

Size Distribution and Firm Dynamics in an Economy with Credit Shocks

In Hwan Jo
The Ohio State University

Tatsuro Senga
The Ohio State University

February 2014

Abstract

A large body of empirical literature documents that firm size distribution is highly-skewed and firms respond differently by size and age over business cycles. We quantitatively investigate macroeconomic implications of matching the empirical size distribution in a model with production heterogeneity. We build an equilibrium model of heterogeneous firms, where each firm faces with a forward-looking collateral constraint for borrowing. We compare the aggregate dynamics of our model economy by changing our specification of firm-level productivity. We identify that the symmetric treatment of idiosyncratic productivity can be misleading, especially upon a financial shock. Aggregation over a skewed firm size distribution implies that recovery from a credit crunch is relatively slower, amplifying resource misallocation from the borrowing constraint. We also study the micro-level employment dynamics upon exogenous shocks. In our model, a credit shock causes more disproportionate responses by firms; net employment growth among young firms falls further. It follows that reallocation is further restricted among small and young firms because of their limited credit, indicating that only productive young firms grow faster.

JEL Classification:

Keywords: Financial frictions, firm size, firm age, business cycles

*We would like to thank Aubhik Khan and Julia Thomas for their helpful comments, support and guidance. All remaining errors are ours.

*Email: jo.34@osu.edu; Address: The Ohio State University, 1945 N. High Street, Columbus, OH 43210.

*Email: senga.1@osu.edu; Address: The Ohio State University, 1945 N. High Street, Columbus, OH 43210.

1 Introduction

The empirical evidence on firm-level heterogeneity is often in contrast with the predictions from the standard models of business cycle analysis with heterogeneous agents. In most cases, individual productivities are assumed to be symmetric in the models of production heterogeneity, and the resulting firm size distribution is not rich enough to be compared with that from data. In fact, the highly-skewed empirical size distribution of firms is not easily obtainable even with financial frictions. Further, the recent empirical studies document that firms respond differently by size, age, and financial structure, and differently by the source of aggregate shocks. The Great Recession in the U.S. is a dramatic example of the above evidence, where small and young firms suffer more from the aggregate contraction originated from a credit crunch. In sum, the evidence on both firm distribution and dynamics in terms of size and age needs to be comprehensively analyzed in a theoretical framework.

The goal of this paper, therefore, is twofold. First, we analyze the aggregate implications of the productivity assumption made on individual firms within a unified framework. To do so, we build an equilibrium model of heterogeneous firms to compare its aggregate dynamics from two different specifications of the idiosyncratic productivity process. In one specification, we exactly match the firm size distribution in Business Dynamics Statistics (BDS) database as well as the aggregate moments. With the correctly specified model for both micro and macro evidence, the other goal of this paper is to quantitatively evaluate firms' differential responses along the aggregate fluctuations. We consider both dimensions of firm size and age, so that we can address the effectiveness of the size- or age-dependent policies in a consistent manner with business cycle analysis.

In our model, firms are heterogeneous in the level of productivity, capital stock, and financial structure. In particular, their decisions on investment and borrowing are endogenously subject to a collateral constraint that is consistent with Kiyotaki and Moore (1997). In addition to exogenous TFP shocks, a credit crunch is modeled as a sudden drop in firms' borrowing capacity in the model. We improve upon the similar framework in Khan and Thomas (2013), by considering a forward-looking borrowing constraint and also by reducing firms' individual state-vector. The latter refinement of the model greatly simplifies the associated decision rules at firm-level, and moreover, it is less costly in applying the computational methods which allows other additional sources of frictions in our model environment. More importantly, while Khan and Thomas (2013) are focused on the aggregate misallocation from financial frictions as well as on the micro-level investment dynamics, our focus in this paper is mainly on the model's implication on the aggregate dynamics when it is capable of generating a more realistic firm size distribution. In addition, we can analyze the firm dynamics upon different sources of shocks at disaggregated levels of size and age, once we match the empirical firm distributions. In fact, to the best of our knowledge, only a few models in the literature on production heterogeneity with financial shocks capture the empirical aspects of both dimensions of firm size and age.

In the U.S. data, firm size distribution is right-skewed and stable over time, as shown in Figure 1. Our results suggest that the conventional symmetric treatment of idiosyncratic productivity relatively well approximates the aggregate dynamics upon the exogenous shocks in the model. When only TFP shocks are considered, our model specification that matches the empirical size distribution leads to identical responses of the aggregate variables with the results from the conventional treatment. However, not only the symmetric identification is not enough to generate the observed size distribution of firms, but we also suspect that the misallocation and recovery from financial shocks may be underestimated in the models with the symmetric individual productivities. This is because a firm's borrowing capacity depends on its collateral value in our model, which takes the form of the firm's capital stock and affects its employment size. When we generate a more realistic firm size distribution where small firms are concentrated in lower tail, a credit crunch in the model severely distorts resource allocation, especially among these small firms. This increased misallocation may further slow down an economy's recovery upon a financial distress, when we further assume that the degree of borrowing condition or the access to finance differs by a firm's size or age as pointed in the related empirical literature.

We also investigate the differential responses by each firm size and age group to an exogenous shock. In particular, we focus on measuring the employment dynamics at disaggregated levels, which characterize the role of size and age in shaping the aggregate dynamics over economic fluctuations. The recent empirical findings by Fort, Haltiwanger, Jarmin, and Miranda (2013) motivate our study on the importance of firm age together with size. We identify the substantial differences in job creation and destruction rates by firm type, through the simulation of our model along its transitional path. In particular, we confirm that small and young firms are more vulnerable to financial shocks, as noted in the literature. For a young firm started with a small amount of capital stock, its borrowing limit is further tightened during a credit crunch. This affects the firm's desired level of employment as well as investment, even without any frictions in labor market. In this regard, a credit shock in our model economy leads to relatively more heterogeneous responses in the net employment growth, when compared to a TFP shock, manifesting itself as a source of misallocation among firms. Our result, therefore, can be considered as a theoretical investigation of the work by Chari, Christiano, and Kehoe (2013), in a more general framework of production heterogeneity that matches their micro-level evidence. Finally, our result suggests the need for evaluating the effectiveness of government policies that aim to support specific firm groups in terms of size and age, with the consideration of financial frictions and the nature of the aggregate shocks.

The rest of this paper is organized as follows. In the next section, we briefly provide a review of the literature that is closely related to our work. We present the model environment in Section 3, and characterize the decision rules of firms by reformulating the model in Section 4. Section 5 summarizes our results from the model. We conclude in Section 6.

2 Related Literature

Our paper is related to several different strands of the literature that studies firm-level heterogeneity and macroeconomic consequences of financial shocks. Following the recent recession in the U.S., there has been a growing research on how financial shocks affect the real activities in an aggregate economy, through the existing financial frictions. As pointed by Ohanian (2010), the standard business cycle model with a typical TFP shock is not able to generate the observed patterns of the macroeconomic variables in the U.S. after 2007. Jermann and Quadrini (2012) investigate the role of financial shocks as another source of business cycles, in an economy with a representative firm that faces with a collateral constraint. Khan and Thomas (2013) open a new prospect of the ongoing research, by blending financial constraints into a business cycle model with heterogeneous firms. They highlight the severe misallocation of resources generated from the borrowing limits imposed on individual firms, and provide a propagation mechanism of financial shocks into the real economy. Buera and Moll (2013) not only confirm this misallocation channel of investment, but also emphasize the importance of underlying heterogeneity within a model that leads to different dynamics of the aggregate wedges. In this paper, our theoretical approach is located on the intersection of the previous two papers. We adopt the general structure of the model environment from Khan and Thoams (2013), and then reformulate the model in a more tractable way that is similar to Buera and Moll (2013). We discuss this improvement in Section 4.

Among the vast literature on firm dynamics and size distribution, our departure from the on-average specification of firm-level productivities is motivated by Gabaix (2011). He studies the endogenous mechanism of idiosyncratic shocks among the largest firms, in amplifying the aggregate fluctuations when a fat-tailed size distribution is considered. Our calibration strategy to match the empirical firm size distribution is related to Guner, Ventura, and Xu (2008). They consider an extreme value of managerial ability to generate the largest establishments of an economy, and use their framework to analyze the cost of size-dependent government policies. In this paper, however, we highlight the importance of financial frictions in shaping a firm size distribution, together with our asymmetric productivity specification. Similar argument is made by Cabral and Mata (2003), and they focus on the evolution of firm size distribution in relation with the selection process among young firms. Regarding the mechanism of firm dynamics, Luttmer (2011) provides the theoretical foundations for generating the observed shape of size distribution.

