup à la Brock and Hommes [1997] where people may choose between different types of operators according to their ability to forecast accurately in past periods. The main difference with my model is that (i) one does not need to select a priori a (small) set of possible expectation operators, and (ii) that rigidities at the intensive margin, which are shown to be significant in the data, may be reproduced.[4] The second example of a state-dependent expectation formation process is Gorodnichenko [2010] who allows firms to endogenously increase their attention after observing a private and a public signal. According to my decomposition into the two margins, the state-dependence arrises in this model at the intensive margin because each firm may change its price at every periods (as long as they are willing to pay a specific fixed-cost) so that there is no extensive margin.[5] Thus, the data tends to indicate that my model is a better representation of the expectation formation process because information rigidities affect expectations at both margins and the state-dependence arrises at the extensive margin, and not at the intensive margin.

Finally, the last, but not least, contribution of the paper is to introduce and resolve a Permanent Income setup with the two-margins state-dependent expectation formation process. When one compares the predictions of this framework to those of the standard full-information rational expectations model of Hall [1978], the sticky information model of Reis [2006] and the noisy information model of Sims [2003] and Luo [2008], we observe major deviations both in terms of the saving behavior and the dynamics of consumption. First, the existence of a state-dependent extensive margin implies that consumption encounters irregular jumps and, second, the stochasticity of durations between two updates prompts a risk-averse consumer to increase her precautionary savings at each date to insure against this greater uncertainty.

The paper is organized as follows. The first section formalizes the expectation formation process with the two margins. The second section shows from the US and European surveys of professional forecasters that the state-dependence of expectations comes only from the extensive margin, and gives

[3] The terms System 1 and System 2 come from Kahneman [2011].

[4] All these models assume that people may access full-information rational expectations so that all the relevant information may be used and processed.

[5] Alternatively stated, there is no extensive margin in Gorodnichenko [2010] because people are systematically doing complex economic tasks so that their attention may not be used for other non-economic tasks.

an estimation of the evolution of overall information rigidities, as well as estimates of rigidities at each margin. The third section introduces the state-dependent process of expectations into a simple behavioral Permanent Income model and underlines the main deviations from other well-known frameworks used in the literature on rational inattention. Finally, the last section extends this simple model by assuming that individuals are perfectly rational, in the sens that they may interact with their automatic System 1 which governs the choice to update their expectations.

1 The Intensive and Extensive Margins of Expectations

Economic agents do not systematically update their expectations (e.g. Andrade and Le Bihan [2013] and Dräger and Lamla [2013]) and don't use all the information available when they revise their expectations (e.g. Coibion and Gorodnichenko [2015, 2012]). Therefore, a complete theory of the expectation formation process must be able to recover these two empirical findings that are well-documented in the literature. However, most studies treat these two components of the expectation formation process as competing theories and test which one reproduces best the data[6]. Hence, it seems urgent to develop the microfundations of an expectation formation model where agents choose simultaneously when to update their expectations and how much information to use when they update. To do so, I shall decompose the expectation formation process in two steps. First, I concentrate on what must be seen as the extensive margin of expectations, namely the individual choice to acquire and process information to identify her best response to an economic problem. Second, I will study the intensive margin of the expectation formation process. The intensive margin represents how much attention an individual is willing to devote to collect and process information. In this section, I develop a simple general framework to replicate the formation process of expectations at both margins.

1.1 Expectation Formation: The Extensive Margin

Let i denote an individual who must decide wether she wants to update her expectations at time t given that her last expectations were formed at time g. Whenever she decides to update her expectations, she must pay a fixed and time-invariant cost K. This monetary cost represents the disincentives to collect, process and interpret information and to do complexe cognitive tasks such as recomputing the best-response to an economic problem given an information set. From a biological point of view, this cost represents the fact that attention and energy are limited (Kahneman [1973]) and therefore that whenever an individual wants to complete an intensive economic task, she must put aside all other intellectually and physically intensive activities. Hence, individuals always face an arbitrage between concentrating on complex economic problems or doing other tasks. The cost K stands for this daily arbitrage.

[6] The only exception that I'm aware of is Andrade and Le Bihan [2013] who estimate a Sticky and Noisy Information Model.

Formally, let $W_{i,t}(.)$ be the utility individual i expects from updating her expectations at time t. Then she will always pay the cost K as soon as $W_{i,t}(.) \geq K$. Let $Y_{i,t}$ be a dummy equal to one when individual i updates her expectations, thus we have:

$$Y_{i,t} = \begin{cases} 1 & \text{if} \quad W_{i,t}(.) \geq K \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The determinants entering the utility $W_{i,t}(.)$ and the form of this fonction are not clearly identified in the literature. A priori, one may think that some of these determinants will be specific to individual i, to the information collected during the last update at time g and to the agent's beliefs since that last update. Let I be a column vector of the idiosyncratic determinants that enter the utility function, $\mathcal{Z}_g$ the variables in the information set up to period g that are relevant for $W_{i,t}(.)$ and $\mathcal{B}_{t,g}$ the variables in the set of beliefs between period g and t entering the utility fonction $W_{i,t}(.)$. Moreover, assuming that the utility fonction is additive with an error term $\varepsilon_{t,i}$ and a constant term a common to all individuals, we can rewrite $W_{i,t}(.)$ as :

$$W_{i,t}(I, \mathcal{I}_{t-g}, \mathcal{B}_{t,t-g}, \varepsilon_{t,i}) = a + \alpha_1 I + \alpha_2 \mathcal{Z}_{t-g} + \alpha_3 \mathcal{B}_{t,t-g} + \varepsilon_{t,i} \tag{2}$$

Thus, the probability that i will update her expectations at time t is $P(Y_{i,t} = 1) = P\big(\varepsilon_{t,i} \geq K - W_{i,t}(I, \mathcal{Z}_{t-g}, \mathcal{B}_{t,t-g}, 0)\big)$. When $\varepsilon_{t,i}$ is an i.i.d. error with extreme value distribution we get that

$$P(Y_{i,t} = 1) = \frac{1}{1 + \exp\left[K - W_{i,t}(I, \mathcal{Z}_g, \mathcal{B}_{t,g}, 0)\right]} \tag{3}$$

The extensive margin of expectations is directly related to the concept of Sticky information in Mankiw and Reis [2002] and Reis [2006]. The main difference is that the probability of an update of expectations is not constant over time[7] and depends on a defined set of variables specific to individual i, the economic shocks before g (through $\mathcal{Z}_g$) and the perceived shocks between g and t (through $\mathcal{B}_{t,g}$). As I will demonstrate in section 2, the state-dependence of the expectation formation process is essential to understand agents' responses to economic fluctuations and is an important feature of the data.

1.2 Expectation Formation: The Intensive Margin

We now turn to the intensive margin of the expectation formation process. Unlike Mankiw and Reis [2002], we don't suppose that people have sufficient cognitive capacities and perfect information to

[7] Branch [2004] develops an alternative setup where the probability that an agent uses the rational operator with full-information to form her expectations is also not constant over time. My approach is however different from that of Branch in many ways. First, it doesn't assume that agents will use cheaper alternative operators to form their expectations (e.g. adaptative expectations) as was first proposed by Brock and Hommes [1997]. Second, in this paper they will face rigidities at the intensive margin so that they will never be able to recover full-information rational expectations. As we will see, the data confirm the existence of rigidities both at the extensive and extensive margins thus favoring the setup developed in this paper to that of Branch.

have Full-Information Rational Expectations (FIRE) at the intensive margin. Instead, I really on the concept of rational inattention, or noisy information, proposed by Sims [2003]. Hence, it is supposed that people know the structure of the model and the parameters driving the economy, but never truly observe the states because of cognitive limitations to process information. Let x be a state variable that the agents want to forecast and $\bar{E}^u_{i,t}x_{t+h}$ be the expectation at time t of this variable x at an horizon h for an individual i who updates her expectations. Because agents know the model, this expectation is the conditional expectation of x_{t+h} given the model that represents the economy and her information set at time t ($\mathcal{I}_{i,t}$). Noisy information implies that each individual i will observe a signal $y_{i,t}$ on the state x_t such that

$$y_{i,t} = x_t + \omega_{i,t} \tag{4}$$

Where $\omega_{i,t}$ is a gaussian noise with zero mean and variance $\sigma^2_{\omega_i}$. According to Sims [2003]'s theory of inattention, we know that the variance of $\omega_{i,t}$ will depend on the cognitive capacities of individual i to process information: if i faces no cognitive constraints, she can choose $\sigma^2_{\omega_i} = 0$ and observe the true state x at each period. Instead of focusing directly on the endogenous optimal choice of $\sigma^2_{\omega_i}$ as in Sims [2003], Woodford [2009] and many others; I assume for simplicity that $\sigma^2_{\omega_i}$ is given but can differ across individuals. Thus, each individual faces a standard gaussian extraction problem and will update her expectation $\bar{E}^u_{i,t}x_t$ using a Kalman filter with constant gain G_i such that :

$$\bar{E}^u_{i,t}x_t = G_i y_{i,t} + (1 - G_i)\bar{E}^u_{i,g}x_t \tag{5}$$

where g is the last time the agent decided to pay the cost K to receive a new signal $y_{i,g}$ (ie. $g = t - \arg\min_{n\in\mathbb{N}^\star} s.t.\ Y_{i,t-n} = 1$). An important feature of the structure of information rigidities at the intensive margin here is that they are constants over time for a given individual. As we will see in the next section of the paper, these rigidities are not affected by economic fluctuations. Hence, for what we are interested in this paper – i.e. the links between the formation process of expectations and economic shocks – it is an acceptable hypothesis.

Even though the setup at the intensive margin is similar to most theories of Sticky Information, there is a major difference regarding the formation of expectations in equation (5): people don't continuously update their expectations and receive a signal $y_{i,t}$. Therefore, the Kalman filter is somehow different that what we usually find in the literature because it doesn't apply on expectations formed at the last period, but on expectations formed $t - g$ periods ago. This is the consequence of the extensive margin since, as there were no updates between g and $t - 1$, we have that $\bar{E}_{i,t-j}x_t = \bar{E}^u_{i,t-g}x_t\ \forall J \in [1, 2, \ldots, g - 1]$.

1.3 Average Forecast Across Agents

We must now put together the extensive and intensive margins of expectations to determine the average forecast across agents in the economy. Recall that the formation process of expectations introduced so far can be summarized for an individual i as :

$$\bar{E}_{i,t}x_t = \begin{cases} \bar{E}^u_{i,t}x_t & \text{if} \quad Y_{i,t} = 1 \\ \bar{E}^u_{i,g}x_t & \text{otherwise} \end{cases} \tag{6}$$

For tractability, we don't take into account the heterogeneity in signal-noise ratios from now on so that the Kalman gains are identical accros individuals.[8] Let $F_t x_{t+h}$ denote the average forecast of the variable x for an horizon h and $F(i)$ the distribution of agents in the economy. We know that at each date t only a share $\lambda_t = \int Y_{i,t} dF(i)$ will update their expectations. Thus, $F_t x_{t+h}$ is a weighted sum of past and present expectations :

$$\begin{aligned} F_t x_{t+h} &= \lambda_t \bar{E}^u_t x_{t+h} + \sum_{j=1}^{\infty} \lambda_{t-j} \prod_{i=0}^{j-1} (1-\lambda_{t-i}) \bar{E}^u_{t-j} x_{t+h} \\ &= \lambda_t \bar{E}^u_t x_{t+h} + (1-\lambda_t) F_{t-1} x_{t+h} \end{aligned} \tag{7}$$

Following Coibion and Gorodnichenko [2015], suppose that the variable x follows an autoregressive process of order one. We get that :

$$x_t = \rho x_{t-1} + v_t \tag{8}$$

where v_t are white noises with variance σ^2_v. Thus, the expectation of an individual who updates at time t is:

$$\begin{aligned} \bar{E}^u_{t,i} x_{t+h} &= \rho^h \bar{E}^u_{i,t} x_t \\ &= G\big(x_{t+h} - v_{t+h,t}\big) + \rho^h G \omega_{i,t} + (1-G) \bar{E}^u_{i,t-g} x_{t+h} \end{aligned} \tag{9}$$

where $v_{t+h,t} = \sum_{j=1}^{h} \rho^{h-j} v_{t+j}$ is the actualized surprise that will occur between t and $t+h$. Looking for the average expectation across those who update amounts to compute $\bar{E}^u_t x_{t+h} = \frac{1}{\lambda_t} \int \bar{E}_{i,t} x_{t+h}\, Y_{i,t} dF(i)$. Thus, we can rewrite $\bar{E}^u_t x_{t+h}$ as:

$$\begin{aligned} \bar{E}^u_t x_{t+h} &= G\big(x_{t+h} - v_{t+h,t}\big) + (1-G) \left[\sum_{g=1}^{\infty} \lambda_{t-g} \prod_{j=1}^{g-1} (1-\lambda_{t-j}) \bar{E}^u_{t-g} x_{t+h} \right] \\ &= G\big(x_{t+h} - v_{t+h,t}\big) + (1-G) F_{t-1} x_{t+h} \end{aligned} \tag{10}$$

[8] A discussion on the implications of this hypothesis on the average forecast can be found in Coibion and Gorodnichenko [2012, 2015].

Introducing this expression in equation (7), the average forecast is

$$F_t x_{t+h} = \lambda_t G\big(x_{t+h} - v_{t+h,t}\big) + (1 - \lambda_t G) F_{t-1} x_{t+h} \tag{11}$$

and the average forecast error

$$x_{t+h} - F_t x_{t+h} \quad = \quad \frac{1 - \lambda_t G}{\lambda_t G}[F_t x_{t+h} - F_{t-1} x_{t+h}] + v_{t+h,t} \tag{12}$$

Thus, when people have full-information rational expectations, $\lambda_t = 1$ and $G = 1$ at every periods and the forecast error is $v_{t+h,t}$ – namely the actualized surprises that will occur in the future; when people have no extensive margin, the model reduces to the standard Noisy Information; and when there are no information rigidities at the intensive margin, the model is equivalent to the Sticky Information of Mankiw et al. [2003] where the share of updates is time-varying. The model developed so far is indeed a general formation process that nests the standard theories of rational inattention as special cases.