The empirical evidence on the responsiveness of small and/or young firms to aggregate shocks further motivates our exploration of the micro-level dynamics of firms in this paper. Gertler and Gilchrist (1994) first document the differential responses by firm size during the periods of monetary contractions. Chari, Christiano, and Kehoe (2013), on the other hand, find that the difference between small and large firms is less clear during the NBER recession periods. Further, they argue that the source of aggregate shocks should be distinguished when comparing

the responses of different firm size groups along business cycles. In this context, a credit shock considered in our paper generates more differential firm-level employment dynamics in relative to a TFP shock, especially among small and young firms. The evidence in Fort, Haltiwanger, Jarmin, and Miranda (2013) adds another dimension of the cyclical behavior of firms; the age distribution. They find that young and small firms were more severely affected by the collapse of the U.S. housing market in the recent recession. Haltiwanger, Jarmin, and Miranda (2013) emphasize the role of young firms in creating jobs, after they find the weak relationship between size and growth after controlling firm age. Schmalz, Sraer, and Thesmar (2013) suggest that business startups and subsequent growth are closely related to the access to external finance and the value of collateral. By considering both size and age distribution of firms in our paper, we can address the above recent evidence in a comprehensive framework.

Finally, we briefly review the recent research on the slow recovery in the U.S. labor market after 2007, in relation with firm size and age dimensions. Siemer (2013) focuses on the employment dynamics among small and young firms during the recent recession, by building a model with financial friction on firm entry and labor adjustment costs. Sedlacek (2013) also considers the endogenous entry of firms in shaping a time-varying firm age distribution. Buera, Fattal-Jaef, and Shin (2013) analyze the interaction between financial and labor market frictions, which generate a substantial increase in unemployment upon a credit shock. They also focus the micro-level dynamics of heterogeneous entrepreneurs in a similar way to our approach in this paper. Our contribution relative to these recent works, however, is that we take a more general approach to both dimensions of firms size and age, without abstracting from the standard elements of business cycle analysis.

3 Model

We model an economy with a large number of heterogeneous firms, which are subject to individual collateral constraints for external financing. Their endogenous decisions of investment, hiring, and borrowing generate a substantial heterogeneity across the firm distribution, together with the persistent differences in productivities. The following subsections first describe the model environment and the optimization problem at firm-level. We complete the model by discussing the household side, then define a recursive competitive equilibrium. In Section 4, we reformulate the model by reducing a firm's individual state-space vector with a new state variable which is consistent with the existing literature. This reformulation not only enables us to derive much simpler decision rules for each individual firm in this general model environment, but also allows us to improve the quantitative investigation of the model even with the rich firm heterogeneity.

3.1 Firms

3.1.1 Production Environment

The production side of the economy is populated by a continuum of firms which produce homogenous output goods. Each firm owns its predetermined capital stock k , and hires labor n in a competitive labor market. Production takes place by using the capital and labor inputs, according to a production technology $y = z\epsilon \cdot F(k, n)$, where $F(\cdot)$ exhibits the *decreasing returns to scale* (DRS) property. There are two different types of exogenous productivity processes; one for the total factor productivity (TFP) z which is common across firms, and the other is for a firm-specific productivity level ϵ . We assume that the shocks to the idiosyncratic productivity component ϵ is from a time-invariant distribution G . And ϵ follows a Markov chain such that $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \dots, \epsilon_{N_\epsilon}\}$ with $\pi_{ij} \equiv \Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_i) \geq 0$, and $\sum_{j=1}^{N_\epsilon} \pi_{ij} = 1$ for each $i = 1, \dots, N_\epsilon$. Independently from ϵ , the aggregate TFP z also follows a Markov chain with the transition probability π_{lm}^z .

To introduce a type of financial friction into the model, we assume that each firm faces with a borrowing constraint for its external financing. Specifically, the amount of newly issued debt b' by a firm is limited by its collateral in the future period, due to the limited enforceability in financial contracts. The amount of collateral is given by the firm's future capital stock k' , that is determined by its current investment. This constraint for intertemporal borrowing endogenously affects the firm's optimal decisions on investment and employment, as will be discussed in the next section. As is conventional in the literature, we adopt the standard one-period discount debt with the price of q . That is, for each unit of newly issued debt b' that needs to be repaid in the next period, a firm only receives q units of output from its borrowing at the current period. Then the borrowing constraint takes the form of,

$$b' \leq \theta \cdot k', \text{ where } \theta \in [0, q^{-1})$$

The parameter θ captures the “*loan-to-value*” ratio which is commonly used in the corporate finance literature, and it measures the relative share of borrowing to the amount of physical asset. Further, θ also reflects the degree of financial friction in the economy. While $\theta = 0$ corresponds to a financial autarky, we have a perfect credit market when $\theta = q^{-1}$. The above collateral constraint is occasionally binding depending on the firm's individual states which will be described below. Also, notice that the collateral constraint itself is *forward-looking* which is consistent with Kiyotaki and Moore (1997), and our modeling approach here neither relies on any exotic timing assumption nor abstracts from the main ingredients of the modern business cycle analysis.

In this model of production heterogeneity, firms are allowed to accumulate enough wealth so that the borrowing constraint does not matter anymore in their decision making. When all firms have such sufficient wealth, the model's aggregate implication becomes analogous to the

standard one-sector growth model with only differences in firm's productivity and capital stock. To avoid this case of Modigliani and Miller (1958), we impose entry and exit of firms in each period. We assume that each firm faces with an exogenous possibility of exit $\pi_d \in (0, 1)$ at the end of a given period, and that the arrival of this exit shock is known before production. Without loss of generality, only surviving firms are able to make intertemporal decisions on its future capital stock as well as borrowing. To maintain a stationary distribution of firms in equilibrium, we replace the exiting firms by an equal measure of new born firms with small enough initial capital stock which is a fraction χ of the average capital stock in the economy. In this way, we can simply prevent the model from converging into the case where the financial consideration becomes irrelevant, as illustrated by Khan and Thomas (2013). Lastly, we abstract from any micro-level adjustment frictions other than the borrowing constraint, in order to focus on the distributional aspects of this model.

At the beginning of a given period, a firm is identified by its individual states (k, b, ϵ) ; its predetermined capital stock, $k \in \mathbf{K} \subset \mathbf{R}_+$, the amount of existing debt issued last period, $b \in \mathbf{B} \subset \mathbf{R}$, and the current idiosyncratic productivity level $\epsilon \in \mathbf{E}$. Just after the arrival of z and ϵ , the firm realizes whether it will survive or exit at the end of the period. Given its aggregate and individual states, the firm maximizes its expected discounted value of all future dividend payments. It first decides its current period labor demand, and start producing. After production is done, it pays the wage bills at ω and its outstanding debt b is cleared. Conditional on survival into the next period, the firm chooses its investment level i for future capital stock k' and the new level of debt b' , and pays out the current period dividends D to its shareholders. Each firm's capital accumulation equation is standard, $i = k' - (1 - \delta)k$. We summarize the distribution of firms over the individual state vector (k, b, ϵ) , using a probability measure μ which is defined on a Borel algebra generated by $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. Because the only economy-wide uncertainty is on z , the aggregate states are given by (z, μ) . The evolution of the firm distribution follows a mapping Γ from the economy's current aggregate states, such that $\mu' = \Gamma(z, \mu)$. Hence, the entire state vector for an individual firm in each period can be described by $(k, b, \epsilon; z, \mu)$. Finally, firms take the wage rate $\omega(z, \mu)$ and the debt price $q(z, \mu)$ as given from the aggregate states (z, μ) .

3.1.2 A Firm's Problem

Now we are able to formulate the firm's dynamic optimization problem recursively. First, we distinguish the timing of the problem by whether the exit shock is arrived. Let $v_0(k, b, \epsilon; z, \mu)$ be the expected discounted value of a firm identified with (k, b, ϵ) at the beginning of a period, before its exit status is known. Once the firm is determined to survive, its *within-the-period continuation value* is given by $v(k, b, \epsilon; z, \mu)$. We illustrate the firm's problem by defining v_0 and v respectively in the following equations.

$$v_0(k, b, \epsilon_i; z_l, \mu) = \pi_d \cdot \max_n [z_l \epsilon_i F(k, n) - \omega(z_l, \mu)n + (1 - \delta)k - b] + (1 - \pi_d) \cdot v(k, b, \epsilon_i; z_l, \mu) \quad (1)$$

With the current realizations of (z_l, ϵ_i) at the beginning of the period, the firm takes a binary expectation over the values before its exit or survival is known. Equation (1) above makes use of the exogenous probability of exit π_d to define the value, $v_0(k, b, \epsilon_i; z_l, \mu)$ of this firm. The first line in (1) corresponds to the case of exit at the end of the period, where the firm maximizes its liquidation value without the intertemporal decisions of k' and b' . The liquidation value is given by the exiting firm's output net of its wage bill payment ωn , debt clearing b , and the undepreciated capital stock $(1 - \delta)k$ after production.

$$v(k, b, \epsilon_i; z_l, \mu) = \max_{n, D, k', b'} \left[D + \sum_{m=1}^{N_z} \pi_{lm}^z d_m(z_l, \mu) \sum_{j=1}^{N_\epsilon} \pi_{ij}^\epsilon v_0(k', b', \epsilon_j; z_m, \mu') \right] \quad (2)$$

s.t. $0 \leq D = z_l \epsilon_i F(k, n) - \omega(z_l, \mu)n + (1 - \delta)k - b - k' + q(z_l, \mu)b'$
 $b' \leq \theta k' \quad , \text{ where } \theta \in [0, q(z_l, \mu)^{-1})$
 $\mu' = \Gamma(z_l, \mu)$

The problem in Equation (2) describes the continuation value of the firm when it is possible to enter the next period. It chooses its optimal level of hiring n , future capital k' , and new debt level b' , to maximize the sum of its current dividends D and the future period *beginning-of-the-period value*, $v_0(k', b', \epsilon_j; z_m, \mu')$. The dividend payment is defined from the firm's budget constraint, and is limited to be non-negative. Due to our assumption of Markov chain on z and ϵ , the future expected value on the RHS of (2) is denoted by the summation operators and the transition probabilities $(\pi_{lm}^z, \pi_{ij}^\epsilon)$, given current (z_l, ϵ_i) . The firm discounts its future value using the stochastic discount factor $d_m(z_l, \mu)$. The collateralized borrowing constraint limits the availability of credit for this firm, so that it also affects the investment decision as well.