Similar to Coibion and Gorodnichenko [2015], I find that the ex-post mean forecast error is related to the ex-ante mean forecast revision and that the relation between these two terms depends only on information rigidities. However, these rigidities come in this framework from both margins of the expectation formation process through the product $\lambda_t G$. Furthermore, because individuals' choices at the extensive margin depend on beliefs about past and present economic shocks, the overall average information rigidities is time-varying.

2 State-Dependent Information Rigidities

From the results presented in the previous section, there is no reason to believe that information rigidities are constant over time. However, most frameworks of information rigidities assume that the expectation formation process is time-dependent and therefore that these rigidities must be constant as long as the structural parameters in the economy remain unchanged. For example, Reis [2006] starts on the basis that people choose ex ante how long they want to remain inattentive. Therefore, they update their expectations only sporadically after a given number of periods and remain totally inattentive between two updates. Hence, Reis [2006]'s setup is built on the strong assumption that people have no access to any information, such as newspapers, TV news or a report on the evolution of their savings[9], when they are not doing complex economic tasks. As a consequence, it must be the case that information rigidities are constants over-time.[10]

This view of a time-dependent (rather than state-dependent) expectation formation process has

[9] In Reis [2006]'s model, if an individual rationally chooses a consumption path between two updates, the shocks (or surprises) that will occur before the next update will be absorbed directly through her savings. Thus, if an agent knows (or has a small idea of) how her savings are evolving, she knows how her wealth is evolving and therefore she may prefer to update her expectations now rather than later.

[10] A similar argument is also true for Sims [2003]'s view of rational inattention : the Kalman gains are constant over time (as long as the distribution of shocks remains the same) so that rigidities are also constants over time.

been challenged by two recent studies from Coibion and Gorodnichenko [2015] and Loungani et al. [2013]. They both show that average information rigidities are lower in recessions that they are in normal times. Therefore, using the framework developed in section 1 at the individual and aggregate levels, I aim to asses in this section whether the rigidities at the extensive or intensive margins are related to economic fluctuations. Alternatively, one might see this exercice as a test of the state-dependence of Noisy and Sticky information when both types of information rigidities simultaneously exist. The results presented hereafter indicate that only the extensive margin responds to past shocks, whereas the intensive margin is not affected by these shocks.

2.1 SPF Data and Identification of Economic Shocks

To asses wether information rigidities are state-dependent, I use two surveys of professional forecasters (SPF) on inflation. (i) The first survey comes from the European Central Bank. Interested readers can find a general presentation of this survey in Andrade and Le Bihan [2013]. The survey covers a period from the first quarter of 1999 to the third quarter of 2015. 50 to 68 professional forecasters were asked to report their quantitative (or point) forecasts for inflation at different horizons each quarter. Here, I focus only on the calendar forecast: in each quarter, forecasters are surveyed about their forecasts for fixed events, namely the current, next and after next[11] years inflation rates. Therefore, we are able to observe successive revisions of a same forecast across time for a given forecaster (or institution). These cross-section panel data are however unbalanced because of missing data and the fact that one cannot observe the same number of revisions (i.e. in normal times we observe 8 revisions for forecast horizons before 2002 and 12 revisions after) at the beginning and at the end of the time period: there are at most 4 revisions per forecaster for forecasts of the year 1999 and at most 11 revisions for forecasts of the year 2015. (ii) The second survey of professional forecasters is currently run by the Federal Reserve Bank of Philadelphia. Each quarter from the fourth quarter of 1968 to the fourth quarter of 2015, on average 41 professional forecasters where asked about their forecasts on inflation. I restrict the analysis to the rolling forecasts. Therefore, these are also cross section panel data (unbalanced for the same reasons as in the ECB SPF) and we observe at most 6 revisions for a same forecast per forecaster (or institution).

A key element of my analysis is the identification of economic shocks. Indeed, we are interested in the response of information rigidities at both margins conditional on economic shocks. It is therefore essential to get correct measures of the major shocks affecting the evolution of inflation. There is a large literature in macroeconomics on how to identify these shocks. Two different methods are usually used: (i) structural and (ii) filtering methods. To ensure that my results will be robust to these two methods, I estimate alternatively inflation shocks with each. (i) The structural estimation of economic shocks is based on Coibion and Gorodnichenko [2015]. I concentrate on productivity and oil price shocks

[11] The after next year forecast horizon is available only for the third and fourth quarters since 2000.

that have been identified as two major components of inflation fluctuations.[12] To identify productivity shocks, I replicate the bivariate structural VAR from Gali [1999] for labor productivity and worked hours. The identifying assumption is that only technology shocks may affect productivity in the long run. Estimations are made separately for the Euro area and the U.S.[13] The second type of structural shocks are oil price shocks. Identification of these shocks is made following Hamilton [1996]: oil price shocks are episodes in which the oil price exceeds the maximum oil price over the last 12 months. When this is the case, the shock is measured as the difference between the current price and the maximum over the last 12 months, and zero otherwise. Data are quarterly from the first quarter of 1985 to the third quarter of 2015. These productivity and oil price shocks have the advantage to be easily interpretable. However, a large part of inflation variations remain unexplained when one uses only these two structural shocks. (ii) Hence, I exploit another statical interpretation of economic shocks: economic shocks are cyclical movements around a trend component. To estimate these high-frequency cyclical variations, I use a high-pass filter widely-used in macroeconomics: the Hodrick-Prescott filter. The smoothing parameter λ is set to 1600 as Hodrick and Prescott [1997] recommend for quarterly data. These inflation shocks are estimated separately for the U.S and the Euro area. Because these shocks may not be interpreted economically, I shall refer to them as undefined shocks.

2.2 State-Dependance of the Extensive Margin

People don't systematically update their expectations. From the European and US SPF we see that respectively only 76 and 96 percent of forecasts are revised from one period to the next on average. These shares are below the 100 percent one may expect when there are no rigidities at the extensive margin. Moreover, as can be seen from figure 1 in appendix, the share of updates is not constant over time. It varies from 51 to 99 percent in the ECB SPF and from 46 to 100 percent in the US SPF. These frequent movements in the share of updates cannot be reconciled with any theory of bounded rationality and are quite puzzling. We need to understand the determinants that prompt individuals to update their expectations. As I have argued in section 1, if there is a cost to update expectations, agents must consider that there is a gain to update and that this gain will overwrite the cost of updating. Therefore, it seems necessary to point out some determinants entering the utility function $W(.)$ (i.e. the expected gain from updating) defined in equation (2). In this paper, I will not give an exhaustive list of all the determinants entering $W(.)$, but I shall assess wether some of these determinants are related to economic fluctuations so that people will update more after an economic shock. Formally, I am trying to see if some beliefs of forecaster i about current economic shocks are in the set $\mathcal{B}_{i,t}$ of beliefs affecting her choice to update her expectations ($Y_{i,t}$).

I focus here on the average share of updates at each period. Estimations on the individual choice to

[12] These shocks have been identified by Coibion and Gorodnichenko [2015] as the ones that explain the most the variance of inflation from the first quarter of 1966 and the third quarter of 2007.

[13] More information on the data and specification of the SVARs are documented in online appendix A.

update are reported in online appendix B and give similar conclusions. Estimations are made separately for the two surveys. Shocks are estimated alternatively by the two methods presented in section 2.1. To allow for a non-linear response to economic shocks, I systematically include a quadratic polynomial for each type of shocks. Results of these estimations are displayed in table 1 and indicate clearly that current economic shocks have an impact on the share of people updating their expectations. More specifically, the quadratic term is positive for all types of shocks in both survey. Thus, the larger is the size of an economic shock, the higher is the share of people updating their expectations. This effect is statistically significant for all types of shocks in the ECB SFP and for the undefined and productivity shocks in the US SPF. These results are strong evidences for the state-dependence at the extensive margin of the expectation formation process and indicate that economic agents are somehow always (partially) attentive to economic fluctuations: when a shock hits the economy, people form new beliefs that incorporate, on average, this shock and adapt their behaviors accordingly.

Table 1: Updates and economic shocks

	European SFP		U.S. SFP	
Shocks	(1)	(2)	(3)	(4)
Oil	-0.123**		0.006**	
	(0.047)		(0.002)	
Oil (quadratic)	0.010***		-0.000	
	(0.003)		(0.000)	
Productivity	-0.070		-0.106	
	(0.138)		(0.497)	
Productivity (quadratic)	0.352***		139.7*	
	(0.130)		(71.44)	
Undefined		0.069		-0.001
		(0.0908)		(0.003)
Undefined (quadratic)		0.157**		0.0014***
		(0.066)		(0.0005)
Constant	0.811***	0.778***	0.966***	0.954***
	(0.096)	(0.068)	(0.005)	(0.006)
Observations	51	66	121	181

Note : Results obtained from an OLS regression on the share of forecasters updating their expectations (λ_t) in the European SPF (for which I use a logistic transformation of λ_t) and the US SPF. Standard errors are estimated with White's estimator. Productivity shocks were computed separately for the U.S. and the Euro zone using structural VARs. Oil price shocks were computed following the methodology of Hamilton [1996]. Undefined shocks were computed using an Hodrick-Prescott filter on inflation (the variable being forecasted). Standard errors are in parenthesis and ***, ** and * respectively denote significance at 1%, 5% and 10% levels.

2.3 State-Independance at the Intensive Margin

It is now necessary to understand if the response of overall information rigidities to economic shocks comes only from the extensive margin or, if people are also more attentive – namely, if the rigidities at the intensive margin also decrease after an economic shock. Alternatively stated, we need to understand if the decrease in global information rigidities – measured though $\lambda_t G_t$[14] – observed in recessions by Coibion and Gorodnichenko [2015] and Loungani et al. [2013] is explained only by an increase in the share of people updating their expectations after large shocks (λ_t) or, if in recessions, people are also more attentive and more efficient in collecting, processing and interpreting information (i.e. G_t increases).

To get a measure of information rigidities at the intensive margin, I shall get ride of rigidities at the extensive margin. An intuitive way to do so is to focus only on periods where individuals update their expectations. Namely, the t such that $Y_{i,t} = 1$ for each individual i. Then, I estimate average rigidities at the individual level from the speed of information transmission. Let $F_{i,t}x_{t+h}$ be the forecast in t of an individual who updates her expectations at period t and whose last update before was in period g. Then, if expectations are FIRE, we expect that all the information available at period g and before is already included in $F_{i,g}x_{t+h}$ and should not affect the forecast revision at period t ($F_{i,t}x_{t+h} - F_{i,g}x_{t+h}$). Therefore, if an economic shock at period g or before does affect the difference $F_{i,t}x_{t+h} - F_{i,g}x_{t+h}$, one can reject the FIRE hypothesis. Moreover, because the size of the effect on the forecast revision is proportional to the speed of information transmission, I obtain an estimate of information rigidities. It is however not clear which type of shocks is the most relevant to estimate these rigidities. Therefore, I use the three types introduced in section 2.1 to estimate :

$$F_{i,t}x_{t+h} - F_{i,g(i)}x_{t+h} = \alpha + \beta_1 z^{\text{Prod}}_{g(i)-1} + \beta_2 z^{\text{Oil}}_{g(i)-1} + \beta_3 z^{\text{Und}}_{g(i)-1} + \varepsilon_{i,t} \tag{13}$$

Where $z^{\text{Prod}}_{g(i)-1}$, $z^{\text{Oil}}_{g(i)-1}$ and $z^{\text{Und}}_{g(i)-1}$ are respectively the productivity, oil price and undefined shocks one period before individual i updated her expectations for the last time before t. Estimations of equation (13) are made separately for the ECB SPF and the US SPF. As can be seen from table 5 in appendix, all coefficients β_1, β_2 and β_3 are significants at the 1% level for both surveys. The FIRE hypothesis is therefore strongly rejected in the data. To asses whether there exists a state-dependence of the intensive margin, I must estimate the evolution of β_1, β_2 and β_3 across time. To do so, I use the panel dimension of the surveys and run rolling estimations: at each date t, I estimate equation (13) on individuals who are updating between[15] t-2 and t+2 to get three sequences $\{\beta_{1,t}\}_{t=0}^{T}$, $\{\beta_{2,t}\}_{t=0}^{T}$ and $\{\beta_{3,t}\}_{t=0}^{T}$. Then, I regress these sequences on the three types of shocks to see if they explain the evolution of the betas. When this is the case, information rigidities at the intensive margin would be

[14] Recall that there are no information rigidities when $\lambda_t G_t$=1.

[15] The width of the window doesn't affect much the results displayed in table 2. Results from alternative estimations with smaller windows of 1 and 0 are available upon request.

state-depend. Otherwise, it would be a strong argument in favor of the state-independence of these rigidities. Table 2 reports the results of these estimations. As we can see, all measures of rigidities are not significally affected by economic shocks; the only exception being undefined and productivity shocks in the US SPF when rigidities are estimated from the speed of transmission of oil price shocks that are find to be significant respectively at the 10% and 1% levels in the US SPF.

Table 2: State-Dependence at the Intensive Margin

	European SFP			U.S. SFP		
	(1)	(2)	(3)	(4)	(5)	(6)
Measures of Rigidities :	Productivity	Oil	Undefined	Productivity	Oil	Undefined
Productivity	0.0391	0.0296	0.0260	1027.3	-34.98*	2.844
	(0.656)	(0.116)	(0.389)	(991.6)	(19.78)	(3.553)
Oil	0.0563	0.0104	0.0195	2.816	-0.0310	0.00510
	(0.0876)	(0.0153)	(0.0519)	(1.767)	(0.0324)	(0.00633)
Undefined	-0.186	-0.0406	-0.0614	2.035	-0.393***	0.0105
	(0.391)	(0.0683)	(0.232)	(5.113)	(0.0995)	(0.0183)
Constant	-0.407	-0.0599	-0.152	-8.576*	0.266**	0.0739***
	(0.327)	(0.0615)	(0.193)	(4.901)	(0.105)	(0.0176)
Observations	50	43	50	111	82	111

NOTES : Results from the OLS regressions of equation $\hat{\beta}_{j,t} = \alpha + \gamma_1 z_t^{\text{Prod}} + \gamma_1 z_t^{\text{Oil}} + \gamma_1 z_t^{\text{Und}} + \text{error}_{j,t}$ where $\hat{\beta}_{j,t}$ $\forall j \in \{1, 2, 3\}$ – the three different measures of information rigidities based on the speed of transmission in agents' forecasts of productivity, oil and undefined shocks – are estimated by regressing equation (13) on people who are updating between $t-2$ and $t+2$. Standard errors are in parenthesis and ***, ** and * respectively denote significance at 1%, 5% and 10% levels.

All things considered, this empirical exercice confirms the existence of rigidities at the intensive margin, but shows that these rigidities are not affected by the business cycle. Thus, as long as we are interested in the links between the expectation formation process and economic fluctuations, it is a good approximation to assume that rigidities at the intensive margin are constants over time, thus confirming the theoretical framework introduced in section 1.

2.4 Overall Information Rigidities

The last empirical question treated in this paper is the decomposition of overall information rigidities – the rigidities when one takes into account both margins – into the two margins. Indeed, since we know that both margins face information rigidities and that only the extensive margin is related to the business cycle, it is essential to highlight which type of rigidities is the main driver of global information rigidities and wether this decomposition into two types is relatively constant over time.

A first intuitive approach to estimate overall rigidities is by regressing the average forecast error on the average forecast revision as in Coibion and Gorodnichenko [2015]:

$$\pi_{t+h} - F_t\pi_{t+h} = \alpha + \beta\big(F_t\pi_{t+h} - F_{t-1}\pi_{t+h}\big) + \varepsilon_t \tag{14}$$

From equation (12) we expect that $\alpha = 0$ and $\beta = \frac{1}{T}\sum_{t=0}^{T}\frac{1-\lambda_t G}{\lambda_t G}$ so that $\hat{\beta}$ is an estimate of average global rigidities over time. Forecast errors $(\pi_{t+h} - F_t\pi_{t+h})$ are measured using real-time data as they were first released. Data are pooled by forecast horizons and dates. Thus, we observe up to five consecutive forecast revisions for a same event in the US SPF and seven in the ECB SPF. As reported in table 3 (Panels A), we find in the European SPF that β is equal to 1.594 and is statistically significant at the 1% level. Regarding the constant term, we find that it is not significant as predicted by theory. Since β maps directly the degree of overall information rigidities, we obtain an estimate of $T^{-1}\sum_{t=1}^{T}\lambda_t G$ equal to 0.39. Note however that, because of Jensen's inequality, this estimate of average global rigidities is biased and should be interpreted as an upper bound;[16] meaning that, at this stage, we can only conclude average information rigidities are such that $T^{-1}\sum_{t=1}^{T}\lambda_t G \leq 0.39$. From the US SPF, we get $\beta = 0.683$ so that $T^{-1}\sum_{t=1}^{T}\lambda_t G \leq 0.594$. These results for the US are almost identical to Coibion and Gorodnichenko [2015] but the interpretation differs since, as long as rigidities are not constant over time, the estimate of beta gives an upper bound for average global rigidities. Overall, rigidities appear to be much more important in the Euro area than in the US. However, one should not overinterpret the difference between estimates from the two samples. Indeed, even though it is not explained by theory, the size of information rigidities is highly sensitive to forecast horizons (e.g. Coibion and Gorodnichenko [2012], Sheng and Wallen [2014] and Dovern et al. [2014]) and likely to forecast types (i.e. rolling or calendar). Hence, a large part of the difference between the two estimates could be due to differences in terms of forecast types and horizons.

Table 3: Estimation of information rigidities

	ECB SPF		ECB SPF	
	(A)	(B)	(A)	(B)
	All sample	Updates only	All sample	Updates only
Forecast revision	1.594***	1.488***	0.683***	0.625***
	(0.363)	(0.340)	(0.132)	(0.125)
Constant	-0.147	-0.144	-0.0751	-0.0861
	(0.0930)	(0.0928)	(0.0542)	(0.0542)
Observations	144	144	713	713

Notes : Results from the OLS regressions of equation (14). Panels A are for all individuals. The estimates β for forecast revision are thus equal to $E(\frac{1-\lambda_t G}{\lambda_t G})$ and are an estimation of global information rigidities. Panels B are only for people who update. The estimates β for forecast revision are thus equal to $\frac{1-G}{G}$ and are an estimation of information rigidities at the intensive margin only. Standard errors are in parenthesis and ***, ** and * respectively denote significance at 1%, 5% and 10% levels.

To get an unbiased estimate of average information rigidities, we must estimate λ_t at each date and G separately. Moreover, this will allow us to decompose global information rigidities into the

[16] Note that because $\frac{1-x}{x}$ is a strictly convex function on $[0,1]$, we have from Jensen's inequality that $\widehat{\lambda G} = \frac{1}{1+\hat{\beta}} > \frac{1}{1+\beta} = \lambda G$.

two margins. It is very intuitive to get an estimate of λ_t at each date t. Indeed, the share of people updating their expectations is directly observable in the data as long as we are willing to assume that an update is always observable (i.e. that whenever someone revises her forecast, she reports a different forecast than the previous one). Estimated λ_t are reported in figure 1. On average, respectively 76 and 96 percent of people update their expectations every quarter in the ECB SPF and the US SPF, so that the mean durations between two updates are of 3,96 and 3,17 months. These durations are lower than what has been find in the literature that tried to estimate them from global rigidities of information without isolating the two margins of information rigidities. Indeed, if one assumes that information rigidities are not sticky but noisy, she might mistakenly conclude from our first estimate of global rigidities that the average duration between two updates is of five months in the US SPF and seven to eight months in the ECB SPF.[17] However, that would be wrong in as long as information rigidities are both sticky and noisy and it gives a strong argument in favor of a direct calibration of sticky information models on the share of updates, instead of relying on non-direct estimations assuming a priori that information is sticky and not noisy.

To estimate G – the rigidities at the intensive margin –, I estimate equation (14) only on the subsample of people who are updating to get ride of the rigidities at the extensive margin. Results of this estimation are reported in table 3 (Panels B). It shows that when we account only for the intensive margin, β is equal to 1.488 in the ECB SPF and 0.625 in the US SPF. Thus, since we assumed that G is constant over time, we get unbiased estimates for G of respectively 0.40 in the ECB SPF and 0.615 in the US SPF. Even though one cannot statistically reject that information rigidities at the intensive margin are different from the global rigidities estimated by regressing equation (14) on everyone, we see that the estimates of rigidities at the intensive margin are, as one may have expected, lower than the (downward biased) estimates of overall rigidities.[18]

Then, I am now able to compute an unbiased estimate of average global rigidities (denoted $\widehat{\lambda G} = T^{-1}\sum_{t=0}^{T}\hat{\lambda}_t\hat{G}$) from the average share of people updating each period and the estimate of G. Doing so, I obtain $\widehat{\lambda G} = 0.307$ in the ECB SPF and 0.589 in the US SPF, thus confirming that the estimates obtained from regressing equation (14) on all individuals are upward biased and that the bias may be large as seems to be the case in the ECB SPF (one can reject that the unbiased estimator equals the biased one at the 10% in the ECB SPF). Moreover, this two-steps estimation of global rigidities permits to measure rigidities volatility. In the ECB SPF, the standard error of estimated rigidities is 0.04 and $\widehat{\lambda_t G}$ varies from 0.21 in the fourth quarter of 2004 to 0.40 in the first quarter of 2009. In the US SPF, the standard error is 0.04 and rigidities vary from 0.28 to 0.615. Thus in both cases, we see that rigidities may double from one period to another. It is therefore necessary to account for this high variability by studying the two margins of the expectation formation process, and not just the

[17] Coibion and Gorodnichenko [2015] show that with noisy information, $\lambda = \frac{1}{1+\beta}$ and the average duration between two updates is λ^{-1}.

[18] Recall that rigidities are low when G is close to unity.

constant intensive margin.

This empirical exercice underlines two main facts which are absents from standard theories of inattention. First, people are inattentive in the sens of both Sims [2003] and Mankiw and Reis [2002]. Namely, they update their expectations only sporadicly and, when they do update, they don't use all the information available. Second, the expectation formation process is state-dependent because people update more after large economic shocks. However, we find no evidence of a more efficient treatment of information in response to economic shocks. Thus, it confirms the view of an expectation formation process where the extensive margin – i.e. the choice to update – is state-dependent, whereas the intensive margin – i.e. the efforts devoted to collect and process information when an individual updates her expectations – is relatively constant over time and not affected by economic fluctuations.

3 A Behavioral Framework with State-Dependent Expectations

Leading theories of inattention are not able to reproduce the complexity of the human behavior and, more specifically, the fact that attention is a scare ressource that people choose, consciously or not, to allocate according to their perception of the environment and its evolutions. Therefore, I propose in this section a simple model of the expectation formation process at the individual level and apply it to a well-known intertemporal problem in economics: the choice between consumption and savings. The results stress the importance of state-dependent expectations to understand individuals behavior and show that in a simple setting it might lead to tremendous deviations from the standard models with FIRE agents, and even inattentive agents as they are described by Sims [2003] or Reis [2006]. The expectation formation process developed here relies on behavioral rules that will be rationalized in the next section. Thus, this section offers a first setup to understand quite easily the main implications of state-dependent expectations for consumption.

3.1 Dual-Process Theory and the Two Margins of the Expectations Formation Process

It is common in psychology to analyze human thinking by distinguishing automatic decisions to more considered decisions. This behavioral dichotomy is central to the dual-process theory which postulates that there are two systems or minds in one brain. The System 1, or automatic system, is costless to use but performs very poorly (though rapidly) compared to the System 2, or rational system, which is costly and slow, but able to resolve complex tasks.[19] Dual-process theory has been first introduced in behavioral economics by the seminal paper of Kanheman and Tversky [1979] on prospect theory and then spread to the general public thanks to Kahneman's bestseller *Thinking, Fast and Slow* (Kahneman [2011]). When applied to the expectation formation process, the dual-process theory rationalizes the existence of the two margins that have been described in the last sections. Indeed, the extensive margin is somehow driven by an automatic – namely a simple, fast, costless in terms of attention and

[19]Interested readers may find an exhaustive presentation of the dual-process theory in Evans and Frankish [2009].

efforts, and likely unconscious – choice based on beliefs that are driven by stimulus. These stimulus represent the news – from newspapers, TV, cheap-talks, etc. – agents get everyday and that may potentially reflect the economic fluctuations (or shocks). Then, when the System 1 takes the decision to update expectations, the system 2 activates. Thus, individuals are willing to pay more attention and efforts to collect and process information in order to do sophisticated economic tasks, such as to identify the best-response to an intertemporel economic problem. This explains why we have fund that information rigidities at the intensive margin (i.e. when the System 2 is at work) are lower than global rigidities in the previous section.

One of the major difficulties when one tries to assess the validity of the behavior of an individual with a dual-process thinking system is that whenever we think about ourselves, "we identify to the System 2, the self-conscious, who is reasoning, has convictions, makes choices and decides what to think and what to do" (Kahneman [2011]). This may somehow explain why most economists are willing to assume fully-informed rational agents who are as smart and informed as they are when they build their models. However, and even if one is willing to assume that it is not to demanding for everyone's System 2, the capacities of the System 1 are not compatible with such demanding tasks. To illustrate the role of the System 1 in our daily economic decisions, one may think of two meaningful examples. First, consider an individual, let say yourself, who is not constantly checking her bank account. She only has some priors about the evolution of her wealth (for example she knows that she is paid at the end of the month, she has a vague idea of her savings return, of how much she spends for housing and eating, etc.). This individual will decide to consult her bank account or to call her banker in order to get a precise evaluation of her wealth only when she will expect a large evolution in her incomes or expenses. Thus, we see that the choice to allocate time and attention to economic choices depend on decision taken when we are least attentive. Another exemple describes well the behavior of most professional forecasters that have been analyzed so far. Most of the time, these professionals don't pay much attention to economic data and their most recent releases. However, when they start to expect that something huge, good or bad, will append to the economy in the next quarters or years, they start to be much more attentive to these newest releases. For some of them, they will even start to collect all the data needed to reestimate the econometric models they have access to in order to get updated forecasts. Once again, we see here that people choose how much attention they are willing to allocate when they are the least attentive and then, do all their best to solve the economic problems they are facing.

Before to introduce the framework in details, it is necessary to clarify the links between the System 1 and System 2 that are going to be at the heart of consumers' behavior. According to the dual-process theory, I assume that the System 1 is automatic, and therefore not optimizing, whereas the System 2 is strategic, and thus optimizing. Hence, the System 2 behaves as a follower in a Stackelberg equilibrium: it chooses its best-response to an economic problem given the behavioral rule governing the System 1.

Alternatively stated, it means that when people are the most attentive and dedicate all their energy to resolve complex economic tasks, they take into account that they will remain inattentive for a possibly long time, and that this time will depend on their beliefs when they will be the least attentive.