3.2 Households and Equilibrium Prices

In the other side of the economy, there is a unit measure of identical households, or a representative household. Each household participates in the labor market activities by supplying a fraction of its time endowment in return for wage income, and holds its wealth as a comprehensive portfolio of one-period firm shares with measure λ and non-contingent bonds ϕ . In addition, households have access to a complete set of state-contingent claims for their consumption smoothing over periods with a subjective discount factor β . Since there is no heterogeneity in households, the equilibrium net quantity of these assets is zero which is redundant. Hence,

we just model a simpler version of the representative household's problem that maximizes the lifetime expected utility by choosing the aggregate consumption C^h and labor supply N^h , and by adjusting the asset portfolio in each period.

$$\begin{aligned}
V^h(\lambda, \phi; z_l, \mu) &= \max_{C^h, N^h, \lambda', \phi'} \left[U(C^h, 1 - N^h) + \beta \sum_{m=1}^{N_z} \pi_{lm}^z V^h(\lambda', \phi'; z_m, \mu') \right] \quad (3) \\
\text{s.t.} \quad C^h + q(z_l, \mu)\phi' + \int_{\mathbf{S}} \rho_1(k', b', \epsilon'; z_l, \mu) \lambda'(d[k' \times b' \times \epsilon']) \\
&\leq \omega(z_l, \mu)N^h + \phi + \int_{\mathbf{S}} \rho_0(k, b, \epsilon; z_l, \mu) \lambda(d[k \times b \times \epsilon]) \\
\mu' &= \Gamma(z_l, \mu)
\end{aligned}$$

As with the common timing issue related with the dividends, $\rho_1(k', b', \epsilon'; z_l, \mu)$ above denotes the ex-dividend prices of firm shares, whereas $\rho_0(k, b, \epsilon; z_l, \mu)$ is the dividend-inclusive values of current share holding. Let $\Phi^h(\lambda, \phi; z, \mu)$ be the household's decision for bond holding, and $\Lambda^h(k', b', \epsilon', \lambda, \phi; z, \mu)$ be the new portfolio choice of firm shares with future states (k', b', ϵ') . We then define a *recursive competitive equilibrium* of the model below.

A *recursive competitive equilibrium* is a set of functions: prices $(\omega, q, (d_m)_{m=1}^{N_z}, \rho_0, \rho_1)$, quantities $(N, K, B, D, C^h, N^h, \Phi^h, \Lambda^h)$, and values (v_0, V^h) that solve the optimization problems, clear each market for labor, output, and asset, and the associated policy functions are consistent with the aggregate law of motion as in the following conditions:

1. v_0 solves Equation (1) and (2), and (N, K, B, D) are the associated policy functions for firms.
2. V^h solves Equation (3), and $(C^h, N^h, \Phi^h, \Lambda^h)$ are the associated policy functions for households.
3. The labor market clears with $N^h = \int_{\mathbf{S}} [N(k, \epsilon; z, \mu)] \cdot \mu(d[k \times b \times \epsilon])$
4. The output market clears with $C^h = \int_{\mathbf{S}} [z\epsilon F(k, N(k, \epsilon; z, \mu)) - (1 - \pi_d)(K(k, b, \epsilon; z, \mu) - (1 - \delta)k) + \pi_d(1 - \delta)k] \cdot \mu(d[k \times b \times \epsilon])$
5. The law of motion for the firm distribution is consistent with the policy functions, where Γ defines the mapping from μ to μ' with $\pi_d, K(k, b, \epsilon; z, \mu)$, and $B(k, b, \epsilon; z, \mu)$.

As noted by Khan and Thomas (2013), it is convenient to modify the value functions v_0 and v by using the equilibrium prices resulting from the market-clearing quantities. Let C and N be the market clearing values for the representative household's consumption and labor hours, satisfying the above recursive competitive equilibrium. The equilibrium prices then can be expressed with the first derivatives of the period utility function $U(C, 1 - N)$, under the assumption of its

differentiability. The real wage $\omega(z, \mu)$ is equal to the marginal rate of substitution between leisure and consumption. Next, the inverse of the discount bond price q^{-1} equals to the expected real interest rate. Finally, the stochastic discount factor $d_m(z, \mu)$ is equal to the household intertemporal marginal rate of substitution across states. The following equations summarize the previous equilibrium prices.

$$\begin{aligned}\omega(z, \mu) &= \frac{D_2U(C, 1 - N)}{D_1U(C, 1 - N)} \\ q(z, \mu) &= \frac{\beta \sum_{m=1}^{N_z} \pi_{lm}^z D_1U(C'_m, 1 - N'_m)}{D_1U(C, 1 - N)} \\ d_m(z, \mu) &= \frac{\beta D_1U(C'_m, 1 - N'_m)}{D_1U(C, 1 - N)}\end{aligned}$$

By using the above definitions of prices, we can solve the equilibrium allocations from the firm's problem in a consistent manner with the household's optimal decisions on consumption, hours worked, and asset quantities. Let $p(z, \mu)$ be the marginal utility from the market-clearing consumption by the representative household, at which firms also value their current period output and dividends. Then the equilibrium price functions are given by,

$$p(z, \mu) \equiv D_1U(C, 1 - N) \tag{4}$$

$$\omega(z, \mu) = \frac{D_2U(C, 1 - N)}{p(z, \mu)} \tag{5}$$

$$q(z, \mu) = \frac{\beta \sum_{m=1}^{N_z} \pi_{lm}^z p(z_m, \mu')}{p(z, \mu)} \tag{6}$$

Letting firms use the price measure of marginal utility enables us to re-write the value functions using β , instead of the stochastic discount factor. Specifically, define new value functions in terms of the utility price $p(z, \mu)$ such that they are identical to the original problem.

$$V_0(k, b, \epsilon; z, \mu) \equiv p(z, \mu) \cdot v_0(k, b, \epsilon; z, \mu)$$

$$V(k, b, \epsilon; z, \mu) \equiv p(z, \mu) \cdot v(k, b, \epsilon; z, \mu)$$

Then the problem solved by a firm with its states $(k, b, \epsilon; z, \mu)$ is given by a pair of value functions as before, augmented with $p(z, \mu)$.

$$V_0(k, b, \epsilon_i; z_l, \mu) = \pi_d \cdot \max_n p(z_l, \mu) [z_l \epsilon_i F(k, n) - \omega(z_l, \mu)n + (1 - \delta)k - b] + (1 - \pi_d) \cdot V(k, b, \epsilon_i; z_l, \mu) \quad (7)$$

$$V(k, b, \epsilon_i; z_l, \mu) = \max_{n, D, k', b'} \left[p(z_l, \mu)D + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij}^\epsilon V_0(k', b', \epsilon_j; z_m, \mu') \right] \quad (8)$$

s.t. $0 \leq D = z_l \epsilon_i F(k, n) - \omega(z_l, \mu)n + (1 - \delta)k - b - k' + q(z_l, \mu)b'$
 $b' \leq \theta k' \quad , \text{ where } \theta \in [0, q(z_l, \mu)^{-1})$
 $\mu' = \Gamma(z_l, \mu)$

For the remainder of this paper, we suppress the aggregate states (z, μ) in the price functions and the decision rules whenever needed for notational simplicity.

4 Analysis

In order to solve the recursive competitive equilibrium presented in the previous section, we first need to characterize the optimal decision rules by distinguishing firm types in a given criteria. In the first subsection below, we illustrate the policy functions associated with labor demand, investment, and debt financing consistent with the existing literature. In the next subsection, we reduce a firm's individual state-space in a novel way by defining its "*cash-on-hand*". This allows us to reformulate the firm's problem so that the associated decision rules can be simply determined by a threshold of the new state variable for each type of firms.

4.1 Firm Types and Unconstrained Decisions

Before we start analyzing the decisions made by firms in the model, it is important to distinguish firms by whether the borrowing constraint is affecting their decisions. This is because the collateral constraint in Equation (8) is not always binding and hence becomes a challenging object to solve. In the following discussion, we follow the definitions of firm types in Khan and Thomas (2013). First, define a firm as *unconstrained* when it already has enough wealth accumulated either in terms of k or $-b$, so that it never worries about the borrowing constraint in its investment decision. Specifically, *an unconstrained firm* is assumed not to experience a binding borrowing constraint in any possible future state. Thus, all the Lagrangian multipliers on the future borrowing constraint become zero. Then the firm becomes indifferent between paying dividends and saving internally because its shadow value of internal saving is equal to $p(z, \mu)$. On the other hand, *constrained firms* are the complement set of all unconstrained firms. A *constrained firm* may have a currently binding borrowing constraint or not. Even though it

does not experience a binding constraint at a given period, the firm puts non-zero probabilities of having a binding constraint in the future. Therefore, its shadow value of retained earnings is greater than the valuation of dividends so that the firm chooses $D = 0$. The remaining subsection establishes the decision rules by the unconstrained firms.