3.2 An Intuitive Framework with State-Dependence at the Extensive Margin

System 2 : Let i denote a consumer living forever in discret time, who consumes c_t each period and whose utility $u(c_t)$ is CARA. This agent discount future utility by the factor β. She earns returns on her assets, w_t, at the gross interest rate $R = \frac{1}{\beta}$ and receives an exogenous stochastic income y_t which is normally distributed with mean $\bar{y}$ and variance σ_y^2. Following Reis [2006], she must pay a monetary cost K whenever she acquires information in order to make optimal decisions. As it has already been discussed in section 1.1, this cost stands for the energy and time devoted to accomplish complex economic tasks. For now, I will assume that the choice to pay K is exogenous and depends on a behavioral rule which reproduces the automatic and unconscious System 1.[20] Moreover, to keep it as intuitive as possible, I will focus only on the extensive margin of the expectation formation process and assume that there are no rigidities at the intensive margin. We are interested in the choice of a consumption path at time t for an individual who updated her information set ($\mathcal{I}_l$) for the last time at period $l \leq t$. Let s_t denote the consumer's wealth, or permanent income, at time t such that $s_t = w_t + \sum_{i=1}^{\infty} E_t\{R^{-i}\ y_{t+i} \mid \mathcal{I}_l\}$, where $\sum_{i=1}^{\infty} E_t\{R^{-i}\ y_{t+i} \mid \mathcal{I}_l\}$ is the expected flow of actualized incomes given the information available at time t and collected in period l. Hence, the consumer solves the following problem at time $t \geq l$:

$$\begin{aligned} \max_{\{c_{t+i}\}_{i=0}^{\infty}} \quad & E_t\left\{ \sum_{i=0}^{\infty} \beta^{t+i} \left(-\frac{e^{-\alpha c_t}}{\alpha} \right) \Big| \, \mathcal{I}_l \right\} \\ \text{s.t.} \quad & s_t = Rs_{t-1} - c_{t-1} + (y_t - \bar{y}) - \iota(t)K \end{aligned} \tag{15}$$

Where $\iota(t)$ is a dummy equal to one when the consumer updates her expectations in t (i.e. in periods such that $t = l$) and $(y_t - \bar{y})$ is the innovation to permanent income at period t.

To clarify the links with the expectation formation process that has been introduced at the begin of this paper, it is essential to understand the main assumptions driving the behavior of the consumer. First, let's focus on her information set. Here, I simply suppose that when the consumer pays the cost K, she observes past incomes. Thus, the information set is the σ-statistics such that $\mathcal{I}_t = \{\{s_i\}_{i=0}^{t}\}$ and the choice to update or not this information set and to recompute the new consumption path is what has been called the extensive margin of the expectation formation process. Therefore, whenever she pays this cost, she gets new information $\mathcal{I}_t \ominus \mathcal{I}_g = \{\{s_i\}_{i=g+1}^{t}\}$ which represents the unpredictable income surprises that occurred between periods g and t and she can compute

[20] In the next section of the paper, I develop an example where the choice of this rule is endogenous and show that the results presented here are not affected.

$\zeta_t = \sum_{u=1}^{\infty} R^{t-g-u}\big(E_t\{y_{g+u} \mid \mathcal{I}_t\} - E_g\{y_{g+u} \mid \mathcal{I}_g\}\big)$ the actualized innovation to permanent income between the new update of the information set in $t = l$ and the update just before at period g. Second, we see that whenever she updates $\mathcal{I}$, she observes perfectly the shocks between periods g and t. This is the consequence of the absence of rigidities at the intensive margin in this setup so that, using her System 2, she may access sufficient cognitive capacities to acquiert and process all the information she needs.

The problem (15) is identical to that of Reis [2006] and its detailed resolution is reported in online appendix C. We can already deduce the consumption dynamics when the consumer doesn't pay the monetary cost K: when the consumer gets no new information about her incomes, she is not going to change her consumption path since it is already optimal. Hence, when $t \neq l$ – i.e. she is not updating $\mathcal{I}$ at time t – it has to be the case with a CARA utility and $R = \beta^{-1}$ that $c_l = c_t$. The main difference in the setup presented here compared to Reis [2006] is that the duration between two updates is stochastic. Thus, a risk-averse consumer would be willing to increase her savings at each date to protect herself against the risk of a long period without updating. However, including the stochasticity of the durations between two updates in the consumer's problem would make the resolution highly intractable because, as we will see, the distribution of these durations is non-standard. Hence, in order to keep this example trackable, I make the strong assumption that the consumer sees the duration between two updates as a deterministic variable which is equal to the ex post average duration. I will relax this assumption in the next section and show that it doesn't alter much savings. Optimal consumption at each date t is therefore[21] :

$$c_t = (R-1)s_l - \frac{(R-1)K}{R^D - 1} - \frac{\alpha\sigma_y^2}{2}\frac{(R-1)(R^D+1)}{R+1} \tag{16}$$

with D the ex post average duration between two updates. The mean duration between updates has two opposite effects on consumption. On the one hand, we see from the second term in equation (16) that an increased duration implies a gain in utility because the individual will pay the cost K less often, but on the other hand it increases precautionary savings (third term) to overcome a higher income risk.

System 1 : After an update, the consumer sticks to her consumption path until she pays another time the cost of updating K and remains partially attentive to the evolution of her incomes during these updates. This lower attention reflects the fact that people devote most of their energy and time to non-economic tasks everyday. In order to introduce inattention, I rely on Sim's [2003] theory of inattention and assume that the consumer is unable to observe her true permanent income between

[21]Note that to obtain this expression for the optimal consumption we need to write s_l as a function of past income surprises and initial conditions s_0 and c_0 for all l. Hence, we must assume that the duration between two updates is finite so that if the consumer consumes more than what her true permanent income allows today, she will have to pay back later when she will update her consumption path (i.e. we impose the transversality condition). As we will see, we do find that the duration between two updates is asymptotically finite.

two updates. Thus, she faces a signal extraction problem identical to the one presented in section 1.2. At each date, she receives a signal on her wealth $s_t^\star = s_t + \xi_t$ where ξ_t is a gaussian white noise with zero mean and variance σ_ξ^2. Following Luo [2008] and Luo et al. [2014], the dynamics of the perceived state variable is for all periods $t > l$:

$$\hat{s}_t = (1-\theta)\ (R\hat{s}_{t-1} - c_{t-1}) + \theta s_t^\star \tag{17}$$

where θ is the Kalman gain when the consumer is the most inattentive. Because the consumer observes the true state when she updates her expectations, the Kalman gain has for initial condition $\hat{s}_l = s_l$. Using equation (17), the fact that from equation (16) consumption is constant between two updates, the history of income surprises $\{\zeta_{l+j}\}_{j=1}^n$ and of idiosyncratic shocks $\{\xi_{l+j}\}_{j=1}^n$, the perceived state n periods after the last update is:

$$\begin{aligned}
\hat{s}_{l+n} &= R^n s_l - \sum_{i=0}^{n-1} R^i c_l + \theta \left[\sum_{i=0}^{n-1} \big[(1-\theta)R\big]^i \xi_{l+n-i} + \sum_{i=0}^{n-1} \sum_{j=0}^{i} (1-\theta)^j R^i \zeta_{l+n-i} \right] \\
&= s_l + \frac{1-R^n}{1-R} B + \frac{1-\big[(1-\theta)R\big]^n L^n}{1-\big[(1-\theta)R\big]L}\ \theta \xi_{l+n} + \sum_{i=0}^{n-1} \left[1-(1-\theta)^{i+1}\right]\ R^i L^i \zeta_{l+n}
\end{aligned} \tag{18}$$

where $B = \frac{(R-1)K}{R^D-1} + \frac{\alpha\sigma_y^2}{2}\frac{(R-1)(R^D+1)}{R+1}$ and L is the lag operator.

In as long as the System 1 is automatic, the consumer is not looking here for the best consumption path given her beliefs, but only trying to derive a simple heuristics to decide whether it would be beneficial to pay the cost K to observe the true state s_t and then deduce a new optimal consumption path through her System 2. For simplicity, I assume that the heuristic rule which drives the choice to update is given by:

$$(\hat{s}_{l+n} - s_l)^2 \geq \rho(K) \tag{19}$$

with ρ a function such that $\rho\colon \mathbb{R}^+ \to \mathbb{R}^+$, $\rho' > 0$ and $\rho(0) = 0$. The choice of this heuristics reproduces the consumer's arbitrage between the utility gains and loses associated to an update. On the one hand, we expect people to be willing to update when they think they are far away from what would have been their optimal path given their beliefs, and this willingness is likely to be non-linear in $|\hat{s}_t - s_l|$. On the other hand, when the monetary cost K of updating is high, they want to update less often. Furthermore, we see that with this heuristics, the model nests the standard FIRE permanent income model as a special case when $K = 0$ and the consumer updates her consumption at each date. To asses whether the heuristics is rational from the consumer's point of view, I will make the choice of $\rho(K)$ endogenous in the next section.

Using the expression of $\hat{s}_{l+n}$ in equation (17), the behavioral rule driving the System 1 becomes:

$$\Big| \underbrace{\frac{1-R^n}{\theta(1-R)}B}_{\Upsilon} + \underbrace{\sum_{i=0}^{n-1} \big[(1-\theta)R\big]^i \xi_{l+n-i}}_{\Omega} + \underbrace{\sum_{i=0}^{n-1} \sum_{j=0}^{i} (1-\theta)^j R^i \zeta_{l+n-i}}_{\Theta} \Big| \geq \frac{\sqrt{\rho(K)}}{\theta} \tag{20}$$

From (20) one may easily identify the three forces driving the choice to update. First, Υ are the returns on savings made since the last update n periods ago. We see that the more the consumer waits between two updates, the more these returns grow and the more the consumer is willing to pay the cost K to use them to increase her consumption. Second, the term Ω stands for the actualized idiosyncratic shocks the consumer wrongly perceives as economic shocks. Finally, Θ are the true actualized income surprises. When the sum of these two actualized shocks is large (in absolute value), the consumer thinks that it would be optimal for her to get a better information on the true shocks so that she can update her consumption path.

Duration between two updates: From equation (19), we know that the consumer will update whenever $(\hat{s}_t - s_l)^2 \geq \rho(K)$. Hence, we can derive the conditional probability of an update n periods since the last update knowing that no update occurred in between, denoted $\mathcal{P}(n)$, by substituting s_{l+n} from its expression in equation (18) to obtain:[22]

$$\mathcal{P}(n) \quad = \quad 1 - \Phi\Bigg(\frac{\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})}{\sqrt{\sigma_\xi^2 + \sigma_y^2}}\Bigg) + \Phi\Bigg(- \frac{\theta^{-1}\sqrt{\rho(K)} + G(\mathcal{H}_{n-1})}{\sqrt{\sigma_\xi^2 + \sigma_y^2}}\Bigg) \tag{21}$$

where $\mathcal{H}_n = \{s_l, \{\zeta_{l+j}\}_{j=1}^n, \{\xi_{l+j}\}_{j=1}^n\}$, $\Phi(.)$ is the cdf of $\mathcal{N}(0,1)$ and $G(\mathcal{H}_{n-1}) = \frac{1-R^n}{\theta(1-R)}B + \big(\sum_{i=1}^{n-1} \big[(1-\theta)R\big]^i \xi_{l+n-i} + \sum_{i=1}^{n-1} \sum_{j=0}^{i} (1-\theta)^j R^i \zeta_{l+n-i}\big)$. Equation (21) indicates that when the sum of the idiosyncratic shock and income surprise at period $l+n$ is larger than $\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})$ or lower than $-\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})$, then the consumer pays the cost K. Moreover, because income surprises and idiosyncratic shocks are i.i.d., the unconditional probability of an update at period $l+n$ is:

$$\mathbf{P}(n=N) \quad = \quad \mathcal{P}(N) \prod_{k=1}^{N-1} \big(1 - \mathcal{P}(k)\big) \tag{22}$$

Note that when $K = 0$, $\mathbf{P}(n=1) = 1$ and we are back to the standard permanent income model with FIRE. The average duration between two updates is $D = \sum_{k=1}^{\infty} \mathbf{P}(n=k)\ k$ where the probabilities $\mathbf{P}(n=k)$ are also functions of D. Therefore, strangely enough, to derive the distribution of durations, we are looking for a fixed point of the first moment of this distribution. This however is economically quite intuitive: on the one hand, the consumer knows that she will remain inattentive for a possibly long time, and prefers to increase her precautionary savings when D increases; however, on the other

[22] See online appendix C.2 for the derivation of this expression.

hand, she receives a return on these savings and wants to reinvest it into consumption as soon as possible. Therefore, the solution of this model has to be a solution of $D = \sum_{k=1}^{\infty} \mathbf{P}(n = k)\ k$ where the probabilities $\mathbf{P}(n = k)\ \forall k$ are given by equation (22).

Proposition 1. *Let $D \in \mathbb{D}$ where $\mathbb{D}$ is a nonempty, compact and convex set. Then, since $\sum_{k=1}^{\infty} \mathbf{P}(n = k)k$ is the sum of continuous functions in D, we know from Brouwer's theorem that $D = \sum_{k=1}^{\infty} \mathbf{P}(n = k)k$ has at least one fixed point.*

Proof. The set $\mathbb{D}$ of durations between two updates is a nonempty and convex subset of $\mathbb{R}^{\star}$. Moreover, since $\lim_{k\mapsto\infty} \mathbf{P}(n = k) = 0$, we can assume a priori that $\infty \notin \mathbb{D}$ so that $\mathbb{D}$ is bounded and closed (i.e. compact). □

Proposition 2. *Let $D = \sum_{k=1}^{\infty} \mathbf{P}(n = k)k$ be a mapping $T : \mathbb{D} \to \mathbb{D}$. Then, if T is a contraction on a non-empty metric space $(\mathbb{D}, d)$, from Banach Fixed Point Theorem T has an unique fixed point that may be fund with a fixed point iteration algorithm.*

In order to solve the model, I arbitrary set $\rho(K) = \left(K + \rho\ (\sigma_\xi + \sigma_y)\right)^2$ for now. The next section will enable the consumer to optimally choose this rule. The constant term ρ allows to calibrate the average duration between two updates. Given the complexity of the model, I shall calibrate it and use simulations to find a solution. I set $K = 30\$$ as in Reis [2006]. Following Pischke [1995], I set the average quarterly revenu to $\bar{y} = 6926\$$ with a standard error $\sigma_y = 1960\$$, $\alpha = \frac{2}{6926}$ and $\beta = 0.95$ so that the consumer represents an american household. The Kalman gain[23] is $\theta = 0.148$. Finally, $\rho = 0.4$ so the average duration between two updates is of 2.67 quarters in my model, a result similar to the estimation of Dräger and Lamla [2013] who find that on average consumers updated their one-year-ahead inflation expectations every 8 months in the Michigan survey. To compute the distribution of durations, I simulate 20 000 series of idiosyncratic and revenu shocks over 100 periods. For each of these series, I use the model to evaluate the number of periods it takes before the consumers choose to pay the cost K for the first time given an exogenous average duration D_0. Then, I simulate the distribution and compute its first moment D_1. If $D_1 \neq D_0$, I recompute the distribution using D_1. I iterate this process until $D_{i+1} = D_i$. The simulated distribution is reported in figure 2 in appendix.

3.3 Consumption Dynamics

Using equations (16) and (15), the consumption dynamics at the individual level is:

$$c_{l+n} - c_l = \begin{cases} 0 & \text{if } l + n < l' \\ (R-1)\zeta_{l'} & \text{if } l + n = l' \end{cases} \tag{23}$$

[23]Following Luo et al. [2014] it corresponds to an information channel capacity of $\kappa = 0.42$ nats and $\sigma_\xi = \sqrt{\frac{\sigma_y^2}{\exp(2\kappa) - R^2}} = 1787.5$.

where l and l' are two successive updating dates and $\zeta_{l'}$ the actualized income surprises between l and l' such that $\zeta_{l'} = \sum_{j=1}^{\infty} R^{-j+(l'-l)}\big(E_{l'}\{y_j \mid \mathcal{I}_{l'}\} - E_l\{y_j \mid \mathcal{I}_l\}\big)$. Thus, the state-dependence of the extensive margin of the expectation formation process affects consumption dynamics at the individual level. To study how optimal consumption is affected by this state-dependence, I compare the consumption path of the consumer in the framework developed in the last section to three benchmark models widely used in the literature: (i) a permanent income model with full-information rational expectations (FIRE), (ii) with inattentive consumers who face information constraints as in Sims [2003] and Luo et al. [2014], (iii) and inattentive consumers who face a cost to update their expectations and consumption and choose ex ante how many periods they want to remain inattentive as in Reis [2006]. Table 4 classifies these models according to their hypothesis on the expectation formation process and recall the predicted dynamics of individual consumption. Note that compared to the existing literature on inattention, the definition of a state-dependent process here is somehow different: by state-dependence, I mean that information rigidities may change even though the structural parameters of the income process remain constants.

Table 4: Classification of Rational Inattention Models

Models	Expectation Formation Process		Consumption Dynamics
	Extensive margin	Intensive margin	
FIRE	none	none	$\Delta c_{t+1} = (R-1)\zeta_{t+1}$
Sims [2003] and Luo et al. [2014]	none	state-independent	$\Delta c_{t+1} = (R-1)\widehat{\zeta_{t+1}}$
Reis [2006]	state-independent	none	$\Delta c_{t+1} = \begin{cases} 0 \\ (R-1)\zeta_{l'} \end{cases}$
State-dependent framework	state-dependent	none	
Data from the US and ECB SPF	state-dependent	state-independent	

NOTES: where $\widehat{\zeta_{t+1}} = G\left[R\left(\frac{1-G(\zeta_t+\xi_t^g)}{1-(1-G)RL}\right) + \zeta_{t+1} + \xi_{t+1}^g\right]$ is the innovation to the imperfect perception of the state variable in Luo et al. [2014], G the Kalman gain and ξ_t^g the idiosyncratic shocks at the intensive margin.

To ensure that the results from the different models remain comparables, I set $\rho = 0.72$ so the population average duration between two updates in the state-dependent framework is of 5 quarters, a duration similar to the theoretical prediction of Reis [2006] with my calibration.[24] The information capacity constraint in the model of Luo et al. [2014] is set to 0.42 nats and consequently the Kalman gain at the intensive margin G is identical to the Kalman gain at the extensive margin θ used in my model.[25] Figure 3 reports an example of different consumption paths predicted by these rational inattention (RI) models. Whereas consumption in the FIRE and Sims [2003]'s RI models is continuous,

[24] In Reis [2006] the duration between two updates is

$$\tilde{D} = \frac{\ln\left(1 + \sqrt{\frac{2R^2(R+1)K}{\alpha\sigma_y^2}}\right)}{\ln(R)} \tag{24}$$

[25] A similar gain at the intensive and extensive may not be reconciled with an interpretation in terms of the Dual-process theory (System 1/System 2). Nevertheless, it allows here to ensure that the differences across the different models come from the expectation formation process, and not from differences in capacities to collect and interpret information.

it encounters regular jumps in models with an extensive margin of the expectation formation process. These jumps are equally spaced in time with Reis' model while, in my setup with state-dependent expectations, they are stochastic and may vary a lot. For example, when income shocks are relatively low, the consumer may rationally choose not to pay the cost K for a long time so that her consumption remains constant.[26] Hence, even though both models include an extensive margin, the implications of a time-dependent versus a state-dependent process are easy to understand here: in Reis' model, it is assumed that people choose ex ante how long they want to remain inattentive. This behavior may be rational if and only if individuals have no access to any information through their System 1 – i.e. when they are the least attentive. Otherwise, it would be optimal for them to deviate from their ex ante commitment to remain inattentive during D periods either because they expect income shocks to be large and want to update their consumption now, or because they think they were smalls and consequently it would be a waste to spend K.

A new feature of the state-dependent model developed in section 3.2 is that the response of consumption to an income shock is non-linear. Given an history of past true and perceived shocks ($\mathcal{H}_{t-1}$), an income shock will not prompt the consumer to deviate from its consumption plan if it is small enough, whereas, if this shock is large, the consumer will choose to pay the cost K and compute its new optimal consumption path. Moreover, a same shock may have opposite effects on consumption depending on past income shocks history ($\mathcal{H}_{t-1}$): if the actualized sum of perceived shocks since the last update is large (in absolute value), then the consumer will choose to update her expectations and consumption path, even though this same shock wouldn't have prompted her to update with another history ($\mathcal{H}'_{t-1}$) such that $G(\mathcal{H}'_{t-1}) < G(\mathcal{H}_{t-1})$. As shown in online appendix D, these non-linearities affect also the dynamics of aggregate consumption. This is a novel feature which may be of interest for economic policies: e.g. as people become more attentive in recession times, the fiscal multiplier might be lower;[27] and conversely, the government may communicate to change people's beliefs in order to enhance the impact of a policy. These questions are left opened for future researches.

4 A Rational Model of State-Dependent Attention

In the framework of section 3.2, the consumer is not rational in the sens that the choice to update is not governed by an optimal rule, but by an exogenously fixed rule. In the present section, I allow the consumer to endogenously choose the rule driving her System 1. Moreover, I relax the assumption that she sees the duration between two updates as a deterministic process so that she can increase her precautionary savings to insure herself against the risk of a long period without changing her consumption path. The results presented so far are only marginally affected by these changes.

[26]See for example the dynamic of consumption between the 20th and the 40th periods on figure 3.
[27]Assuming that the ricardian equivalence holds.

4.1 A General Framework

The distribution of durations is non-standard and therefore it seems reasonable to suppose a priori that individuals may not be aware of the exact distribution, but just of its first moment. However, one may argue that, because we are studying a repeated game with an infinitely living agent, the consumer may learn the distribution of durations from her own experience and from others. Thus, she will be able to estimate some higher moments of this distribution and, because she is risk-averse, to adapt her behavior in order to protect herself against greater uncertainty. Moreover, assuming that agents are able to learn this distribution, we can also imagine that they will be able to infer how their behaviors affect this distribution. Indeed, as we have seen, the distribution of durations is a fonction of savings and the rule driving the System 1. Hence, a super-consumer with very strong optimizing skills will be able to optimally select her best-response taking into account that the distribution of durations is a function of these two components. Formally, this super-consumer solves:

$$\begin{aligned} \max_{\{c_{t+i}\}_{i=0}^{\infty},b} \quad & E_t\Big\{ \sum_{i=0}^{\infty} \beta^{t+i} \Big(- \frac{e^{-\alpha c_t}}{\alpha}\Big)\Big| \, \mathcal{I}_l \Big\} \\ \text{s.t.} \quad & s_t = Rs_{t-1} - c_{t-1} + \iota(t)\big(\zeta_t - K\big) \\ & (s_l - \hat{s}_t)^2 \geq b \qquad \Longrightarrow \; t = l' \end{aligned} \tag{25}$$

where the second constraint implies that when $(s_l - \hat{s}_t)^2 \geq b$, she updates her expectations (i.e. $t = l'$). The link between this second constraint and the maximization problem is not obvious in (25) and will be explicitly exposed later. However, knowing the results from the last section, we expect that the upper bound b affects the probability of an update at date t. Thus, it changes the consumer's intertemporal utility through the expectation operator E_t. One may make the resolution of this problem simpler using the method of undetermined coefficients. I make the educated guess that the optimal consumption at each period $l \leq t < l'$ takes the form $c_t = (R-1)s_l + a$ where a is the sum of precautionary savings and the decrease in consumption due to the cost K. Therefore, we expect $a < 0$. From the consumer, it is now equivalent to choose $\{c_{t+i}\}_{i=0}^{\infty}$ or a. At date t, the Euler equation writes $\exp(-\alpha c_t) = E_t\big\{ \exp(-\alpha c_{t+n}) | \, \mathcal{I}_l\big\}$ and $c_t = c_{t+n}$ as long as $t + n < l'$. Using the guess for c_t and the budget constraint, I rewrite the Euler equation for two successive updating dates l and $l' = l + d$ as :

$$\exp(-\alpha c_l) \;\; = \;\; E_l\Big\{ \exp\Big(- \alpha[(R-1)(s_l - \frac{1-R^d}{1-R}a + \zeta_{l,d} - K) + a]\Big)\Big| \, \mathcal{I}_l \Big\} \tag{26}$$

with $\zeta_{l,d}$ the actualized income surprised since the last update at period l such that $\zeta_{l,d} = \sum_{i=0}^{d} R^{d-i}\zeta_{l+i}$. Furthermore, since $\zeta_t \rightsquigarrow \mathcal{N}(0, \sigma_y^2)$ and is i.i.d across time, we know that $\zeta_{l,d} \rightsquigarrow \mathcal{N}(0, \sum_{i=0}^{d} R^{2(d-i)}\sigma_y^2)$. From this last expression, it is clear that when d, the duration between l and l', increases, the income risk from not updating increases to. If F_{ζ_d} denotes the cdf of $\zeta_{l,d}$, the Euler equation becomes:

$$\exp(-\alpha c_t) = \sum_{D=1}^{\infty} \mathbf{P}(d=D) \int_{-\infty}^{\infty} \exp\Big(-\alpha\big[(R-1)(s_l + \zeta_{l,d} - K) + (2-R^d)a\big]\Big)\, dF_{\zeta_d}(\zeta_{l,d}) \tag{27}$$

where $\mathbf{P}(d=D)$ is the unconditional probability of an update after D periods from equation[28] (22). Furthermore, we know that $E\big[(R-1)(s_l + \zeta_{l,d} - K) + (2-R^d)a\big] = (R-1)(s_l - K) + (2-R^d)a$ and $Var\big[(R-1)(s_l + \zeta_{l,d} - K) + (2-R^d)a\big] = (R-1)^2\sigma^2_{\zeta_d}$. Next, using that for a Gaussian random variable X and a constant z, $E(e^{zX}) = e^{zE(X)+\frac{z^2}{2}Var(X)}$. The Euler equation finally writes :

$$\exp(-\alpha c_t) = \sum_{D=1}^{\infty} \mathbf{P}(d=D) \exp\Big(-\alpha\big[(R-1)(s_l - K) + (2-R^d)a\big] + \frac{\alpha^2}{2}(R-1)^2\sigma^2_{\zeta_d}\Big) \tag{28}$$

Proposition 3. *A solution to the consumer's problem (25) is a duo $\{a^\star, b^\star\}$ such that,*

$$\begin{aligned}
c_t &= -\alpha^{-1}\ln\Big[\sum_{D=1}^{\infty} \mathbf{P}(d=D)\exp\Big(-\alpha\big[(R-1)(s_l-K) + (2-R^d)a^\star\big] + \frac{\alpha^2}{2}(R-1)^2\sigma^2_{\zeta_d}\Big)\Big] \\
\mathbf{P}(d=D) &= \mathcal{P}(d)\prod_{k=1}^{d-1}\big(1-\mathcal{P}(k)\big) \\
\mathcal{P}(d) &= 1-\Phi\Big(\frac{\theta^{-1}\sqrt{b^\star} - G(\mathcal{H}_{n-1})}{\sqrt{\sigma^2_\xi + \sigma^2_y}}\Big) + \Phi\Big(-\frac{\theta^{-1}\sqrt{b^\star} + G(\mathcal{H}_{n-1})}{\sqrt{\sigma^2_\xi + \sigma^2_y}}\Big) \\
G(\mathcal{H}_{d-1}) &= \frac{1-R^d}{\theta(1-R)}a^\star + \Big(\sum_{i=1}^{d-1}\big[(1-\theta)R\big]^i \xi_{l+d-i} + \sum_{i=1}^{d-1}\sum_{j=0}^{i}(1-\theta)^j R^i \zeta_{l+d-i}\Big)
\end{aligned}$$

and the initial guess holds ex post: $c_t = (R-1)s_l + a^\star$.