We first begin with the decision rule of labor demand. Since there is no adjustment friction in employment in this model for brevity, all firms with the same (k, ϵ) choose the unconstrained labor demand $n = N^w(k, \epsilon; z, \mu)$ that solves the static condition $z\epsilon D_2 F(k, n) = \omega$. With a specific functional form of the production function, we show the analytic solution of $N^w(k, \epsilon)$ in the appendix section. Next, we consider the unconstrained choice of the future capital k' , when the borrowing constraint is not relevant for an unconstrained firm. Since there is no adjustment friction in capital stock in this case, we can easily derive the unconstrained choice of $k' = K^w(\epsilon; z, \mu)$ from the optimization problem below. In this problem, we define a firm's current earnings net of the wage bill as $\Pi^w(k, \epsilon; z, \mu) \equiv z\epsilon F(k, N^w) - \omega N^w$, using the unconstrained labor choice $N^w(k, \epsilon)$. And the analytic solution of k' is also available in the appendix once we assume the Markov property of the productivity processes.

$$\max_{k'} \left[-p(z_l, \mu)k' + \beta \sum_{m=1}^{N_z} \pi_{lm}^z p(z_m, \mu') \sum_{j=1}^{N_\epsilon} \pi_{ij}^\epsilon (\Pi^w(k', \epsilon_j; z_m, \mu') + (1 - \delta)k') \right]$$

With the unconstrained policy functions N^w and K^w on hand, now we solve for the unconstrained choice of the new debt b' . We define an unconstrained decision rule for the new debt level by using the definition of unconstrained firms itself and the indifference result on dividend payments as mentioned earlier. *The minimum savings policy* of an unconstrained firm is defined recursively as below, where the firm chooses $b' = B^w(\epsilon; z, \mu)$ that ensures the firm remains unconstrained in any future path of (z, ϵ) .

$$B^w(\epsilon_i; z_l, \mu) = \min_{(\epsilon_j, z_m)_{j,m}} \left(\tilde{B}(K^w(\epsilon_i), \epsilon_j; z_m, \mu') \right) \quad (9)$$

$$\begin{aligned} \tilde{B}(k, \epsilon_i; z_l, \mu) &= z_l \epsilon_i F(k, N^w) - \omega N^w + (1 - \delta)k - K^w(\epsilon_i) \\ &\quad + q \min \{ B^w(\epsilon_i; z_l, \mu), \theta K^w(\epsilon_i) \} \end{aligned} \quad (10)$$

$\tilde{B}(k, \epsilon_i; z_l, \mu)$ in (10) is the maximum level of debt that ensures the firm can adopt the minimum savings policy at the current period without paying negative dividends. On the RHS of Equation (10), the minimum operator imposes the borrowing constraint. In turn, Equation (9) defines the minimum savings policy $B^w(\epsilon; z, \mu)$ by considering all possible future realizations of the productivities, when its existing debt level at the beginning of the next period $\tilde{B}(K^w(\epsilon_i), \epsilon_j; z_m, \mu')$, is given from its unconstrained decisions made at the current period. Finally, the dividend payment from an unconstrained firm is then the residual from the firm's budget constraint in (8) when adopting the unconstrained choices, (N^w, K^w, B^w) .

4.2 Reformulated Firm's Problem and Simplified Decision Rules

Now, we consider another approach of solving the firm's problem illustrated in the previous section. Since the state vector of the problem is multi-dimensional including the two continuous individual states (k, b) , it is always desirable to reduce the state space for computational ease as well as for adding more ingredients for further investigation of the model's implications. In this subsection, we define a new individual state variable that encompasses a firm's information contained in (k, b) without changing the solution of the original problem in (8). Further, the decision rules by firm type can be simply characterized by defining a threshold in this new state variable. This reformulation of the original problem is relatively new in the literature on production heterogeneity with financial frictions.

For a simple illustration, substitute the unconstrained labor choice N^w into (8) since the borrowing constraint does not affect this decision. Then the budget constraint for a firm with (k, b, ϵ) is given by,

$$0 \leq D = z\epsilon F(k, N^w) - \omega N^w + (1 - \delta)k - b - k' + qb'$$

From the above, let $m(k, b, \epsilon; z, \mu) \equiv z\epsilon F(k, N^w) - \omega N^w + (1 - \delta)k - b$ be *the cash-on-hand* of a firm with (k, b, ϵ) . In particular, it is the amount of the firm's current output less the wage bills and the outstanding debt with the value of its capital stock when uninstalled. Since it does not contain the information regarding the current intertemporal decisions on k' and b' , we can summarize the individual states (k, b) by $m(k, b, \epsilon)$. And the evolution of $m(k, b, \epsilon)$ is affected by those current choices. Specifically, given the firm's current decisions on k' and b' , the level of its cash-on-hand in the next period is determined as $m(k', b', \epsilon'; z', \mu')$. Now, we reformulate the equations (7) and (8) using $m(k, b, \epsilon)$ and define new value functions W_0 and W as below.

$$W_0(m, \epsilon_i; z_l, \mu) = \pi_d \cdot p \cdot m(k, b, \epsilon_i) + (1 - \pi_d) \cdot W(m, \epsilon_i; z_l, \mu) \quad (11)$$

$$W(m, \epsilon_i; z_l, \mu) = \max_{D, m', k', b'} \left[pD + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij}^\epsilon W_0(m(k', b', \epsilon_j; z_m, \mu'), \epsilon_j; z_m, \mu') \right] \quad (12)$$

s.t.

$$0 \leq D = m(k, b, \epsilon_i) - k' + qb'$$

$$b' \leq \theta k'$$

$$m(k', b', \epsilon_j; z_m, \mu') = z_m \epsilon_j (k')^\alpha (N^w(k', \epsilon_j))^\nu - \omega(z_m, \mu') N^w(k', \epsilon_j) + (1 - \delta)k' - b'$$

$$\mu' = \Gamma(z, \mu)$$

Notice that we only modify the previous value functions by substituting the new state variable $m(k, b, \epsilon)$, without changing the firm's problem itself. The problem of choosing k' and b' now corresponds to that of choosing the optimal level of future m , in which the composition of the portfolio between k' and b' should be properly determined with the borrowing limit. However,

this reformulation greatly simplifies the decision rules among both the unconstrained and the constrained firms which are endogenously determined by the level of the cash-on-hand held by each type of firms. As discussed before, the unconstrained firms are able to conduct the efficient level of investment for $k' = K^w$ and debt financing $b' = B^w$. From this fact, we define a threshold value of m such that a firm with (k, b, ϵ) is identified to be unconstrained. From the budget constraint above, the unconstrained dividend policy, $D^w(k, b, \epsilon) \equiv m - K^w + qB^w$, implies that an unconstrained firm's current level of m should be greater than or equal to a certain threshold value \tilde{m} defined as,

$$\tilde{m}(\epsilon; z, \mu) \equiv K^w(\epsilon; z, \mu) - qB^w(\epsilon; z, \mu) \quad (13)$$

Regardless of a firm's current state of k and b , we now recognize the constrained firms such that $m < \tilde{m}$. For constrained firms, we further distinguish them by whether the current borrowing constraint is binding or not, because the bindingness affects the investment decision made by these firms. For the constrained firms with currently non-binding borrowing constraint (*Type-1 firms*), they are able to achieve the target capital stock K^w as the unconstrained firms do. Their debt policy then is determined from the zero-dividend policy for the constrained firms. On the other hand, firms with binding borrowing constraint at the current period (*Type-2 firms*) invest until the maximum level that their borrowing limits allow. This constrained choice of future capital is also determined by m . Since all constrained firms do not pay dividends, $D = 0$ implies that their portfolio structure between k' and b' should be within the available cash-on-hand m , as long as negative dividends are not allowed. Therefore, the upper bound of future capital for a constrained firm with (k, b, ϵ) is determined by $\bar{K} \equiv \frac{m}{1-q\theta}$, and we are now able to distinguish Type-1 and Type-2 firms by comparing each firm's unconstrained capital decision K^w with the upper bound \bar{K} . Notice that the upper bound of k' becomes the infinity when $\theta = q^{-1}$, which corresponds to the case of perfect credit market, where all constrained firms still choose K^w without any misallocation of investment resources. The following summarizes the decision rules made by each firm type in this model.

- Firms with $m(k, b, \epsilon) \geq \tilde{m}(\epsilon)$ are unconstrained, and adopt $K^w(\epsilon)$ and $B^w(\epsilon)$.
- Constrained firms with $m(k, b, \epsilon) < \tilde{m}(\epsilon)$ face with the upper bound of $k' \leq \bar{K} \equiv \frac{m}{1-q\theta}$.
 - Type-1 firms with $K^w(\epsilon) \leq \bar{K}$ choose $k' = K^w(\epsilon)$ and $b' = \frac{1}{q}(K^w(\epsilon) - m)$.
 - Type-2 firms with $K^w(\epsilon) > \bar{K}$ choose $k' = \bar{K}$ and $b' = \frac{1}{q}(\bar{K} - m)$.

Our characterization of the decision rules above is similar to that of Buera and Moll (2013). Here, we consider a more general environment with heterogeneous firms, and the advantage from reducing the state-space of the model is enormous while preserving the implications from a credit shock model as will be shown in the next section.

5 Results

5.1 Parameters

We parameterize our model to match the key aggregate moments and the size distribution of firms in the U.S. data. First, we set a time period in the model to one year. For the representative household's period utility, we assume the standard functional form (Rogerson (1988)), $U(C, 1 - N) = \log C + \psi(1 - N)$. The log of idiosyncratic productivity ϵ is assumed to follow an AR(1) process, $\log \epsilon' = \rho_\epsilon \log \epsilon + \eta'$ with $\eta' \sim G(\eta)$. We assume that $G(\cdot)$ is time-invariant, and discretize the ϵ process for our numerical exercises. The persistency (ρ_ϵ) and the standard deviation (σ_ϵ) values of ϵ are from Khan and Thomas (2013). In the standard models of production heterogeneity, $G(\cdot)$ is assumed to be a normal distribution with mean zero. In this section, however, we assume that $G(\cdot)$ takes a Pareto distribution to generate more skewed firm size distribution. We report the resulting size distribution with the U.S. data in the next subsection.

The parameters of the model are set to be consistent with the existing literature and data. We begin with setting the subjective discount factor β to imply the annual real interest rate of 4 percent. The production parameter ν is set to be the average labor share of income at 0.6, following Cooley and Prescott (1995). We set the depreciation rate δ to match the average investment to capital ratio during the postwar U.S. periods, where the private capital stock is measured in BEA Fixed Asset Tables. In turn, the capital share of output α is determined to yield the observed average capital to output ratio of 2.3, given the value of δ . The preference parameter of disutility from labor, ψ is set to imply the total hours of worked to be one-third.

We set the period exogenous exit rate π_d to 0.1 targeting the average firm exit rate in BDS database, and assume that the total measure of firms is constant over time. In each period, exiting firms are assumed to be replaced by the same number of new firms starting with a small capital stock and zero debt. The initial capital stock is given as a χ fraction of the average capital stock held by the incumbent firms of the period. We set χ to be 0.1. Lastly, the steady state level of loan-to-value ratio θ is set to imply the aggregate debt to asset ratio of 0.31, which is less than its counterpart (0.37) of nonfarm nonfinancial businesses in the Flow of Funds. In the model, the aggregate asset holding is the sum of capital stock and negative debt over the distribution of firms. Table 1 below summarizes the parameter values used in calibrating our model.

Table 1

Parameter Vaules			
α	0.273	β	0.960
ν	0.600	ψ	2.100
δ	0.069	χ	0.100
θ	0.700	ρ_ε	0.659
π_d	0.100	σ_ε	0.135

5.2 Steady State

We begin with the stationary distribution of firms in the model. Figure 2 shows the entire firm distribution over capital and debt level (k, b) . It displays all three types of firms that we considered in the previous section; unconstrained, Type-1, and Type-2 firms. The unconstrained firms are located in the area on negative debt starting from the front mass near zero capital stock in the figure. Their levels of b reflect the minimum savings policy $B^w(\epsilon)$. They are apart from the body of the distribution with most firms, and the connected areas toward the unconstrained firms indicate Type-1 firms accumulating assets to be unconstrained. About 50 percent of all firms in our model are distinguished as Type-2 with currently under binding constraints. They are mainly on the diagonal straight line that represents the inverse relationship between these firms' capital and financial savings.

The above firm types are more easily distinguishable when we look at the decision rules on capital k' , debt b' , and dividends D , based on a firm's the cash-on-hand m . Figure 3 shows the decision rules of the firms with the same median value of ϵ . The level of cash-on-hand, $m(k, b, \epsilon)$ is denoted in the horizontal axis, and we add the two vertical lines to distinguish each firm type where the one near $m = 7$ represents the threshold of being unconstrained, $\tilde{m}(\epsilon)$. When a firm has accumulated enough wealth such that $m \geq \tilde{m}(\epsilon)$, it is considered as unconstrained. The firm then adopts the unconstrained choice of capital $K^w(\epsilon)$ as well as its minimum savings policy $B^w(\epsilon)$, and pays positive dividends. Constrained firms with m less than the threshold value, on the other hand, follow the zero-dividend policy to accumulate their wealth toward being unconstrained, as discussed earlier. Type-1 firms in the middle part of the figure, between the two vertical lines, are able to adopt the optimal level of capital, and gradually save more as their m increase. Lastly, Type-2 firms with very small m are only able to invest until their borrowing limits allow.

Now, we compare the model's firm size distribution with that of Business Dynamics Statistics (BDS) database. BDS is constructed using the Census from Longitudinal Business Database (LBD), and covers about 90 percent of U.S. private employment starting from 1977. Firms and establishments are categorized by size and age, in addition to their annual job creation and destruction. To generate the corresponding firm distribution from the model, we simulate a large panel of firms at the steady state. Both of the empirical and model-generated size distributions

are reported in Table 2. The first column of the table shows the average share of firms in each size group between 2003 and 2011. The empirical distribution exhibits the well-known shape, where more than 75 percent of firms hire less than 10 employees in a given year. The share of firms in each size group decreases in the number of employees, and only 4.4 percent in the database can be considered as relatively large with more than 50 employees. The steady state size distribution of the model is on the next column of the table. With our asymmetric treatment of ϵ using a Pareto distribution, the model matches the BDS distribution relatively well, with small firms are slightly more concentrated on the lower tail. This realistic firm size distribution comes from the combination of the model's financial friction with a fat-tailed distribution of firm-level productivities. It becomes clear when we re-calibrate our model after introducing the standard identification of ϵ process with a log-normal distribution as in the literature. In the last column of Table 2, the size distribution from the model is solely driven by the borrowing constraint. With the symmetric values of ϵ , we now have relatively more shares of mid-sized firms in the distribution. We further investigate the aggregate implications of this specification issue of ϵ in the next subsection, when we report the impulse responses upon the aggregate shocks in the model.

Table 2

Size group (Employees)	BDS database (% shares)	Model	
		Pareto ϵ	Log-normal ϵ
0 to 9	75.9	81.3	52.5
10 to 19	12.2	12.0	21.9
20 to 49	7.5	4.6	18.6
50 to 99	2.4	1.6	5.7
100+	2.0	0.5	1.3

Even though our model does not feature endogenous firm entry and exit dynamics, we also report the age distribution of firms in Table 3. Due to the availability of firm age data, we exclude the censored firms in the data and the empirical age distribution is also the average between 2003 and 2011. The model also well captures the age distribution of firms in BDS. Since the exogenous probability of exit commonly applies to all firms regardless of size and age in the model, we can indirectly identify that most of the exiting firms are small, once a skewed firm size distribution is generated. Therefore, our model is also consistent with the evidence on volatile entry and exit among small firms.

Table 3

Age group	BDS database	Model
(Age)	(% shares)	
0	10.5	9.5
1	8.0	8.6
2	7.0	7.7
3	6.2	7.1
4	5.6	6.3
5	5.1	5.8
6 to 10	19.8	21.7
11 to 15	13.7	13.1
16+	24.2	20.3

5.3 Aggregate Dynamics

In this subsection, we first briefly discuss the transitional dynamics of the model in relation with the existing literature. Then we analyze the aggregate implication of our specification of ϵ that matches the empirical size distribution. All the impulse response functions presented in this paper are from our application of perfect foresight transition. We assume that an economy is initially at the steady state of the model, and an unexpected shock hits the economy at date 1 while the future path of the shock is known to the agents. In particular, we follow the numerical method to generate transitional dynamics in the models of heterogeneous agents, as in Guerrieri and Lorenzoni (2011) and Khan (2011).

First, we report the results from our preferred specification of ϵ . Figure 4 shows our model's aggregate dynamics upon a persistent TFP shock. The initial magnitude of the shock is about 2 percent below the steady state level of z , and gradually recovers with the persistency of $\rho_z = 0.91$. As confirmed in Khan and Thomas (2013), a shock to the aggregate TFP in the models of production heterogeneity generates almost isomorphic responses to the results from the standard business cycle model with representative agents. As in the literature, TFP shocks neither generate the observed fluctuations of the aggregates in the recent U.S. recession, nor aggravate the reallocation of factors across firms. The measured TFP in the upper-left panel of the figure exhibits small differences from the dynamics of z .