4.2 Cost to Update, Savings and the Choice to Update

I solve the model numerically with the following three-steps algorithm: (i) I arbitrary choose a duo $\{a_0, b_0\}$ and compute the distribution of durations. (ii) Then, I compute the Euler equation to find c_t. (iii) Finally, I check if the initial guess holds. When it holds, the duo $\{a_0, b_0\}$ is a solution to the consumer's problem and I restart the algorithm with another duo $\{a_0', b_0'\}$. When it doesn't hold, I restart the algorithm using the duo $\{a_1, b_0\}$ where a_1 is the a that would have been a solution to the initial guess at the last step. Thus, for a given vector B of b's, I find a unique vector of a's such that for each b_i, a_i solves the equations in proposition 3. The optimal solution is the one such that $\{a^{opt}, b^{opt}\} = \arg\max_{\mathbf{S}} c_t$ where $\mathbf{S}$ is the set of solutions $\{b_i, a_i\}$. Figure 4 reports the solutions for different values of the cost to update K and gives an intuition of how they are obtained with this algorithm. When the cost to update is 30$ per quarter, as in Reis [2006], the super-consumer behaves

[28] Now we have $\rho(K) = b$.

closely to the one introduced in the less sophisticated model of the previous section.[29] He will rationally chooses $a^\star = -40.6\$$ and $b^\star = 4000$ such that the average duration between two updates is a year.

However, when the consumer accounts for the stochasticity of durations between two updates, she is willing to increase her savings to insure herself against this greater uncertainty. The figure 5 reports the policy function of a as a function of the average duration between two updates. The decrease in consumption a is systematically larger when the consumer internalizes the behavior of her System 1. Hence, the state-dependence of the expectation formation process does not only affect consumption dynamics, but also the level of consumption each period. The sparser is the distribution of durations, the larger are the precautionary savings. Another interesting feature of the policy function in figure 5 emerges: the optimal response of a to the average duration is concave. Indeed, if a consumer prefers to update very often, she will choose a low threshold b and thus will not need a lot of precautionary savings. However, a low b implies in return that she will have to pay the cost K very often so that her wealth will decrease a lot in fine. Hence, the consumer faces an arbitrage between the costs in utility of the precautionary savings and K. This arbitrage is central to the consumer and guides her choice of the optimal threshold b, and therefore the distribution of durations in this model. Finally, the results also indicate that even when the cost to update is small, the consumer would prefer not to update her expectations every periods. For example, if K is equal to 20$, the consumer will rationally choose a threshold b such that she updates her expectations every two quarters on average. In comparaison to the time-dependent model of Reis [2006], the average duration between two updates is however systematically smaller (figure 6) in the state-dependent model presented in this paper. This may suggest that with state-dependent expectations, the cost to acquire and process information must be relatively large to reproduce the rigidities fund in the data.

Conclusion

Reis [2006] developed a Noisy Information model in response to Tobin's critics (Nobel Lecture, 1989) : "Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer except when extraordinary events compel revisions. It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous. But at present, models of such realism seem beyond the power of our analytical tools."

This paper goes one step further by developing an expectation formation process where the duration between two updates of expectations is an endogenous process determined by past and current economic shocks – thus taking explicitly in account the exception for "extraordinary events" from Tobin's critics. It may be rationalized by the fact that, even though people pay little attention to their

[29] Recall that in the last section a was equal to -34.8$ and b was 4958 so that the average duration between two updates was equal to 5 quarters.

economic tasks on a daily basis, they are still able to form beliefs from the news communicated through newspapers, TV news, and cheap talks. Thus, when they feel that it would be beneficial for them, they are willing to allocate all their attention to better interpret information and resolve complex economic tasks. Therefore, the expectation formation process behaves like a dual-process similar to Kahneman [2011]'s System 1/ System 2.

The second question treated in this paper is the link between the business cycle and the expectation formation process. The data clearly indicates that information rigidities decrease after a shock. More specifically, I show that this decrease comes from the increased share of people updating their expectations, and not from a better use of information. Thus, it seems urgent to account for the state-dependence of expectations. To illustrate that the state-dependence of expectations is critical to understand economic behaviors, I analyse a well-known economic model: the Permanent Income Model. Even in this simple setup, we see that the introduction of state-dependent expectations results in major deviations from standard results when consumers have full-information rational expectations, but also when they are inattentive as modeled by Sims [2003] and Reis [2006]. More specifically, consumers face more uncertainty and shall increase their precautionary savings. Also, and more importantly, they react differently after an unexpected income shock: if this shock is large enough, they want to update their expectations and incorporate this news to increase consumption today; however, if they feel that this shock was not so important for them, they are not going to update their consumption path. Hence, consumers' behavior become highly non-linear because their response to an income shock depends on their beliefs about the size of this shock. From this example, we expect that the role of state-dependent expectations for fiscal and monetary policies might be essential, and worth being studied thoroughly.

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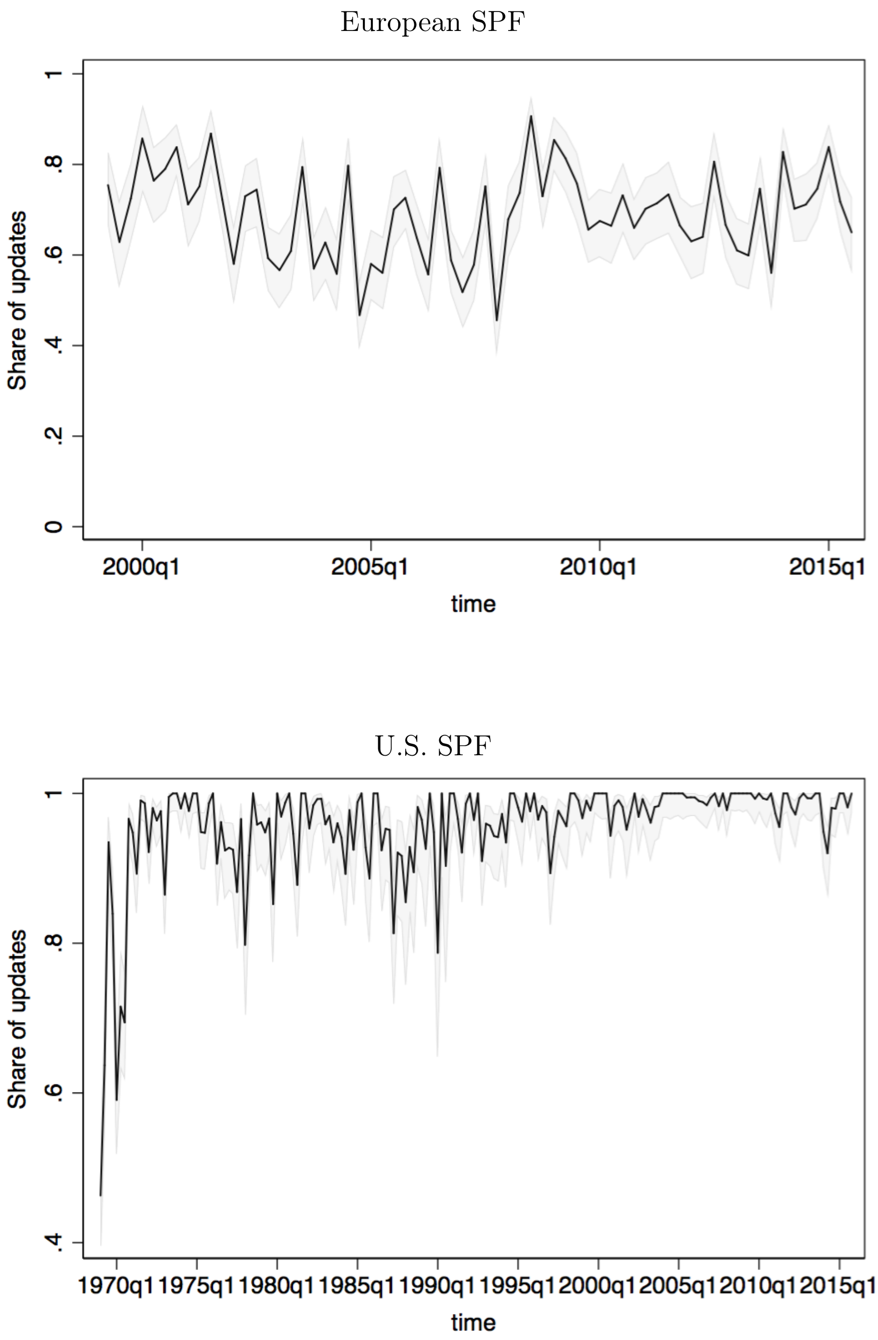
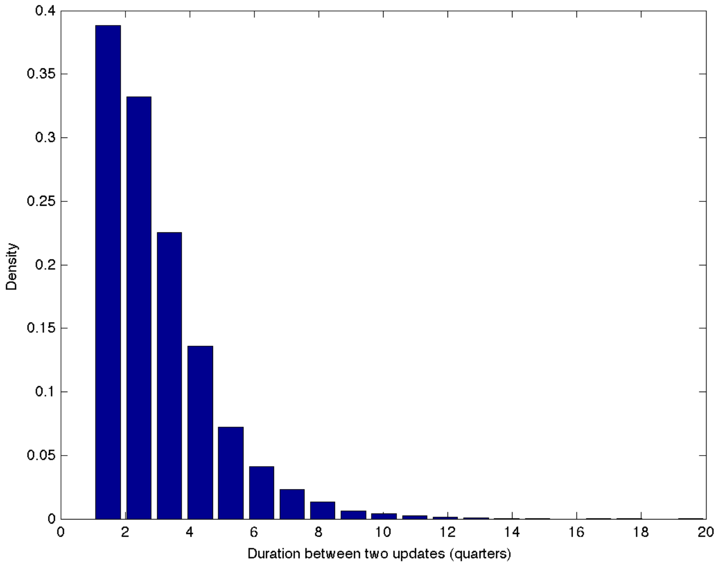
Figure 1: Shares of Updates



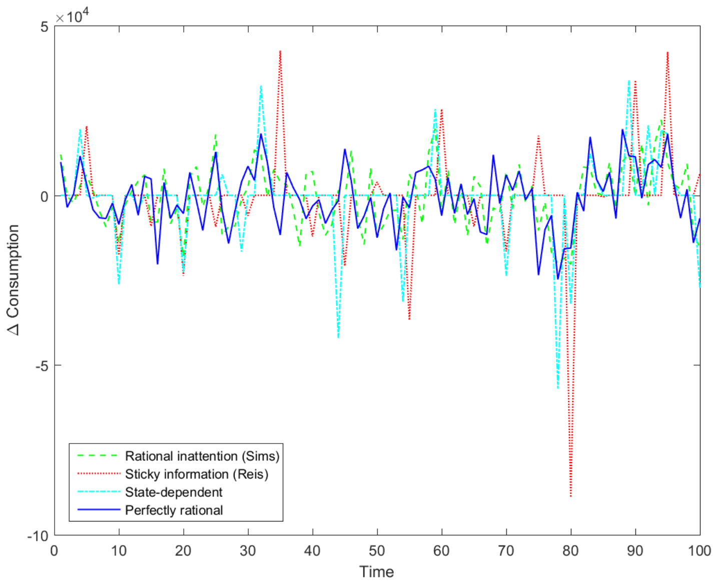
Note: Shares of forecasters updating their forecasts from one period to the next (λ_t) in the European Survey of Professional Forecasters (SPF) run by the European Central Bank (top panel) and the American SPF run by the Federal Reserve Bank of Philadelphia (bottom panel). The grey area is the 95% confidence interval computed as a Wilson score interval.

Figure 2: Distribution of Durations Between Two Updates



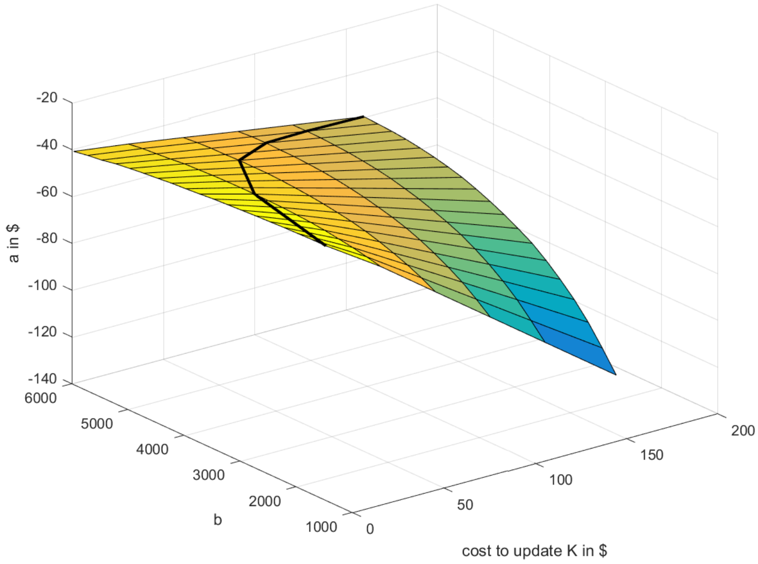
NOTE: This distribution was simulated for 20 000 series of random idiosyncratic and income shocks for the model introduced in section 3.2.The average duration is of 2.65 quarters and the standard error is 1.81.

Figure 3: Consumption Dynamics in RI Models



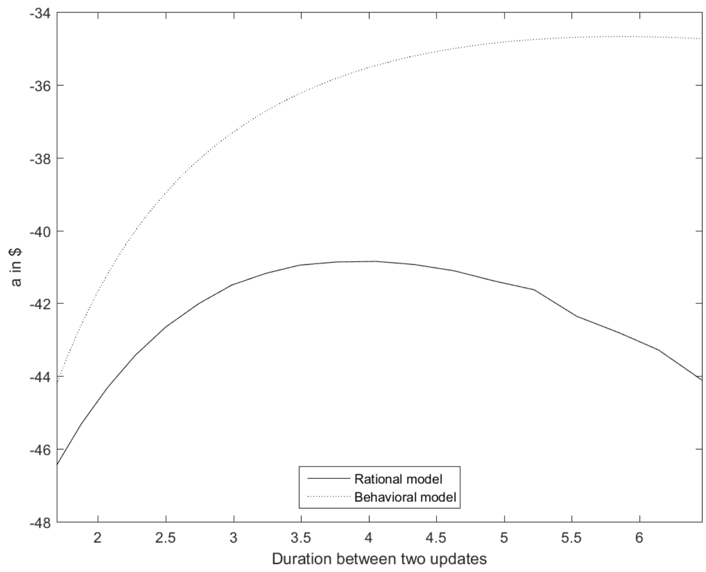
NOTE: Simulated consumption paths for different models of inattention. The revenu and idiosyncratic shocks are the same across models. Thus, the differences in consumption dynamics reflect disagreements about the expectation formation process.