To implement a credit shock in our model, we calibrate the parameter θ in the collateral constraint. Specifically, a credit crunch is represented by a sudden tightening of θ for 3 periods that generates a decline of about 50 percent in aggregate lending. The resulting fall in the debt to asset ratio is 41 percent at the impact date. We assume that θ gradually reverts to its pre-crisis level from date 4. This magnitude of a credit shock is relatively large when compared to Khan and Thomas (2013), but it is still within the range of the empirical evidence on the reduction in corporate loans around the 2007 recession (Ivashina and Scharfstein (2010)). In addition, we abstract from any micro-level adjustment frictions in labor and capital, which further amplify the effect of the credit crunch. This is indicated in Figure 5, where the responses of aggregate

employment and investment are relatively volatile upon a credit shock. In the figure, however, our results on the initial drops in measured TFP, output, and employment are consistent with the observed data. Upon the decrease in θ , the measured TFP falls about 2 percent below its steady state level, and gradually recovers after date 3. This is because of the increased misallocation across firms from tighter borrowing constraints. In overall, our model generates consistent dynamics of the aggregates with the existing literature on production heterogeneity and financial shocks.

From Table 2 and the previous figures (Figure 4 and 5), our model of heterogeneous firms generates both business cycles and firm size distribution as observed in the U.S. data. We examine how the model's aggregate dynamics change with the specification of idiosyncratic productivities. As in the previous subsection, we re-calibrate and simulate the model with the conventional symmetric ϵ process, where shocks to ϵ are from the standard normal distribution. The comparison between the two specifications is reported in Figure 6 and 7, respectively to each aggregate shock. Figure 6 illustrates the responses from a TFP shock, and our specification of ϵ in this case is not relevant in shaping the aggregate dynamics. Again, this is because of the nature of a TFP shock from which all firms are symmetrically affected, so that it does not further distort resource allocation.

In response to a credit shock in Figure 7, the aggregate variables in the symmetric specification of ϵ (Log-normal) are very similar to the results from our benchmark specification (Pareto). However, all responses of measured TFP, output, and investment are relatively less volatile, and we can identify that the recovery with the symmetric ϵ is slightly faster after date 5. Although the difference is small in this exercise, the result implies that the aggregate responses and the recovery from financial shocks may be underestimated in the models with symmetric idiosyncratic individual productivities. This is a natural result from our specification of ϵ when we consider the misallocation channel from the borrowing constraint in our model. That is, only a small number of firms are productive enough to become very large, and the recovery from the credit crunch is mainly driven by these firms. In addition, most firms are small in our model-generated size distribution as reported in Table 2, and the possibility of their acquiring high individual productivity in the future is low. In order to recover from the recession, these firms need to borrow enough externally. However, most of them suffer from the tightening of credit, because they also have insufficient collateral to borrow. Therefore, the interest rate falls further and the recovery becomes slower, if we take a realistic firm size distribution into the model. In this regard, we suspect that the business cycle analysis of the existing models in matching the second moments of the empirical targets also needs to be re-evaluated, once a firm size distribution becomes consistent with its empirical counterpart. Moreover, when we introduce additional frictions or distorting taxes into our model, the observed difference in the aggregate dynamics with the asymmetric treatment of ϵ will be further distinguishable. In summary, we confirm that the case with the symmetric ϵ well approximates the aggregate dynamics of a model, but

we also identify that the results from financial shocks can be different when the empirical firm size distribution is matched.

5.4 Micro-level Dynamics

We now switch our attention to disaggregated dynamics at firm-level. In this subsection, we maintain our calibrated parameters of the model that matches both firm size and age distributions. To analyze the firm-level employment dynamics from our model, we use the simulation method along the transitional dynamics solved in the previous subsection. The number of firms is set to 150000, and the reported results are the average values taken over 200 iterations of the simulation. As an overview of this subsection, we begin with defining the firm-level job flows following the literature. We then focus on the differential responses by each firm size and age group upon a TFP shock or a credit shock, respectively.

Job creation and destruction rates are constructed using the micro-level data on firms and establishments. When a firm increases its employment between the two adjacent periods, for instance, the additional jobs are counted as job creation. We follow the definitions from Business Dynamics Statistics (BDS) (Haltiwanger, Jarmin, and Miranda (2009)), which are originated from Davis, Haltiwanger, and Schuh (1996). Given a group index s for a firm i , job creation rate (JC) and job destruction rate (JD) in each size or age group are defined as,

$$\begin{aligned}
 JC_{s,t} &= \sum_{i \in s} \frac{n_{i,t} - n_{i,t-1}}{0.5(N_{s,t} + N_{s,t-1})} \quad , \text{ if } n_{i,t} \geq n_{i,t-1} \\
 JD_{s,t} &= \sum_{i \in s} \frac{|n_{i,t} - n_{i,t-1}|}{0.5(N_{s,t} + N_{s,t-1})} \quad , \text{ if } n_{i,t} < n_{i,t-1}
 \end{aligned}$$

, where $n_{i,t}$ is the employment of firm i at date t and $N_{s,t}$ is the aggregate employment in group s . The net employment growth rate (NEG) in group s is well approximated from the difference of JC and JD rates, $NEG_{s,t} = JC_{s,t} - JD_{s,t}$.

First, we briefly discuss about the dynamics from each size group following a TFP shock. Figure 8 shows the differential responses by firm size. Due to the lack of labor adjustment friction in our model, large firms hiring more than 50 employees immediately adjust upon a negative shock. After date 3, their employment drives the observed overshooting in the aggregate employment as seen Figure 4. In contrast, small firms rather increase employment slightly. This unique dynamics is driven mainly by a larger share of old firms among them, because Figure 9 indicates that most young firms respond quickly upon the same shock. In our model economy, young firms are mostly small exogenously, but the opposite is not true because there can exist some old firms with relatively low levels of productivity. This distinction between size and age dimension is also emphasized in Haltiwanger, Jarmin, and Miranda (2013). Other than the smallest group, all firm size groups respond in a similar way by destroying jobs.

Next, we look at the responses from a credit shock in both dimensions of size and age. Since

the credit shock itself directly affects the borrowing constraint in our model, the differential responses in Figure 10 dramatically highlight the non-convexity from the friction. At the impact, the initial contraction of employment in each size group is almost similar about 5 percent below their steady state levels. Along the recovery stage after date 3, the employment growth becomes more differential between the smallest, mid-sized (10 to 49 employees), and largest groups. Unlike the case of a TFP shock, the net employment growth in the smallest group of firms becomes volatile, and exhibits a more persistent response along the transition. As discussed in the decision rules at the steady state (Figure 3), small firms tend to borrow externally to accumulate their capital stock over time. Now that the borrowing condition gradually recovers, they rapidly increase employment to obtain larger amount of cash-on-hand more quickly. The largest firms, on the other hand, also increase hiring from the initial drop to prevent themselves from becoming constrained. But once the recovery begins, they do not need to hire more because the gradual increase in θ implies that they can borrow enough with the same level of collateral. In sum, our finding in this exercise suggests that firms respond differently by size over business cycles, depending on the source of the aggregate shocks. Due to the increased misallocation, a credit shock generates more differential employment dynamics among firms. Therefore, we are not able to confirm whether small firms suffer more during any recession periods when we consider both type of shocks. This result is also consistent with the empirical evidence in Chari, Christiano, and Kehoe (2013), where they consider more broader empirical measures of firm size

The age dimension of a credit shock is illustrated in Figure 11. Upon a credit shock, young firms display the sharpest contraction in net employment growth. We further focus on the group of young firms in Figure 12, where firms with age less than 3 show more differential responses. This is because the new firms (age 0) in our model are born very small gradually accumulating wealth, and therefore, they are mostly subject to tighter credit conditions. Together with Figure 10, we confirm the evidence in Fort, Haltiwanger, Jarmin, and Miranda (2013) that young and small firms relatively destroy more jobs during the recent financial crisis. Older firms in Figure 11 are relatively unstable during the transition because substantial shares of them are also small in our firm size distribution. When compared to the TFP shock case (Figure 9), however, the response of each age group is more persistent and the magnitude of employment adjustment is also larger. From the results in this subsection, we identify the differential employment dynamics at firm-level from our model economy, which is consistent with the recent empirical evidence. In addition, we confirm that the increased misallocation from a financial shock affects firms in a different way from a TFP shock, even at micro-level adjustments.

6 Concluding Remarks

We have analyzed the macroeconomic implications of matching the empirical firm size distribution in a model with production heterogeneity. We have built an equilibrium model with

a forward-looking collateral constraint and reduced the state space of the model by introducing a new individual state variable. From that, we were able to specify the idiosyncratic productivity process that generates a rich distribution of firms. We have calibrated our model to both macroeconomic moments and firm size distribution observed in the U.S. data.

Our results suggest that the conventional symmetric treatment on firm-level productivity in a model can be misleading when both a highly-skewed size distribution and a financial shock are considered. Because tight borrowing conditions mainly apply to a relatively large number of small firms, recovery from a credit crunch is only driven by a few large firms in our model specification. Moreover, misallocation among small and young firms further slows down the recovery of endogenous aggregate productivity.

While investigating the macroeconomic dynamics from an alternative productivity specification, we also identified the differential responses of firms in size and age dimensions. Our model economy delivers more disproportionate adjustments in employment at firm-level, which are consistent with the recent evidence. In particular, net employment growth by young and small firms falls further upon a credit shock as seen in the recent recession. In contrast to an aggregate TFP shock that affects all firms' incentives to adjust their labor demand, a sudden tightening in borrowing conditions affect firms differently. It is, therefore, the nature of a shock that determines the micro-level firm dynamics of an economy.

References

- [1] Buera, F. J., R. Fattal-Jaef and Y. Shin (2013) "Anatomy of a Credit Crunch: From Capital to Labor Markets" Working paper
- [2] Buera, F. J. and B. Moll (2013) "Aggregate Implications of a Credit Crunch" Working paper
- [3] Cabral, L. M. B. and J. Mata (2003) "On the Evolution of the Firm Size Distribution: Facts and Theory" *American Economic Review* 93(4), 1075–1090
- [4] Chari, V. V., L. Christiano and P. J. Kehoe (2013) "The Behavior of Small and Large Firms over the Business Cycle" Working paper
- [5] Cooley, T. F. and E. C. Prescott (1995) "Economic Growth and Business Cycles" *Frontiers of Business Cycle Research*, Princeton University Press
- [6] Davis, S. J., J. C. Haltiwanger and S. Schuh (1996) *Job Creation and Destruction*, The MIT Press
- [7] Fort, T. C., J. C. Haltiwanger, R. S. Jarmin and J. Miranda (2013) "How Firms Respond to Business Cycles: The Role of Firm Age and Firm Size" , *IMF Economic Review*
- [8] Gabaix, X. (2011) "THE GRANULAR ORIGINS OF AGGREGATE FLUCTUATIONS" *Econometrica* 79(3), 733–772
- [9] Gertler, M. and S. Gilchrist (1994) "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms" *Quarterly Journal of Economics* 109 (2): 309-340
- [10] Guerrieri, V. and G. Lorenzoni (2012) "Credit Crisis, Precautionary Savings and the Liquidity Trap" Working paper
- [11] Guner, N., G. Ventura and Y. Xu (2008) "Macroeconomic implications of size-dependent policies" *Review of Economic Dynamics* 11(4), 721–744
- [12] Haltiwanger, J. C., R. S. Jarmin and J. Miranda (2009) "Business Dynamics Statistics: An Overview" Business Dynamics Statistics Database
- [13] Haltiwanger, J. C., R. S. Jarmin and J. Miranda (2013) "WHO CREATES JOBS? SMALL VERSUS LARGE VERSUS YOUNG" *The Review of Economics and Statistics* 95(2), 347–361
- [14] Ivashina, V. and D. Scharfstein (2010) "Bank lending during the financial crisis of 2008" *Journal of Financial Economics* 97(3), 319–338

- [15] Jermann, U. and V. Quadrini (2012) "Macroeconomic Effects of Financial Shocks" *American Economic Review* 102(1), 238–271
- [16] Kiyotaki, N. and J. Moore (1997) "Credit Cycles" *Journal of Political Economy* 105(2), 211–248
- [17] Khan, A. (2011) "The solution of competitive equilibrium of the lumpy investment model under perfect foresight" Mimeo
- [18] Khan, A. and J. K. Thomas (2013) "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity" *Journal of Political Economy* 121(6), 1055–1107
- [19] Luttmer, E. G. J. (2011) "On the Mechanics of Firm Growth" *Review of Economic Studies* 78(3), 1042–1068
- [20] Modigliani, F. and M. H. Miller (1958). "The Cost of Capital, Corporation Finance and the Theory of Investment". *American Economic Review* 48 (3), 261–297
- [21] Ohanian, L. E. (2010) "The Economic Crisis from a Neoclassical Perspective" *Journal of Economic Perspectives* 24(4), 45–66
- [22] Rogerson, R. (1988) "Indivisible labor, lotteries and equilibrium" *Journal of Monetary Economics* 21(1), 3–16
- [23] Schmalz, M. C., D. A. Sraer and D. Thesmar "HOUSING COLLATERAL AND ENTREPRENEURSHIP" *NBER Working Paper Series* No. 19680
- [24] Sedlacek, P. (2013) "Firm Age, Business Cycles and the Aggregate Labor Market" Working Paper
- [25] Siemer, M. (2013) "Firm Entry and Employment Dynamics in the Great Recession" Working Paper