Figure 4: Cost to Update and Optimal Behavior



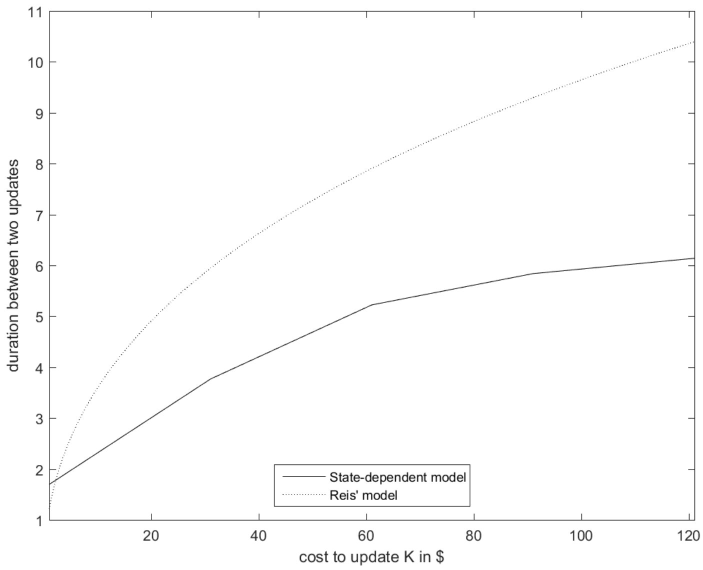
Note: Set of all possible solutions $\{a, b\}$ for different values of the cost to update K. The black line shows the subset of these solutions which maximize the intertemporal utility of the consumer. The algorithm used to obtain these solutions proceeds as follow: (i) for a given value of K and b it looks for the a which solves the equations of proposition 3. (ii) The optimal solution for a given K is then simply the couple $\{a, b\}$ that maximizes the intertemporal utility of the consumer (i.e. the black line).

Figure 5: Precautionary Savings



Note: Values of a in the optimal consumption $c_t = (R-1)s_l + a$ obtained from the Behavioral Framework in section 3 (dotted line) and the Rational Model in section 4 (plain line) for different average durations between two updates.

Figure 6: Cost to Update and Duration Between Two Updates



NOTE: Optimal average duration between two updates in the Rational Model of section 4 and in Reis [2006]'s model with time-dependent expectations.

Table 5: Information Rigidities at the Intensive Margin

	(1) ECB SPF	(2) US SPF
$z^{\text{Prod}}_{t-g(i)-1}$	0.176***	10.17***
	(0.0183)	(2.144)
$z^{\text{Oil}}_{t-g(i)-1}$	0.0237***	0.0292***
	(0.00181)	(0.00295)
$z^{\text{Und}}_{t-g(i)-1}$	0.174***	29.58***
	(0.0104)	(1.803)
Constant	-0.0496***	-0.0480***
	(0.00620)	(0.0104)
Observations	4102	4234

NOTE: Results from the OLS regression of equation (13). The variables $z^{\text{Prod}}_{t-g(i)-1}$, $z^{\text{Oil}}_{t-g(i)-1}$ and $z^{\text{Und}}_{t-g(i)-1}$ are respectively the productivity, oil price and undefined shocks one period before individual i updated her expectations for the last time before t. The sample contains only individuals who are updating their forecasts. Regression (1) is for the survey of professional forecasters from the European Central Bank and regression (2) is for the survey of professional forecasters from the Federal Reserve Bank of Philadelphia. We can strongly reject the FIRE hypothesis since all coefficients are significant at the 1% level. Standard errors are in parenthesis and ***, ** and * respectively denote significance at 1%, 5% and 10% levels.

Online Appendix

Not for publication

A Stuctural VARs for Productivity Shocks

Productivity shocks are estimated using the methodology of Gali [1999]. Estimations are made separately for the US and the Euro zone.

For the US, I use the log of seasonally adjusted monthly hours worked in non farming sectors from the Current Employment Statistics survey. Quarterly data are the average across months. The productivity index is the difference between the log of the quarterly real GDP (seasonally adjusted) and the log of hours worked. The bivariate structural VAR of order 4 is on the differentiated series. The identifying assumption is that only technology shocks may affect productivity in the long run.

For the Euro zone, I concentrate only on the Euro area 19 fixed composition (see the definition from the ECB to get a list of all 19 countries). Data on hours worked are for all activities and seasonally adjusted. For productivity, I use the index directly released by the ECB. The structural VAR is of order 5 on the productivity and the differentiated log of hours. The identifying assumption is the same as for the US.

B Shocks and Updates at the Micro Level

In this appendix I report the results from regressions of the individual choice to update on economic shocks. One may see this exercice as a robustness check for the analysis presented in section 2.2 at the agregate level.

Let's focus on the choice to update expectations. From section 1.1, we may expect that this choice depends on present beliefs about recent economic shocks. Let $b_{i,t}$ denotes these beliefs at period t. If beliefs are additive, we have $b_{i,t} = z_t + \omega_t$ where $\omega_{i,t}$ is a white noise with mean zero and z_t is the true shock. Thus, one may express the latent variable $W_{i,t}$ such that $Y_{i,t} = 1$ if $W_{i,t} \geq 0$ and zero otherwise as:[30]

$$\begin{aligned} W_{i,t} &= a + \beta b_{i,t} + \varepsilon_t \\ &= a + \beta z_t + \beta \omega_{i,t} + \varepsilon_t \end{aligned} \tag{29}$$

Therefore, as long as ω and ε are independent white noises, one may estimate equation (29) directly on z_t and get an unbiased estimate of β. Moreover, the impact of economic shocks may be non-linear on the latent variable: the marginal gain to update may be larger when the shocks are large. To account for this possibility, I introduce a quadratic term for beliefs and get:

$$\begin{aligned} W_{i,t} &= a + \beta_1 b_{i,t} + \beta_2 b_{i,t}^2 + \varepsilon_t \\ &= a + \beta_1 z_t + \beta_2 z_t^2 + 2\beta_2 z_t \omega_{i,t} + \beta_1 \omega_{i,t} + \beta_2 \omega_{i,t}^2 + \varepsilon_t \end{aligned} \tag{30}$$

[30] The notation is identical to section 1.1.

Even though ω is not observable in the data, it is a white noise with zero mean such that $E(z_t\omega_{i,t}|z_t) = 0$. Therefore, standard estimates of β_1 and β_2 are unbiased when the estimation equation is:

$$W_{i,t} = a + \beta_1 z_t + \beta_2 z_t^2 + \nu \tag{31}$$

where $\nu = 2\beta_2 z_t\omega_{i,t} + \beta_1\omega_{i,t} + \beta_2\omega_{i,t}^2 + \varepsilon_t$ is a white noise with zero mean. Thus, when $\nu \rightsquigarrow \text{Logistic}(0,1)$, the model reduces to a logistic regression on $Y_{i,t}$. Table 6 displays the results from these logistic regressions for the US and the European SPF. Complementary regressions with fixed-effects and a control for the horizon of the forecast are also included. The fixed-effects regression controls for possible time-invariant idiosyncratic characteristics that may be correlated with z, whereas the quadratic polynomial for the forecast horizon aims to control for the effect of horizon on the probability of an update that have been identified by Andrade and Le Bihan [2013] and Dräger and Lamla [2013].

Table 6: Updates and Shocks at the Micro Level

	ECB SPF			US SPF		
Shocks	(1)	(2)	(3)	(4)	(5)	(6)
Productivity	-0.188**	-0.199**	-0.184*	-9.165	21.96	21.71
	(0.0883)	(0.0988)	(0.0983)	(14.13)	(22.95)	(23.20)
Productivity (quadratic)	1.007***	1.028***	1.025***	3433.8	-355.5	-293.6
	(0.137)	(0.148)	(0.149)	(2195.9)	(3576.5)	(3615.9)
Oil	-0.127***	-0.141***	-0.151***	0.296***	0.185*	0.186*
	(0.0251)	(0.0296)	(0.0298)	(0.0642)	(0.108)	(0.108)
Oil (quadratic)	0.0124***	0.0136***	0.0145***	-0.0172***	-0.0124	-0.0125
	(0.00255)	(0.00298)	(0.00299)	(0.00567)	(0.00885)	(0.00899)
Undefined	0.0398	0.0416	0.0309	-0.103	-0.0750	-0.0749
	(0.0463)	(0.0506)	(0.0509)	(0.0730)	(0.106)	(0.108)
Undefined (quadratic)	0.162***	0.174***	0.168***	0.0119	0.158*	0.159*
	(0.0359)	(0.0360)	(0.0361)	(0.0587)	(0.0847)	(0.0857)
Horizon			-0.0989***			-0.821***
			(0.0361)			(0.187)
Horizon (quadratic)			0.00243			0.174***
			(0.00370)			(0.0509)
Constant	0.986***			3.517***		
	(0.0393)			(0.0802)		
Controls	None	Fixed-effects	Fixed-effects	None	Fixed-effects	Fixed-effects
Observations	6736	6660	6660	14157	7564	7564

Note: Logistic regressions on the dummy variable $Y_{i,t}$, which is equal to one when an update in forecasts is observed at period t for individual i. The fixed-effects regressions are conditional logistic regressions (clustered by individuals). Standard errors are in parenthesis and ***, ** and * respectively denote significance at 1%, 5% and 10% levels.

Overall, the results confirm the conclusions of section 2.2: large economic shocks prompt professional forecasters to update their expectations. This effect appears to be much more important for european forecasters than for american forecasters. This may be due to the fact that the latter update much more often than the former, so that there is less room for beliefs for them when one looks at the evolution of forecast one quarter to the next.

C Detailed Resolution of the Simple Behavioral Model

In this appendix I derive step-by-step the solution of the simple behavioral model presented in section 3.2 and prove the main propositions.

C.1 Optimal consumption

The consumer's problem writes:[31]

$$\begin{array}{ll} \max\limits_{\{c_{t+i}\}_{i=0}^{\infty}} & E_t\left\{ \sum\limits_{i=0}^{\infty} \beta^{t+i} \left(-\frac{e^{-\alpha c_t}}{\alpha} \right) \Big| \, \mathcal{I}_l \right\} \\ \text{s.t.} & s_t = Rs_{t-1} - c_{t-1} + (y_t - \bar{y}) - \iota(t)K \end{array}$$

The Lagrangian associated to this problem is thus

$$\mathcal{L}(c_{t+i}, s_{t+i}) = \sum_{i=0}^{\infty} \beta^{t+i} E_t \left\{ -\frac{e^{-\alpha c_t}}{\alpha} - \lambda_{t+i}\big(Rs_{t+i-1} - c_{t+i-1} + \iota(t)\big(\zeta_{t+i} - K\big) - s_{t+i}\big) \right\}$$

and the first order conditions are

$$\begin{aligned} \beta^t(E_t\{\exp(-\alpha c_t)\} - E_t\{\lambda_t\}) &= 0 \\ \beta^t E_t\{\lambda_t\} - R\beta^{t+1} E_t\{\lambda_{t+1}\} &= 0 \end{aligned}$$

and $\beta R = 1$. Thus, the Euler equation is

$$\exp(-\alpha c_t) = E_t\{\exp(-\alpha c_{t+1})\}$$

From the Euler equation we see that consumption has to be constant when the consumer does not update her expectations from one period to the next. To derive the optimal consumption, I make the educated guess that consumption has the form $c_t = A + Bs_t$. Furthermore, given the initial conditions c_0 and s_0, the permanent income at date t of a consumer who updates for the first time is:

$$s_t = R^t s_0 - \sum_{i=0}^{t-1} R^i c_{t-i} + \zeta_t - K$$

[31] The notation is introduced in section 3.2.

with $\zeta_t = \sum_{i=0}^{t-1} R^i (y_{t-i} - \bar{y})$ and because consumption is constant between two updates, I can rewrite it as:

$$s_t = R^t s_0 - \frac{1-R^t}{1-R} c_0 + \zeta_t - K$$

Thus, since income surprises are normally distributed, so is the permanent income and, given our initial guess, so is also consumption. Then, $\exp(-\alpha c_t)$ is log-normally distributed and its first moment is:

$$E_0\Big(\exp(-\alpha c_t) \Big) = \exp\Big(-\alpha E_0(c_t) + \frac{\alpha^2}{2} V_0(c_t) \Big)$$

where the first moment of consumption is:

$$\begin{aligned}
E_0(c_t) &= E_0(A + Bs_t) \\
&= A + BE_0(R^t s_0 - \frac{1-R^t}{1-R} c_0 + \zeta_t - K) \\
&= A + B(R^t s_0 - \frac{1-R^t}{1-R} c_0 - K + E_0(\zeta_t)) \\
&= A + B(R^t s_0 - \frac{1-R^t}{1-R} c_0 - K)
\end{aligned}$$

and the second moment is:

$$\begin{aligned}
V_0(c_t) &= V_0(A + Bs_t) \\
&= B^2 V_0(\zeta_t) \\
&= B^2 V_0(\sum_{i=0}^{t-1} R^i (y_{t-i} - \bar{y})) \\
&= B^2 \sum_{i=0}^{t-1} R^{2i} \sigma_y^2 = B^2 \frac{1-R^{2t}}{1-R^2} \sigma_y^2
\end{aligned}$$