Figure 1

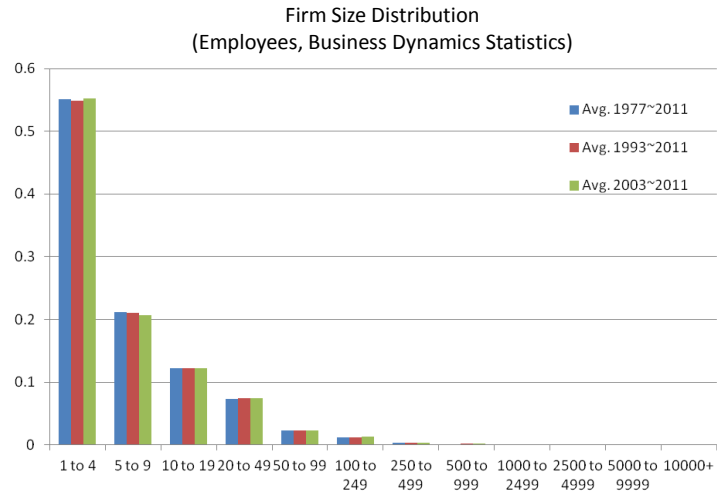


Figure 2

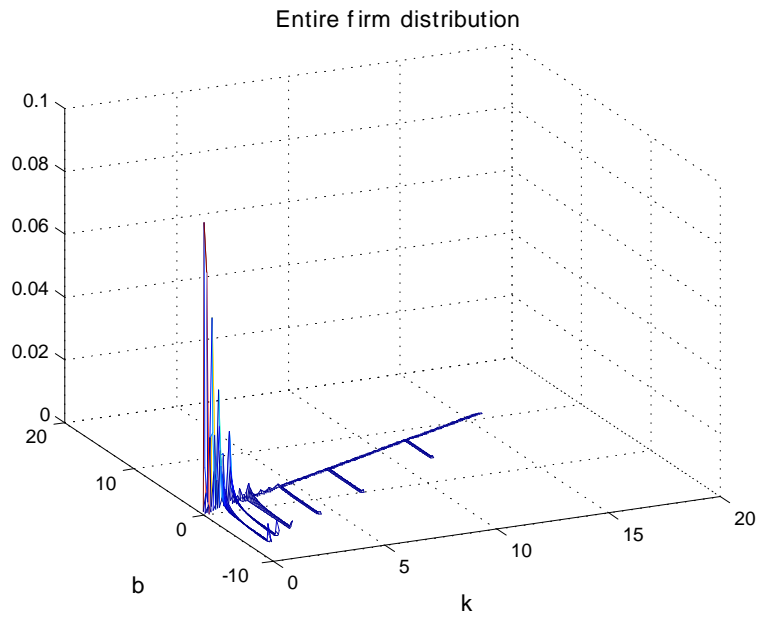


Figure 3

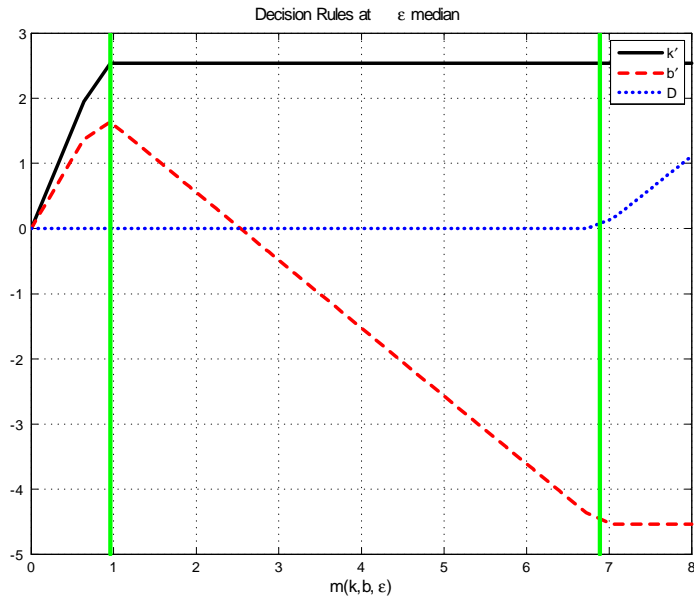


Figure 4

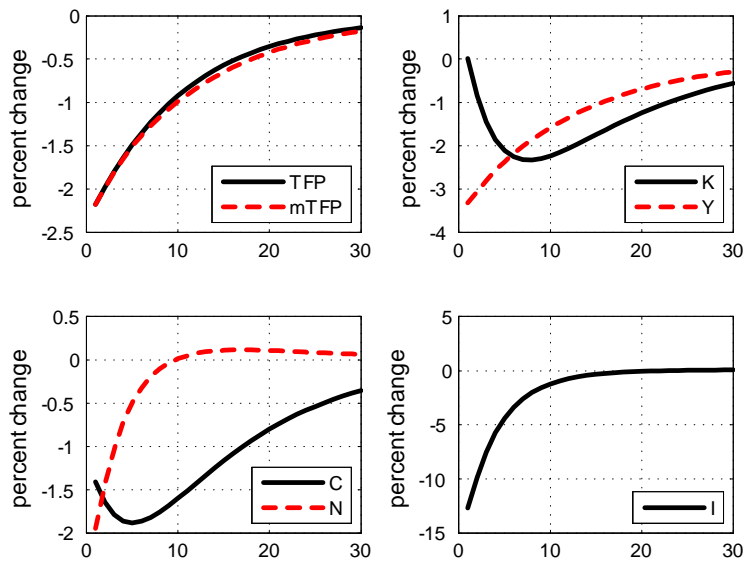


Figure 5

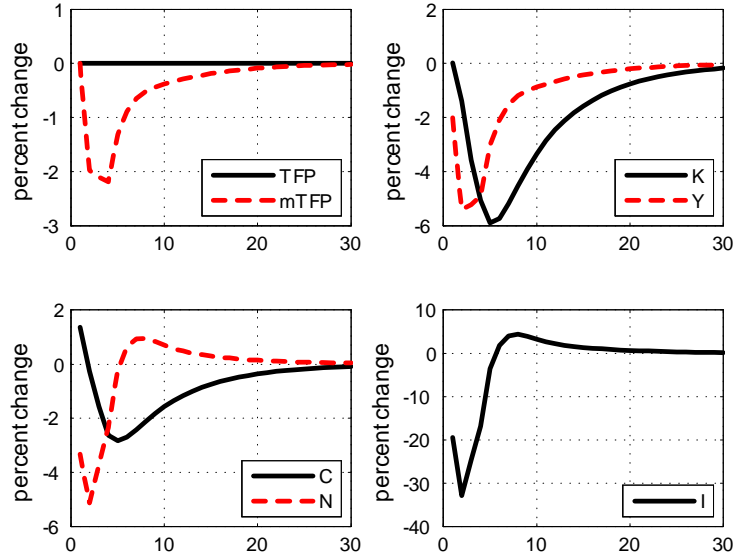


Figure 6

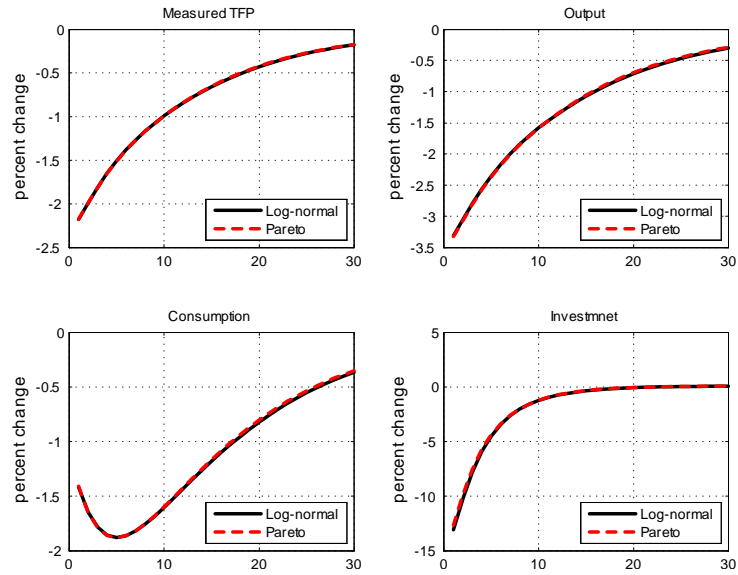


Figure 7

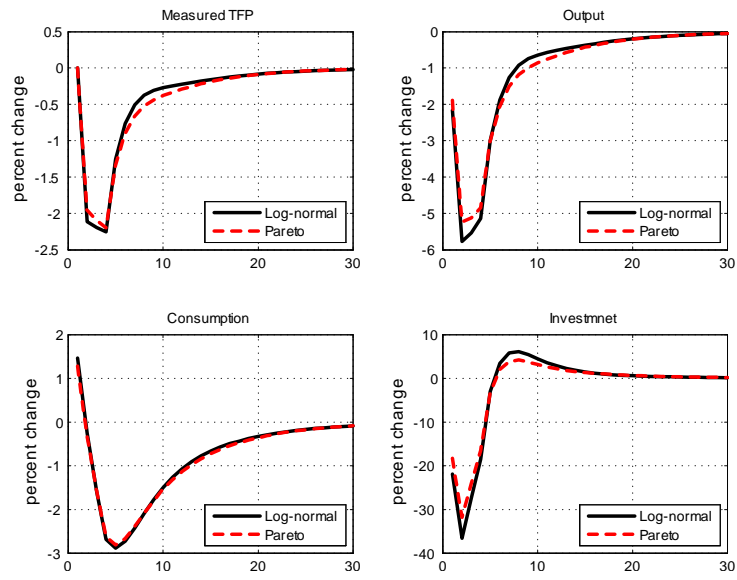


Figure 8

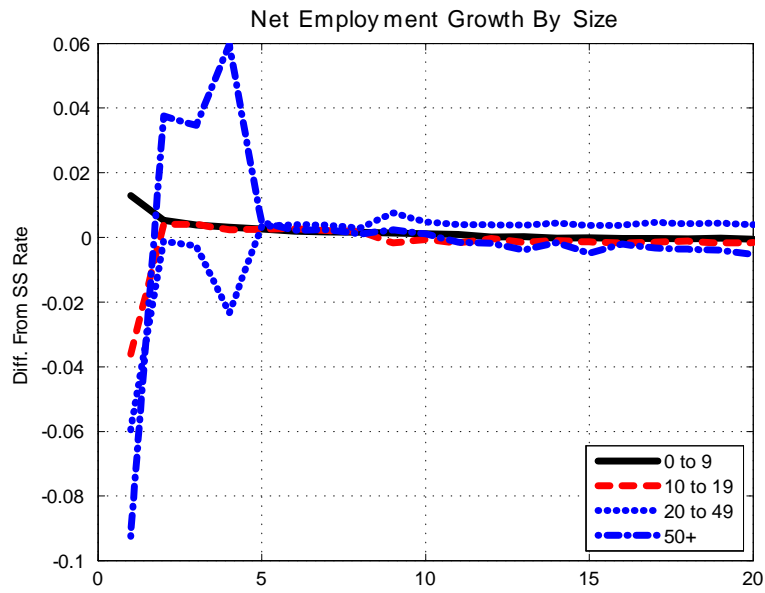


Figure 9

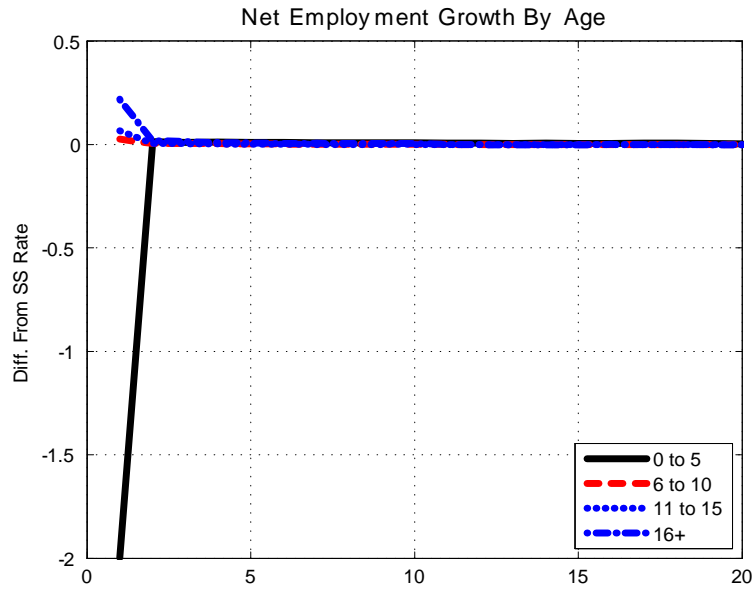


Figure 10



Figure 11

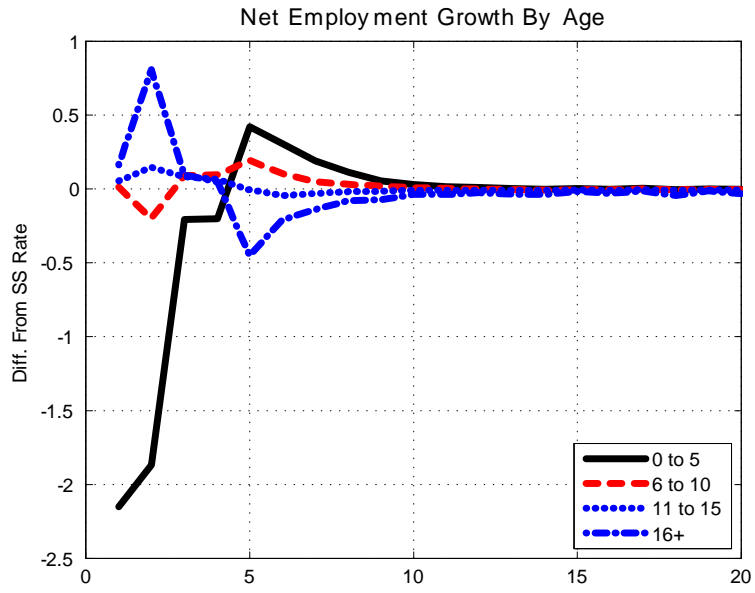


Figure 12