Thus, from the Euler equation between 0 and the date of the first update (t), I get:

$$\begin{aligned}
\exp(-\alpha c_0) &= \exp\Big(-\alpha[A + B(R^t s_0 - \frac{1-R^t}{1-R} c_0 - K)] + \frac{\alpha^2}{2} B^2 \frac{1-R^{2t}}{1-R^2} \sigma_y^2 \Big) \\
\Longleftrightarrow \quad c_0 &= A + B(R^t s_0 - \frac{1-R^t}{1-R} c_0 - K) - \frac{\alpha}{2} B^2 \frac{1-R^{2t}}{1-R^2} \sigma_y^2 \\
\Longleftrightarrow \quad A + Bs_0 &= A + B(R^t s_0 - \frac{1-R^t}{1-R} (A + Bs_0) - K) - \frac{\alpha}{2} B^2 \frac{1-R^{2t}}{1-R^2} \sigma_y^2 \\
\Longleftrightarrow \quad 0 &= \frac{((1-R) + B)(R^t - 1)}{1-R} s_0 - \frac{1-R^t}{1-R} A - K - \frac{\alpha}{2} B \frac{1-R^{2t}}{1-R^2} \sigma_y^2 \\
\Longleftrightarrow \quad A &= -(1 - R + B) s_0 - \frac{1-R}{1-R^t} K - \frac{\alpha}{2} B \frac{1-R^{2t}}{1-R^t} \frac{1-R}{1-R^2} \sigma_y^2
\end{aligned}$$

and $B = (R-1)$. Note that the subscript t refers here to the number of periods between the time of the choice (period 0) and the next update. Recall also that in this simple model, it is assumed that the consumer sees this duration as deterministic and exogenous. To make the distinction between consumption at period t and the number of periods between two updates, I use the notation introduced in the main content of the paper and denote by d the duration between two updates. Next, I get optimal consumption at updating dates $t = l$ as

$$c_t = (R-1)s_l - \frac{R-1}{R^d-1}K - \frac{\alpha}{2}(R-1)\frac{(1+R^d)}{(1+R)}\sigma_y^2$$

Therefore, optimal consumption at the next period is either $c_{t+1} = c_t$ if there is no update or $c_{t+1} = (R-1)s_{l'} - \frac{R-1}{R^d-1}K - \frac{\alpha}{2}(R-1)\frac{(1+R^d)}{(1+R)}\sigma_y^2$ if there is an update and $l' = t+1$.

C.2 Duration Between Two Updates

Starting from equation (18) rewritten as:

$$\begin{aligned}
\hat{s}_{l+n} &= s_l + \frac{1-R^n}{1-R}\Bigg(\frac{(R-1)K}{R^D-1} + \frac{\alpha\sigma_y^2}{2R^2}\frac{(R-1)(R^D+1)}{R+1}\Bigg) \\
&\quad +\theta\Bigg[\sum_{i=0}^{n-1}\big[(1-\theta)R\big]^i\xi_{l+n-i} + \sum_{i=0}^{n-1}\sum_{j=0}^{i}(1-\theta)^j R^i\zeta_{l+n-i}\Bigg]
\end{aligned}$$

and given the expression for the behavioral rule (19), we get that the conditional probability of an update n periods after the last update, knowing that the consumer did not update inbetween, is :

$$\begin{aligned}
\mathcal{P}(n) &= \mathbf{P}\big[n = D' \,\big|\mathcal{H}_{n-1}\big] \\
&= \mathbf{P}\big[(\hat{s}_{l+n} - s_l)^2 \geq \rho(K) \,\big|\mathcal{H}_{n-1}\big] \\
&= \mathbf{P}\big[\hat{s}_{l+n} - s_l \geq \sqrt{\rho(K)} \,\big|\mathcal{H}_{n-1}\big] + \mathbf{P}\big[\hat{s}_{l+n} - s_l \leq -\sqrt{\rho(K)} \,\big|\mathcal{H}_{n-1}\big] \\
&= \mathbf{P}\big[\xi_{l+n} + \zeta_{l+n} \geq \theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})\big|\mathcal{H}_{n-1}\big] \\
&\qquad + \mathbf{P}\big[\xi_{l+n} + \zeta_{l+n} \leq -\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})\big|\mathcal{H}_{n-1}\big] \\
&= \mathbf{P}\Big[\frac{\xi_{l+n} + \zeta_{l+n}}{\sqrt{\sigma_\xi^2 + \sigma_y^2}} \geq \frac{\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})}{\sqrt{\sigma_\xi^2 + \sigma_y^2}}\big|\mathcal{H}_{n-1}\Big] \\
&\qquad + \mathbf{P}\Big[\frac{\xi_{l+n} + \zeta_{l+n}}{\sqrt{\sigma_\xi^2 + \sigma_y^2}} \leq -\frac{\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})}{\sqrt{\sigma_\xi^2 + \sigma_y^2}}\big|\mathcal{H}_{n-1}\Big] \\
&= 1 - \Phi\Bigg(\frac{\theta^{-1}\sqrt{\rho(K)} - G(\mathcal{H}_{n-1})}{\sqrt{\sigma_\xi^2 + \sigma_y^2}}\Bigg) + \Phi\Bigg(-\frac{\theta^{-1}\sqrt{\rho(K)} + G(\mathcal{H}_{n-1})}{\sqrt{\sigma_\xi^2 + \sigma_y^2}}\Bigg)
\end{aligned}$$

with $\mathcal{H}_n = \{s_l, \{\zeta_{l+j}\}_{j=1}^n, \{\xi_{l+j}\}_{j=1}^n\}$ the history of past shocks, $\Phi(.)$ the cdf of $\mathcal{N}(0,1)$ and

$$G(\mathcal{H}_{n-1}) = \frac{1-R^n}{\theta(1-R)}\left(\frac{(R-1)K}{R^D-1}+\frac{\alpha\sigma_y^2}{2R^2}\frac{(R-1)(R^D+1)}{R+1}\right)+\sum_{i=1}^{n-1}\left[(1-\theta)R\right]^i\xi_{l+n-i}+\sum_{i=1}^{n-1}\sum_{j=0}^{i}(1-\theta)^jR^i\zeta_{l+n-i}$$

Using these conditional probabilities and the fact that $\xi_t, \zeta_t, \xi_{t+k}$ and ζ_{t+k} are jointly independent $\forall k \neq 0$, letting $f_{\zeta+\xi}(x)$ be the pdf of the random variable[32] $x_t = \xi_t + \zeta_t$, the unconditional probability of an update after one period is,

$$\mathbf{P}(n=1) \;=\; \int_{\theta^{-1}(\sqrt{\rho(K)}-B)}^{\infty} f_{\zeta+\xi}(x_1)\;dx_1 + \int_{-\infty}^{-\theta^{-1}(\sqrt{\rho(K)}+B)} f_{\zeta+\xi}(x_1)\;dx_1$$

after two periods,

$$\mathbf{P}(n=2) \;=\; \int_{-\theta^{-1}(\sqrt{\rho(K)}+B)}^{\theta^{-1}(\sqrt{\rho(K)}-B)}\left[\int_{\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_1)}^{\infty} f_{\zeta+\xi}(x_2)\;dx_2 + \int_{-\infty}^{-\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_1)} f_{\zeta+\xi}(x_2)\;dx_2\right] dF_{\zeta+\xi}(x_1)$$

after N periods,

$$\begin{aligned}\mathbf{P}(n=N) \;=\; & \int_{-\theta^{-1}(\sqrt{\rho(K)}+B)}^{\theta^{-1}(\sqrt{\rho(K)}-B)}\int_{-\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_1)}^{\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_1)}\cdots\int_{-\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_{N-2})}^{\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_{N-2})}\\ & \left[\int_{\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_{N-1})}^{\infty} f_{\zeta+\xi}(x_N)\;dx_N + \int_{-\infty}^{-\theta^{-1}\sqrt{\rho(K)}-G(\mathcal{H}_{N-1})} f_{\zeta+\xi}(x_N)\;dx_N\right]\\ & dF_{\zeta,\xi}(x_{N-1})\ldots dF_{\zeta+\xi}(x_1)\end{aligned}$$

Or similarly,

$$\mathbf{P}(n=N) \;=\; \mathcal{P}(N)\prod_{k=1}^{N-1}\big(1-\mathcal{P}(k)\big)$$

D Dynamics of Aggregate Consumption with State-Dependent Expectations

This appendix shows that the response of consumption to an income shock depends on the size of this shock. This feature of aggregate consumption comes from the state-dependence of expectations at the micro level.

To simulate the impulse-response function of aggregate consumption to an aggregate income shock

[32]With $f_{\zeta+\xi}(x) = \frac{1}{\sqrt{\sigma_\xi^2+\sigma_y^2}\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x}{\sqrt{\sigma_\xi^2+\sigma_y^2}}\right)^2\right)$.

in the partial equilibrium model developed in section 3.2, I assume that the economy is composed of a finite number of consumers (N) who are identical. The only heterogeneity comes from the signals they receive at each period. In order to make the simulations as clear as possible, all individuals have the same cognitive capacities so that their signals are identically distributed. Defining aggregate consumption (C_t) as the sum of individual consumptions across the economy, its dynamics is simply:

$$C_{t+1} - C_t = \sum_{j=0}^{\infty} e_{t+1}(j)$$

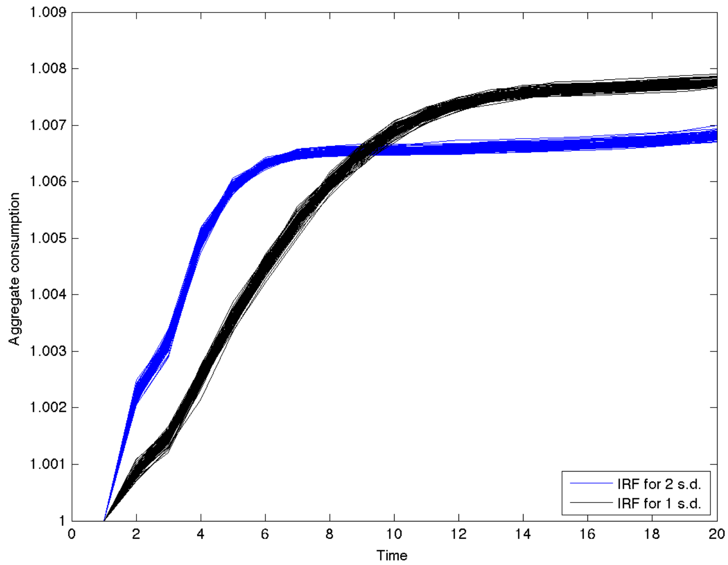
where $e^j_{t+1} = \sum_{i:l=t+1 \text{ and } g=t-j}(1-R)\zeta_{t+1,i}$ is the sum of the actualized income surprises (times $1-R$ the marginal propensity to consume out of these surprises) of people updating in $t+1$ and whose last update before was in $t-j$.

The state-dependent expectation formation process developed in this paper tells us that, at the individual level, the larger is the size of this income shock, the higher is the probability of an update. Thus, when the economy encounters a large exogenous income shock affecting each consumer, one may expect that the share of people updating their expectations will be large. Therefore, since aggregate consumption reflects individual behaviors, one may wonder how these micro-reactions will affect the dynamics of macro-variables. To answer this question, I have computed different impulse-response functions of aggregate consumption for different income shocks and reported the results in figure 7. These impulse-response functions were simulated first at the individual level for an income shock, identical for all consumers, of one or two standard deviations of the income process (σ_y). The series for aggregate consumption were then recovered by summing consumption over individuals.

Technical remark on the simulated IRFs: The dynamics of aggregate consumption is stochastic in my simulations. Indeed, to compute the e^j_{t+1}, which are random variables by definition, I need to draw a sequence of idiosyncratic shocks for each individual. Thus, as long as N – the number of consumers in my simulations – is finite, the IRF of aggregate consumption will be sample specific: the draws of idiosyncratic shocks are not the same from one sample to another, even though they all come from a same distribution. More precisely, I find that the empirical variance of the simulated IRFs is decreasing in the number of individuals N. This problem is common in agent-based simulations and I shall account for it when interpreting the results. Note however that this question is only technical and should not be interpreted as an economic phenomenon: in the real world, the economy is composed of a sufficiently large number of consumers so that the variance of IRFs is almost zero.

Figure 7 indicates that the response of aggregate consumption is significally affected by the size of the income shock: not only the long run value of aggregate consumption is different from one simulation to the other, but also the rate of convergence to this value. Regarding the rate of convergence, we

Figure 7: IRF of Aggregate Consumption to an Aggregate Income Shock



NOTE: IRFs of aggregate consumption simulated for two different income shocks of one or two standard errors of y. Each black (blue) line reports an IRF simulated for one (two) income shock. Thus, the differences between the lines with a same color is an indication of the variance of the IRFs explained by the fact that the simulation is based on a finite number of agents (N=1000). Therefore, the difference between the blue lines and the black lines reflects only the impact of the size of the income shock. Aggregate consumption is normalized by its value just before the shock.

know that the larger is a shock (in absolute value), the higher is the share of people who update their expectations and the faster they revise their consumption path. Thus, the smoothness of consumption to unexpected income shock decreases with the size of the shock.

The difference regarding the long run value might be quite puzzling: How can the level of aggregate consumption in the long run be negatively correlated to the size of the income shock ? However, the answer to this question is very intuitive. When a consumer benefits from a positive unexpected income shock and do not update her expectations, this supplementary wealth will grow at the gross interest rate each period until she realizes its existence and updates her consumption path.[33] Therefore, she will be able to chose a higher level of consumption in the future, when she will update her new consumption path.

[33]Note however that this delayed increase in consumption, even though it implies a higher level of consumption in the future, doesn't mean an increase in her intertemporal utility since $\beta = R^{-1}$.